# Semi-analytical/Semi-numerical modeling of reionization & 21 cm probes

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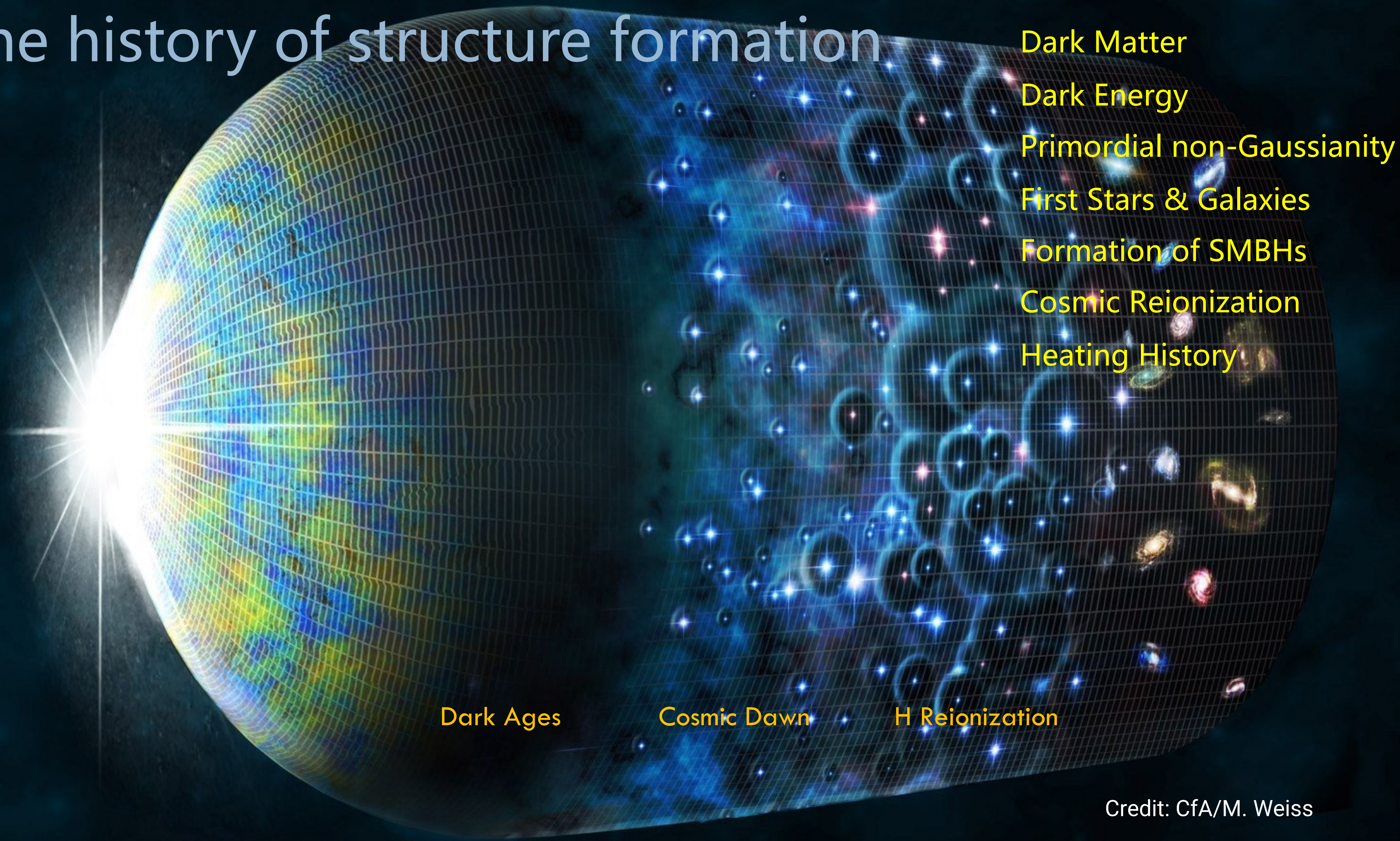
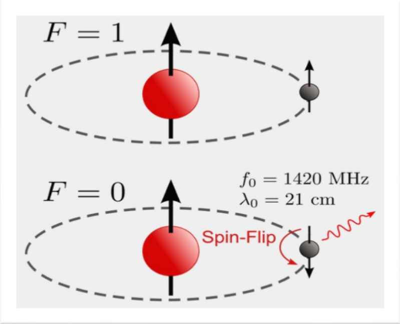
# The history of structure formation

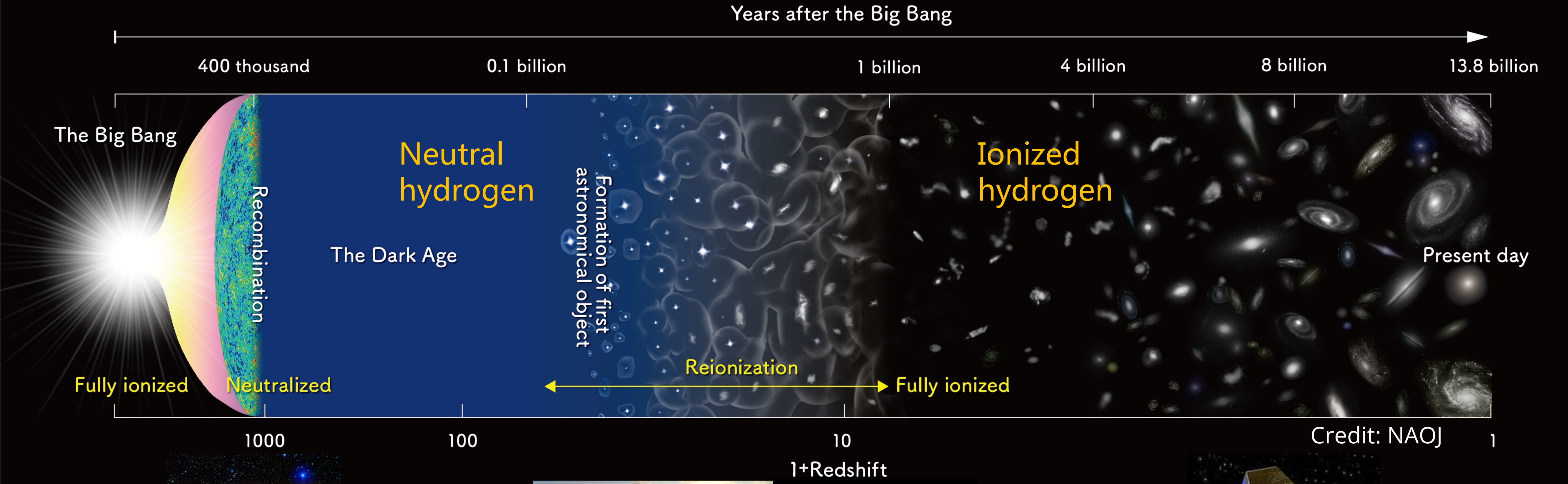
- Dark Matter
- Dark Energy
- Primordial non-Gaussianity
- First Stars & Galaxies
- Formation of SMBHs
- Cosmic Reionization
- Heating History

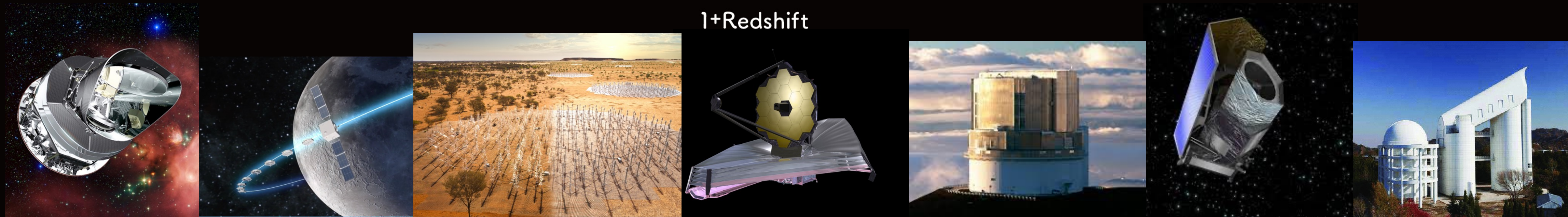



Credit: CfA/M. Weiss

# THE 21CM LINE OF HI:
# EXPLORING THE LAST DESERT IN THE OBSERVATIONAL UNIVERSE



# The 21 cm probes to CD/EoR

- Using CMB as background
  - → **1.** The sky-averaged 21-cm brightness -- the global 21cm spectrum

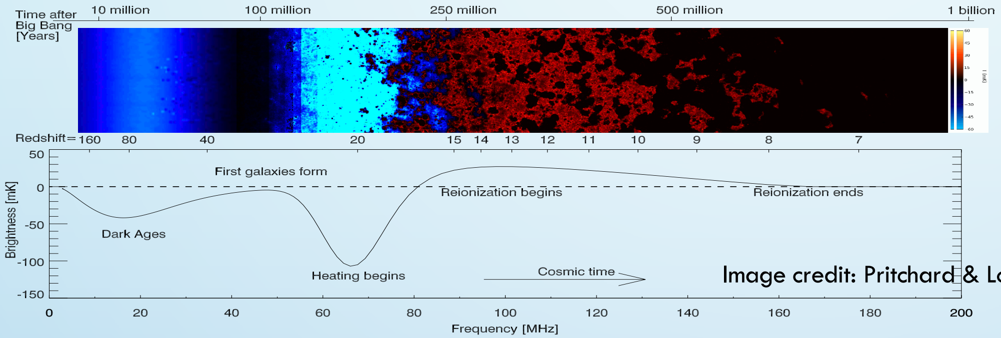
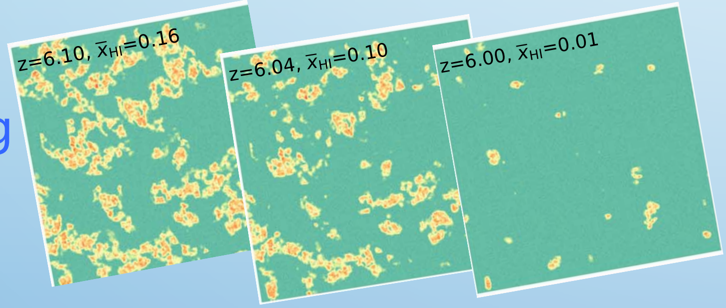


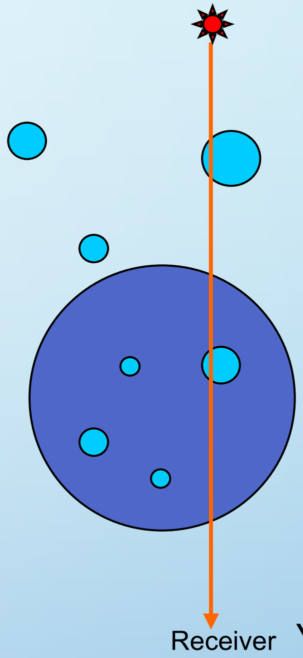

Image credit: Pritchard & Loeb 2012

  - → **2.** 21 cm tomography

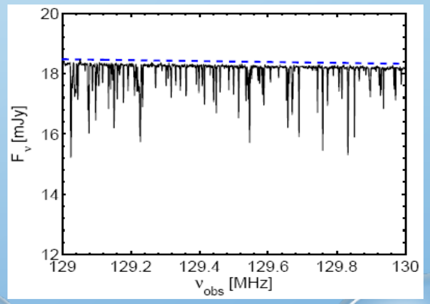
21 cm imaging




→ 21 cm statistics




**YX** et al. 2011 MNRAS

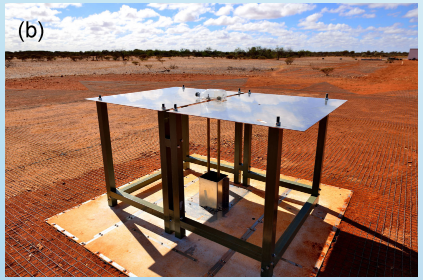


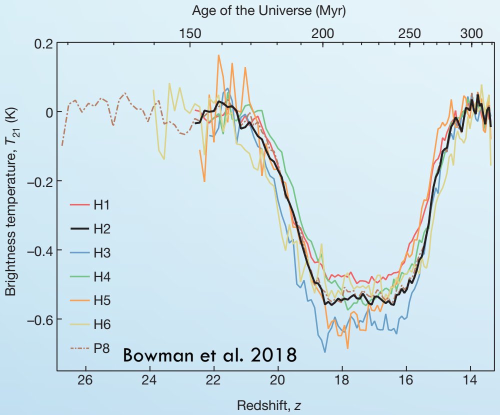

- Using high-z radio point sources as background
  - → **3.** 21 cm forest (absorption lines) (e.g. Carilli et al. 2002; **YX** et al. 2009, 2010, 2011)

# Current upper limits of 21 cm signals from CD/EoR

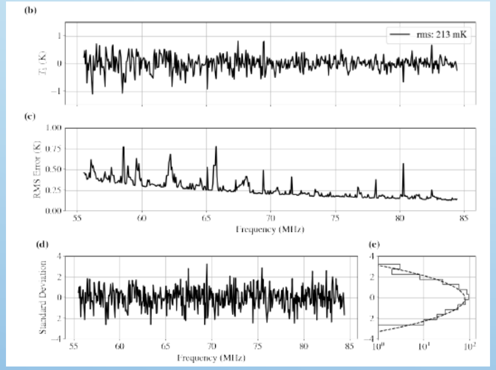
1. 21-cm global spectrum

EDGES-Low-band



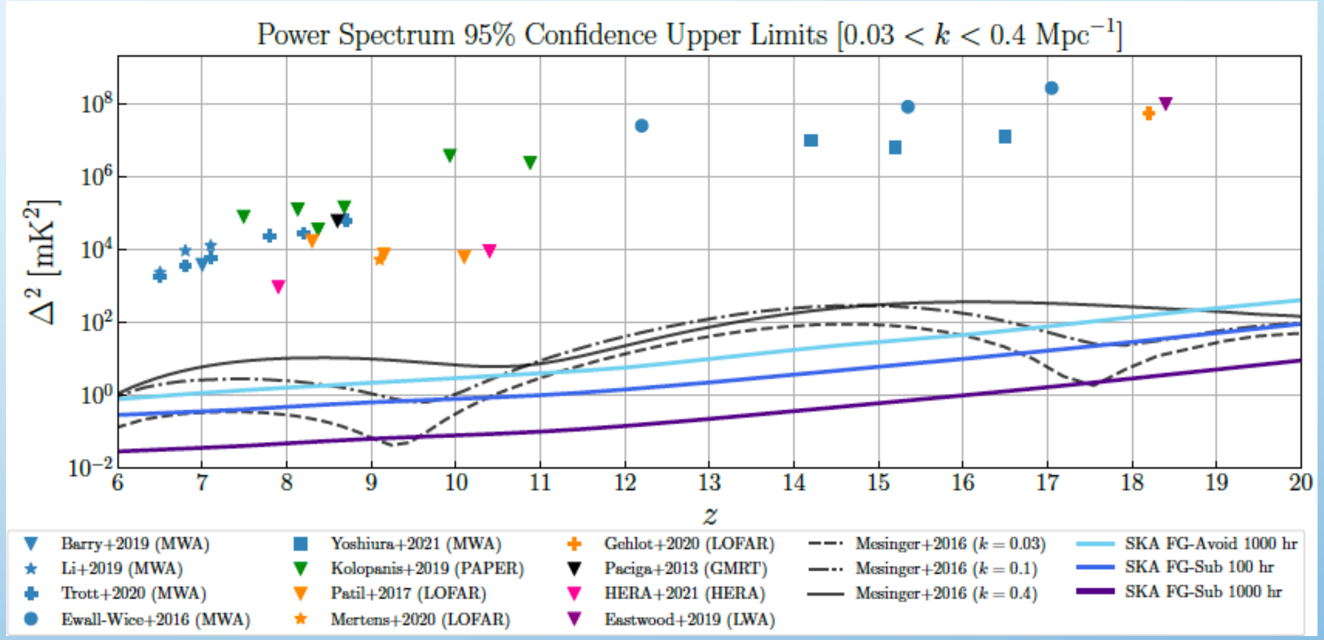




Singh et al. 2112.06778

2. 21-cm tomography


Interferometers




Barry et al. arXiv:2110.06173

# Modeling structures during the CD/EoR

- Analytical modeling
- Semi-numerical simulation/Semi-analytical modeling
- N-body + semi-analytical galaxy evolution
- Numerical simulation

| | |
|---|---|
| Global evolution | 1. Global 21-cm spectrum |
| Large-scale structure | 2. 21-cm tomography |
| Small-scale structure | 3. 21-cm forest |

Modeling the large-scale structures of the IGM during EoR

➔ 21-cm tomography

# Analytical models of reionization

- ***Excursion set models of reionization (ESMR)***

Early stage – the “bubble model” (Furlanetto et al. 2004)

-- growing ionized bubbles

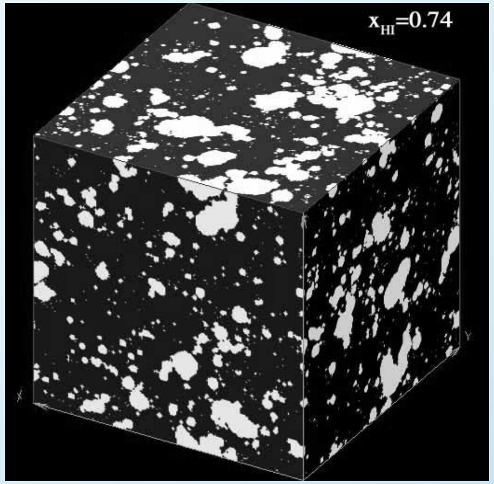
Late stage – the “island model” (Xu et al. 2014)

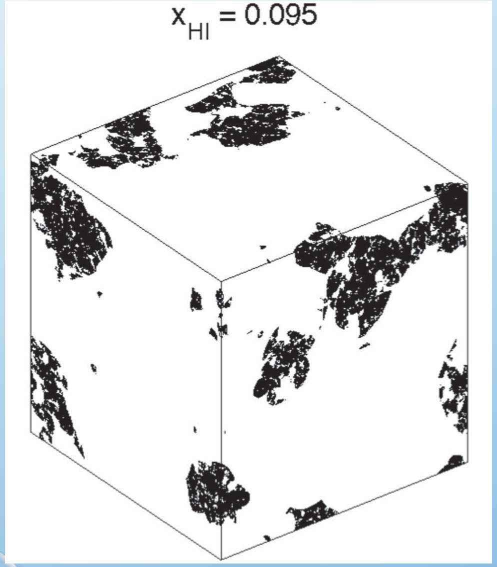
-- shrinking neutral islands

- ***Linear perturbation theory of reionization (LPTR)***

-- Yi Mao’s talk

(Zhang et al. 2006; reformulated in Mao et al. 2015)

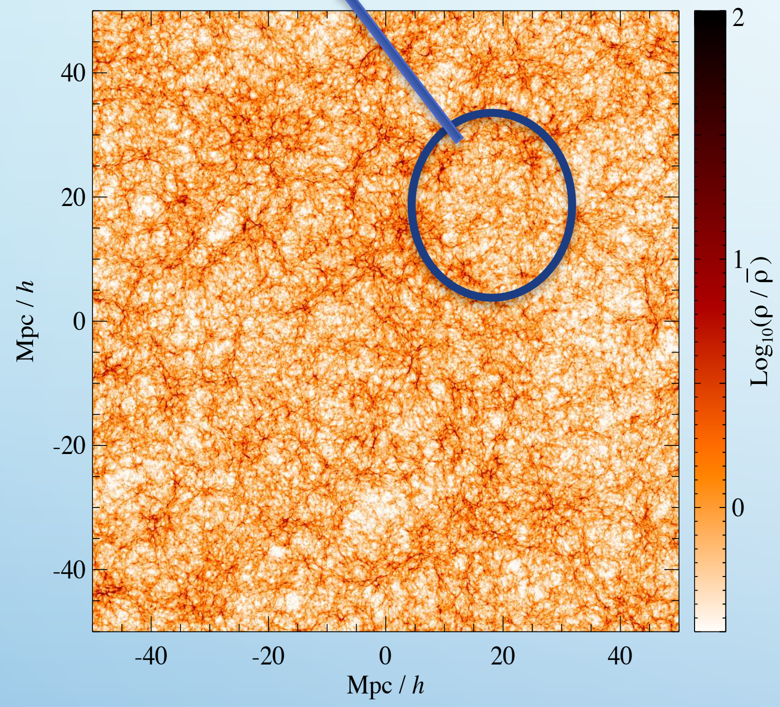


# The Excursion Set Theory of Halo Model

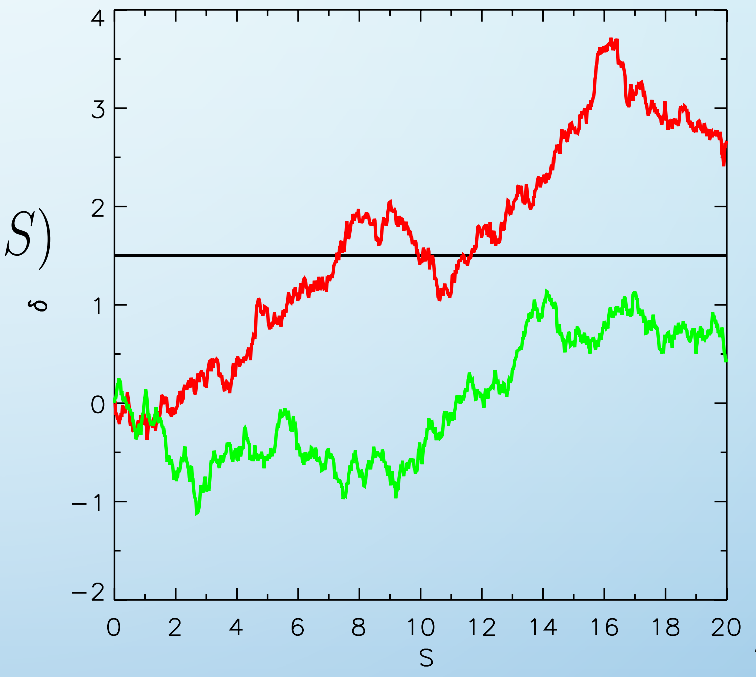
(Bond et al. 1991, Lacey & Cole 1993)

$$\delta(\vec{x}; R) \equiv [\rho(\vec{x}) - \rho_{\rm M}]/\rho_{\rm M}$$




- Halo density barrier

$\delta(S)$



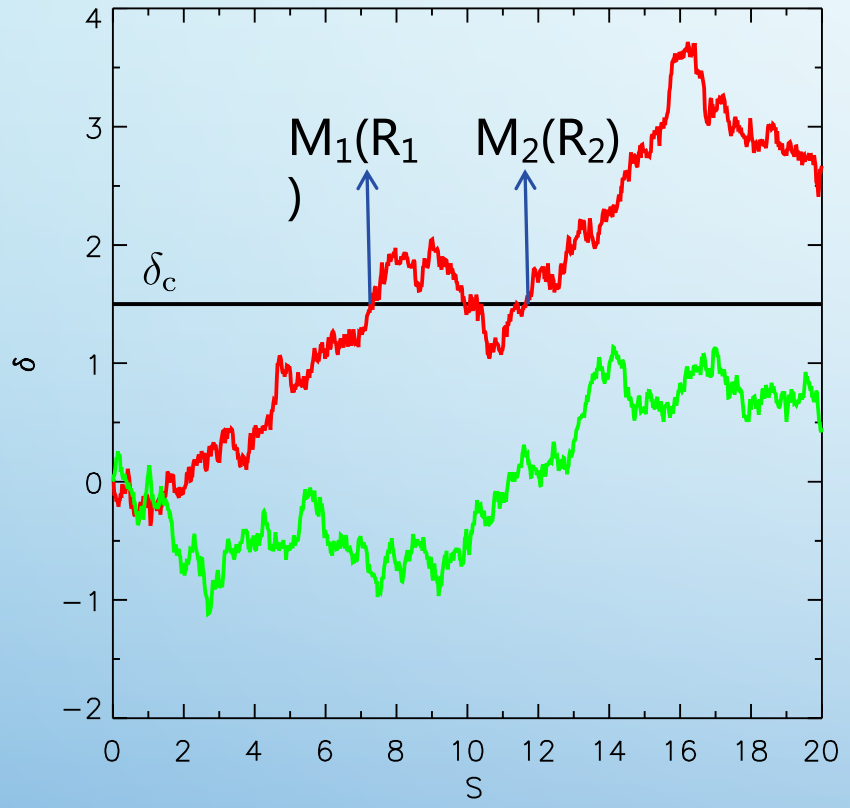

$S \equiv \sigma^2(R)$

- Solving a diffusion equation → *"first-crossing distribution"*

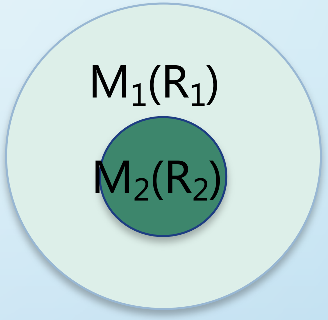
→ halo mass function

# The Excursion Set Theory of Halo Model

(Bond et al. 1991, Lacey & Cole 1993)




- *The cloud-in-cloud problem*




- Solving a diffusion equation

→ *"first-crossing distribution"*

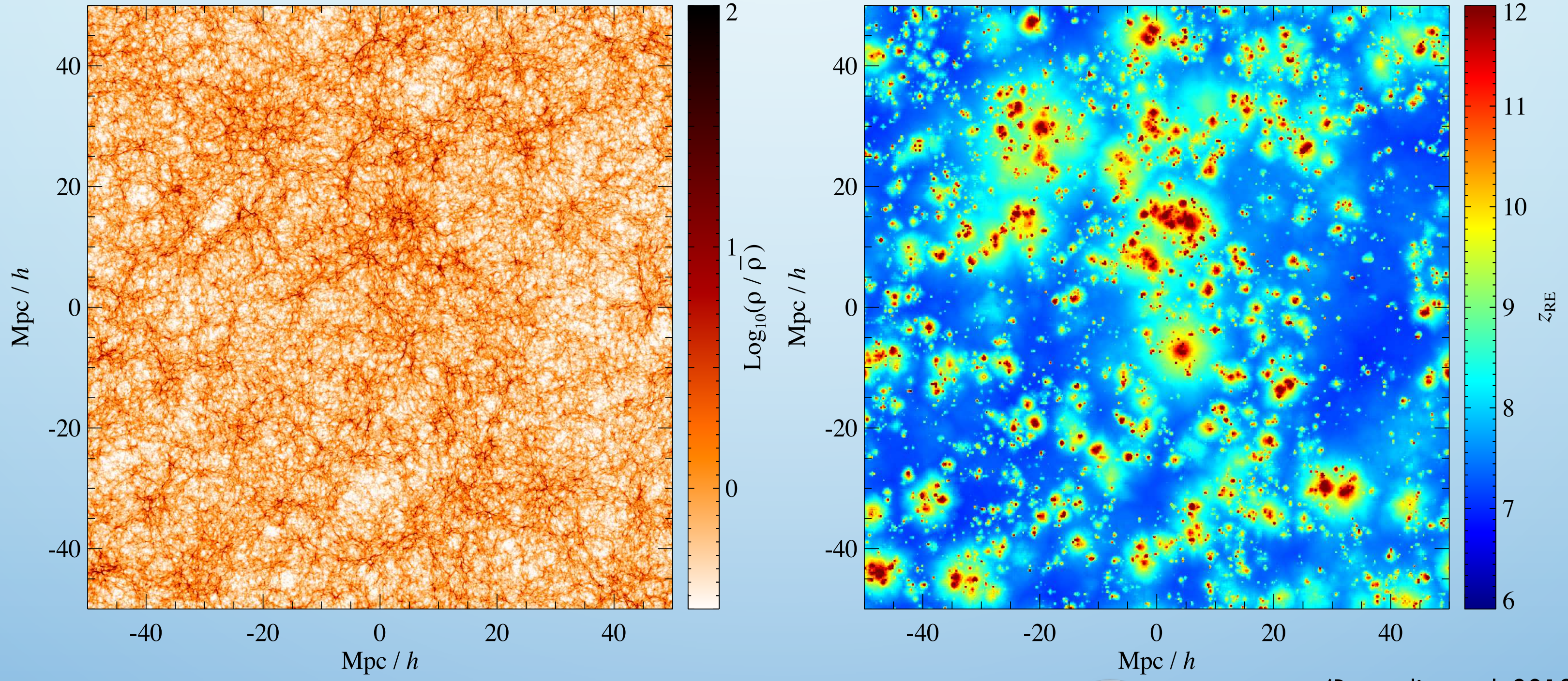
→ halo mass function

➢ The excursion-set model of voids

# Why excursion set theory?

→ ***Full RT-simulations are computationally expensive***

→ ***The reionization field follows the density field on large scales***

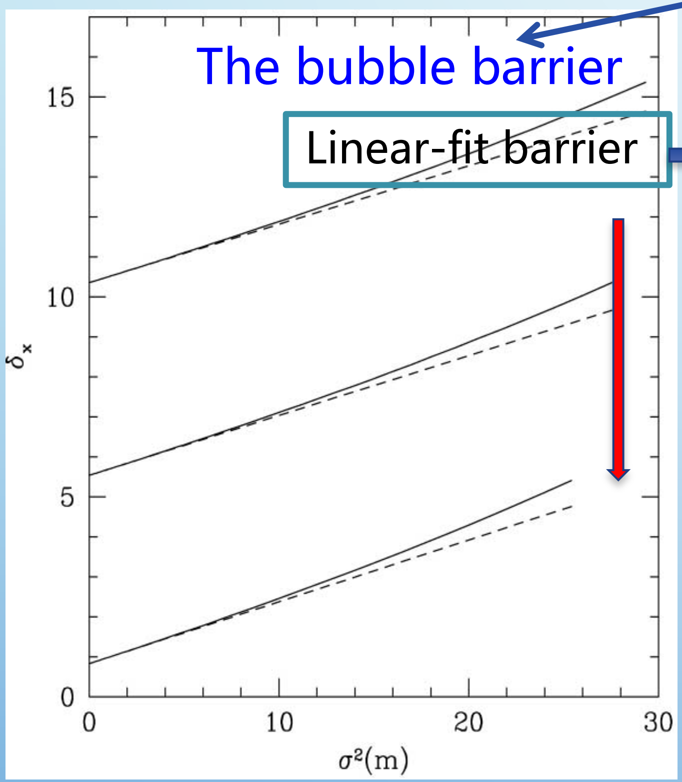



(Battaglia et al. 2013 ApJ, 776, 81)

# The Excursion Set Approach for ionized bubbles – The bubble model of reionization (Furlanetto et al. 2004)

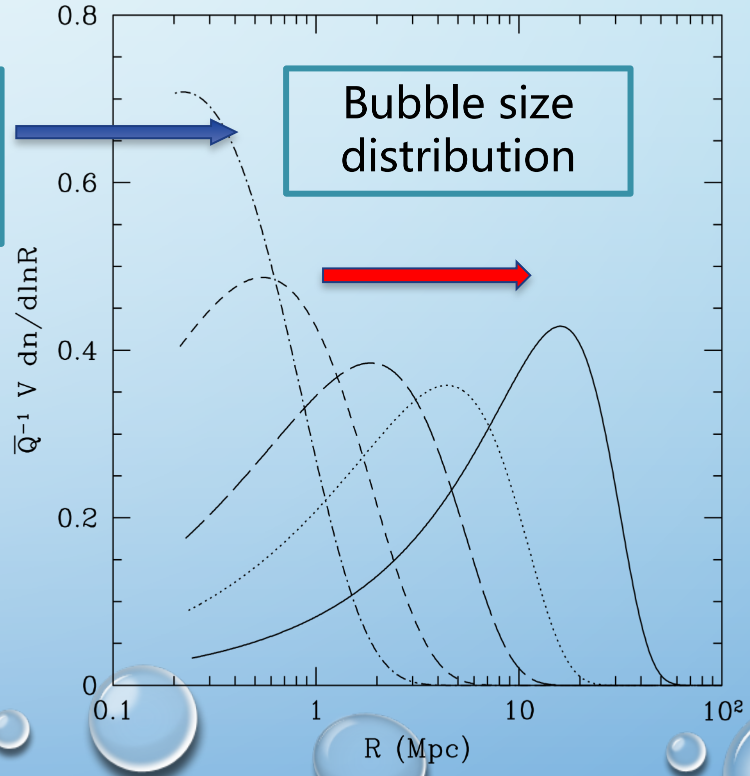
- Relate the ionization field to the initial density field
- Ask whether an isolated region of mass M can be fully self-ionized.

$$f_{\rm coll} \ge f_x \equiv \zeta^{-1}. \longrightarrow \boxed{\delta_m \ge \delta_x(m,z) \equiv \delta_c(z) - \sqrt{2}K(\zeta)[\sigma^2_{\rm min} - \sigma^2(m)]^{1/2}} \quad K(\zeta) = \mathrm{erf}^{-1}(1-\zeta^{-1}).$$




First-up-crossing distribution (analytical)

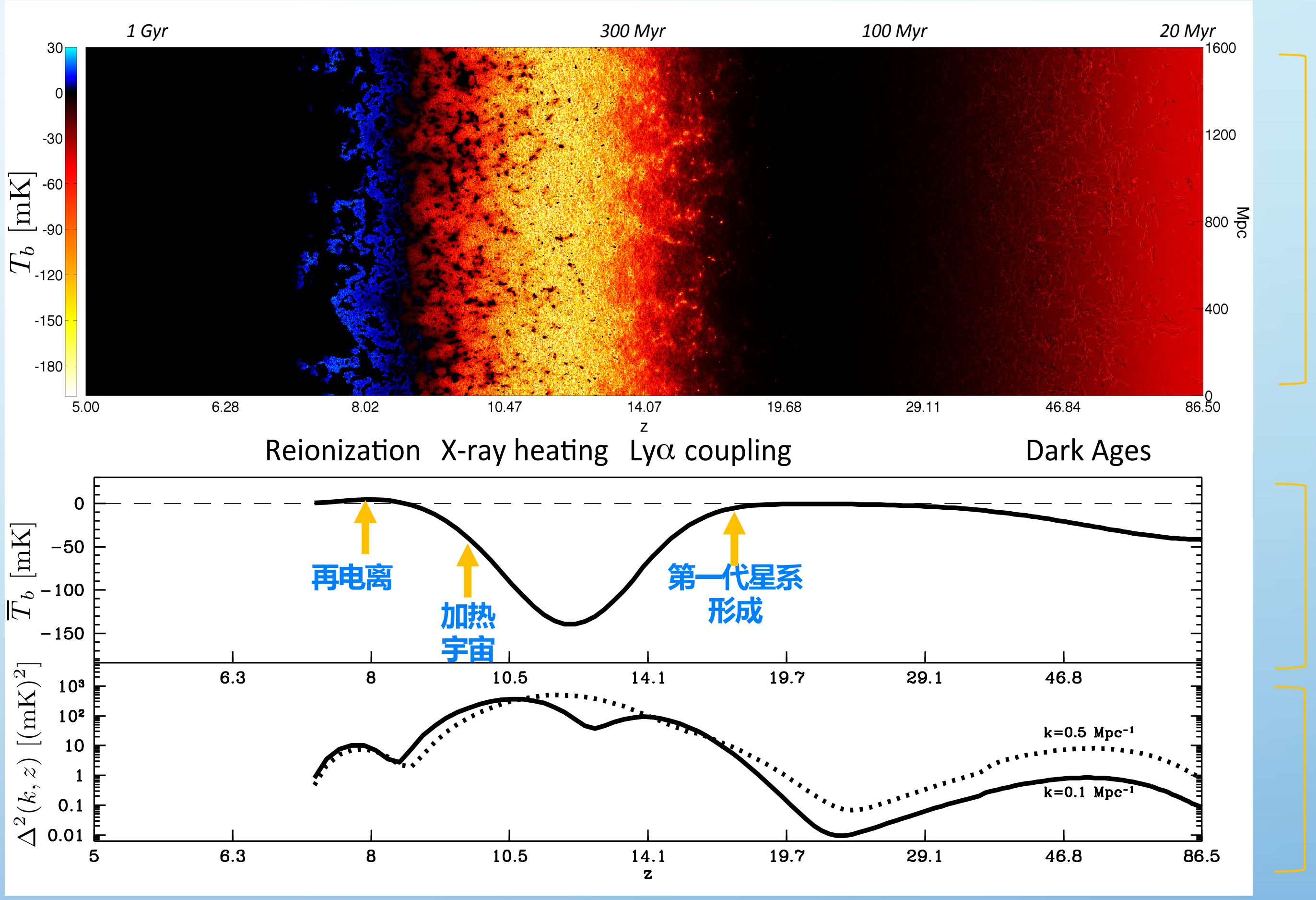
*Bubble-in-bubble problem*



# Semi-numerical simulations based on the bubble model

FFT → filter on R → iFFT → x_HI (**x**;R)

- 21cmFAST (Mesinger et al.)
- simFAST21 (Santos et al.)
- Fialkov & Barkana
- photon-conserving SCRIPT (Choudhury et al.)




Credit: 21cmFAST team

# However, after percolation…

1. The isolated and spherical assumption for the ionized bubbles breaks down
   → the neutral islands are more isolated
2. The existence of an ionizing background
   → the shape of barriers could be changed
   (the linear fit may not apply)

**The island model**

It would be relatively easier for the upcoming instruments to probe the signal at the late reionization stages.

# The Island Model

(Xu et al. 2014)

- Negative island barrier ( "inside-out" reionization)
- Island mass scales are identified by ***first-down-crossings*** through the island barrier (but not the "never-up-crossing" distribution).

With the inclusion of an ionizing background, the condition of keeping from being ionized:

$$\xi f_{\rm coll}(\delta_{\rm M}; M, z) + \boxed{\frac{\Omega_m}{\Omega_b} \frac{N_{\rm back} m_{\rm H}}{M X_{\rm H}(1+\bar{n}_{\rm rec})}} < 1,$$

The contribution of background ionizing photons

➔ The island barrier:

$$\delta_{\rm M} < \delta_{\rm I}(M,z) \equiv \delta_c(z) - \sqrt{2[S_{\rm max} - S(M)]}\,{\rm erfc}^{-1}\left[K(M,z)\right],$$

$$K(M,z) = \xi^{-1}\left[1 - \boxed{N_{\rm back}}(1+\bar{n}_{\rm rec})^{-1}\frac{m_{\rm H}}{M(\Omega_b/\Omega_m)X_{\rm H}}\right].$$

# The ionizing background

* Considering the effect of ***Lyman limit systems*** on the mean free path of ionizing photons, the comoving number density of background ionizing photons is

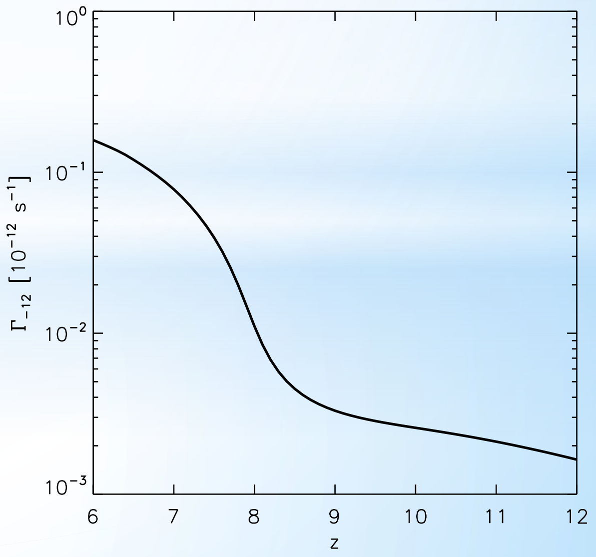
$$n_\gamma(z) = \int_z \bar{n}_{\rm H} \left| \frac{{\rm d}f_{\rm coll}(z')}{{\rm d}z'} \right| f_\star N_{\gamma/{\rm H}} f_{\rm esc} \exp\left[ -\frac{l(z,z')}{\lambda_{\rm mfp}(z)} \right] {\rm d}z',$$

* With the MHR00 model for the volume-weighted density distribution of the IGM (Miralda-Escude et al. 2000),

$$P_{\rm V}(\Delta)\, d\Delta = A_0 \exp\left[ -\frac{(\Delta^{-2/3} - C_0)^2}{2\,(2\delta_0/3)^2} \right] \Delta^{-\beta}\, d\Delta$$

the mean free path of ionizing photons can be written as

$$\lambda_{\rm mfp} = \frac{\lambda_0}{[1 - F_{\rm V}(\Delta_{\rm crit})]^{2/3}},$$



* Solving for the first-down-crossing distribution (Zhang & Hui 2006):

(the "island-in-island" problem is naturally solved)

$$f_{\rm I}(S_{\rm I}) = -g_1(S_{\rm I}) - \int_0^{S_{\rm I}} {\rm d}S' f_{\rm I}(S') \left[ g_2(S_{\rm I}, S') \right],$$

$$g_1(S_{\rm I}) = \left[ \frac{\delta_{\rm I}(S_{\rm I})}{S_{\rm I}} - 2\frac{{\rm d}\delta_{\rm I}}{{\rm d}S_{\rm I}} \right] P_0[\delta_{\rm I}(S_{\rm I}), S_{\rm I}], \quad P_0(\delta, S) = \frac{1}{\sqrt{2\pi S}} \exp\left( -\frac{\delta^2}{2S} \right)$$

$$g_2(S_{\rm I}, S') = \left[ 2\frac{{\rm d}\delta_{\rm I}}{{\rm d}S_{\rm I}} - \frac{\delta_{\rm I}(S_{\rm I}) - \delta_{\rm I}(S')}{S_{\rm I} - S'} \right] P_0[\delta_{\rm I}(S_{\rm I}) - \delta_{\rm I}(S'), S_{\rm I} - S'],$$

* The mass function of islands:

$$\frac{{\rm d}n}{{\rm d}\ln M_{\rm I}}(M_{\rm I}, z) = \bar{\rho}_{\rm m,0} f_{\rm I}(S_{\rm I}, z) \left| \frac{{\rm d}S_{\rm I}}{{\rm d}M_{\rm I}} \right|.$$

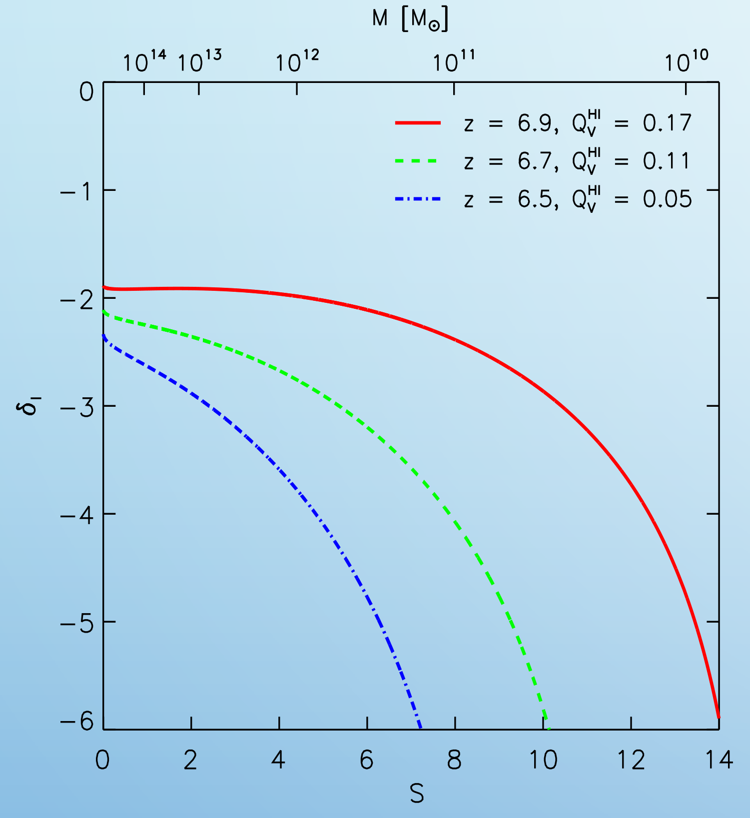
* The volume fraction of neutral regions:

$$\mathbf{Q}_{\rm V}^{\rm I} = \int {\rm d}M_{\rm I} \frac{{\rm d}n}{{\rm d}M_{\rm I}} V(M_{\rm I}).$$

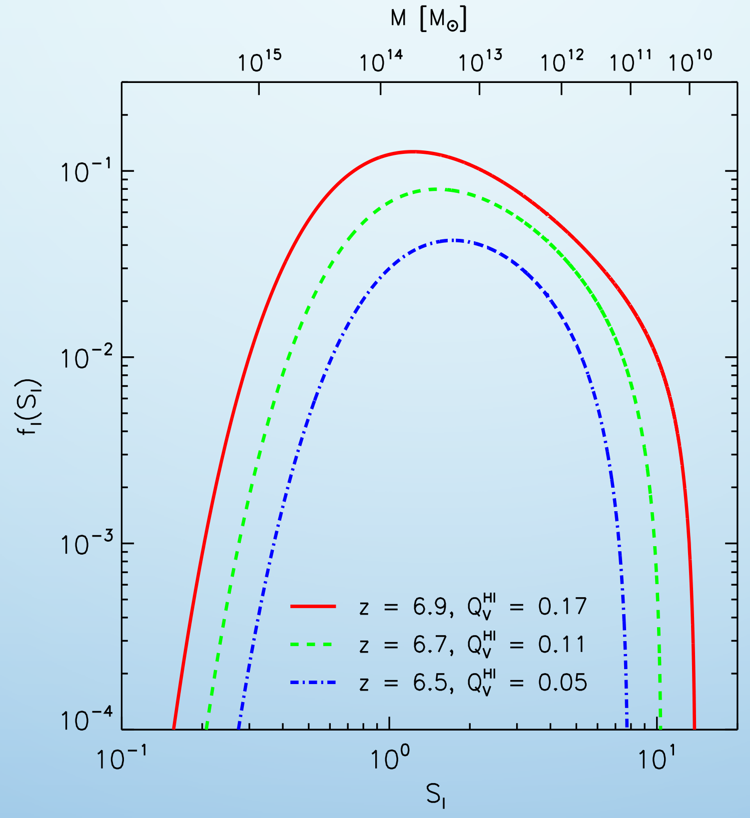
(Xu et al. 2014)

# The island-vS model – varying surface area

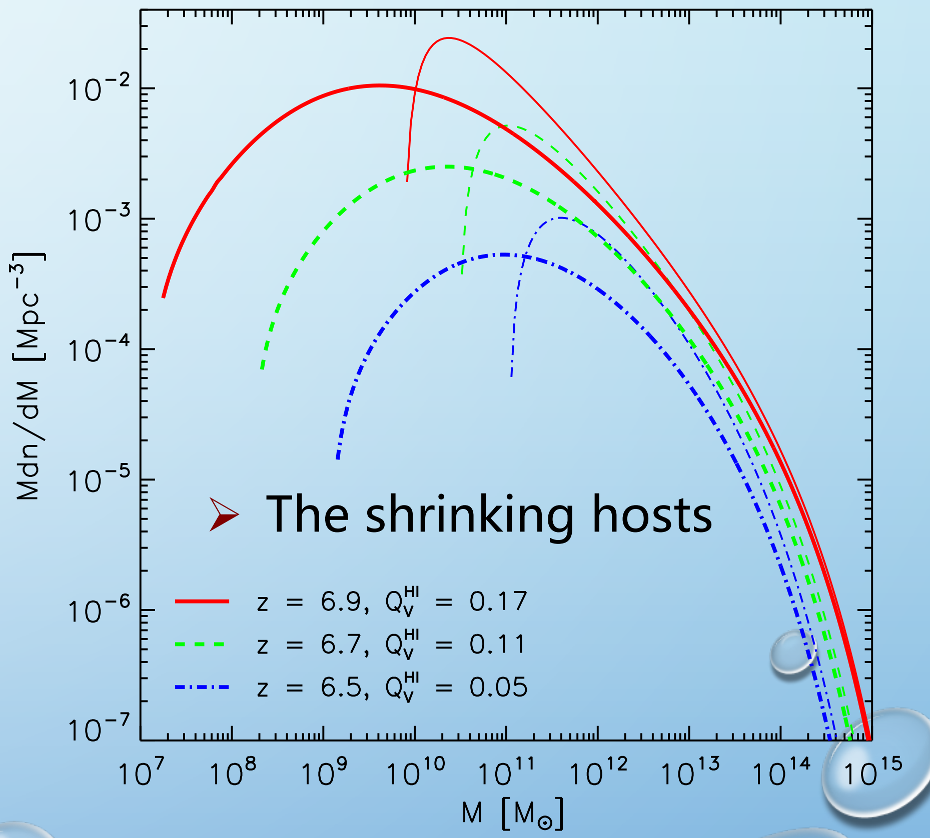
## island barrier




## first down-crossing distribution




## host island mass function




$$\frac{M_f}{M_i} = (1 - \frac{\Delta R}{R_i})^3$$

# The bubbles-in-island effect

Solving for a two-barrier problem:

1 - The first down-crossing distribution of random walks w.r.t. ***island barrier***.

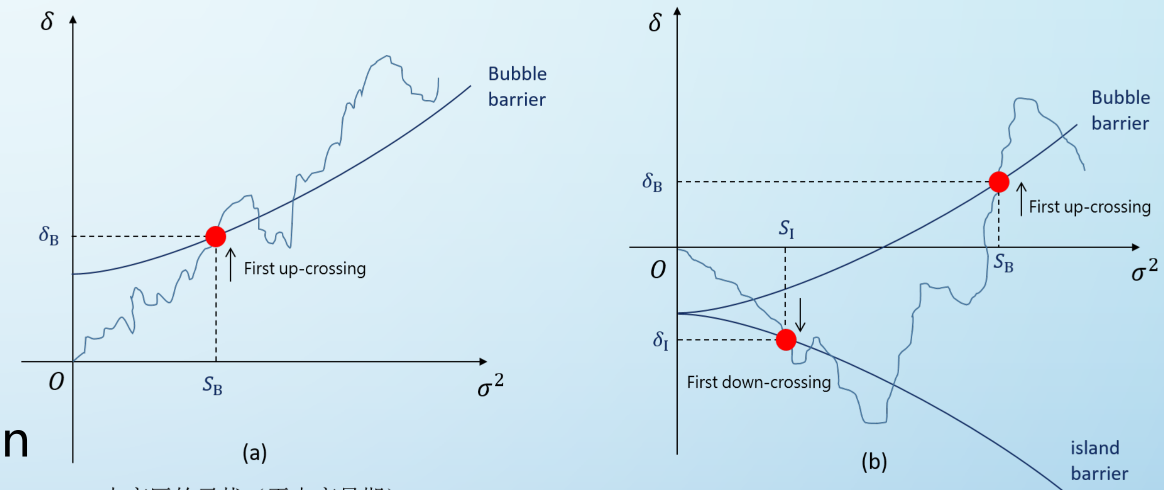
$$f_{\mathrm{I}}(S_{\mathrm{I}}, z)$$

2 - The conditional first up-crossing distribution w.r.t. ***bubble barrier***.

$$f_{\mathrm{B}}[S_{\mathrm{B}}, \delta_{\mathrm{B}} | S_{\mathrm{I}}, \delta_{\mathrm{I}}]$$

The effective bubble barrier:

$$\delta'_{\mathrm{B}} = \delta_{\mathrm{B}}(S + S_{\mathrm{I}}) - \delta_{\mathrm{I}}(S_{\mathrm{I}}) \qquad \text{where } S = S_{\mathrm{B}} - S_{\mathrm{I}}.$$




电离区的寻找（再电离早期）

中性区及其内部的电离区寻找（再电离晚期）

(Xu et al. 2014)

# The bubbles-in-island effect

* The bubbles-in-island fraction:

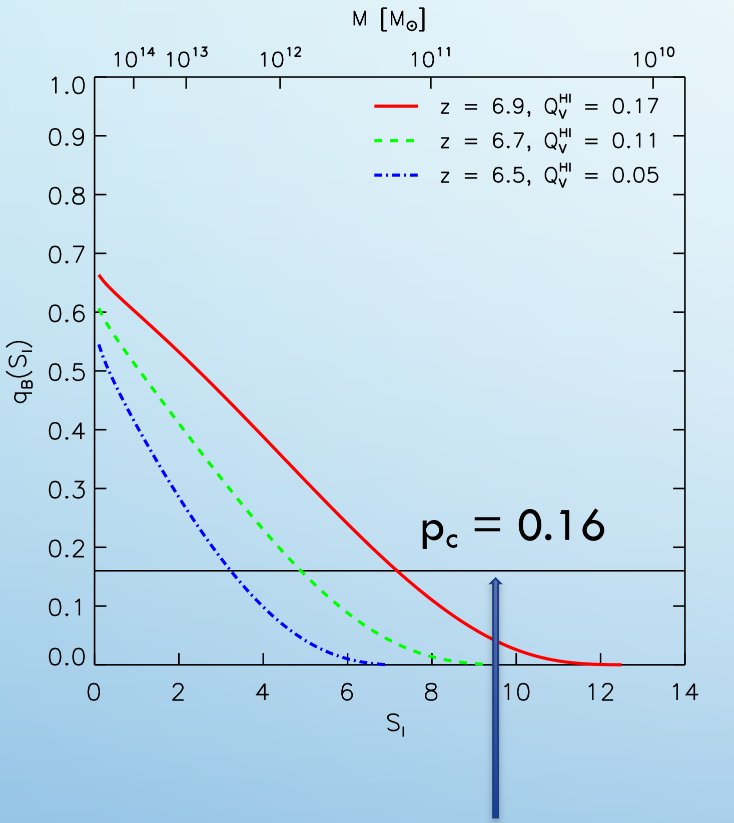
$$q_{\rm B}(S_{\rm I},\delta_{\rm I};z)=\int_{S_{\rm I}}^{S_{\rm max}(\xi\cdot M_{\rm min})}[1+\delta_{\rm I}\,D(z)]\;\;f_{\rm B}[S_{\rm B},\delta_{\rm B}|S_{\rm I},\delta_{\rm I}]\;{\rm d}S_{\rm B}.$$

* The neutral island mass function:

$$\frac{dn}{dM}(M,z)=\frac{dn}{dM_{\rm I}}\frac{dM_{\rm I}}{dM}=\frac{\bar{\rho}_{\rm m,0}}{M_{\rm I}}f_{\rm I}(S_{\rm I},z)\left|\frac{dS_{\rm I}}{dM_{\rm I}}\right|\frac{dM_{\rm I}}{dM}.$$

$$M\;=\;M_{\rm I}(S_{\rm I})\,[1-q_{\rm B}(S_{\rm I},\delta_{\rm I};z)]$$

# The problem of large bubbles-in-island fraction




for Gaussian random fields

Host islands → overestimate the neutral fraction

Neutral islands → not the real image

Difficult to visually identify the host islands

Break down of bubble model inside islands

# The role of percolation threshold $p_c$

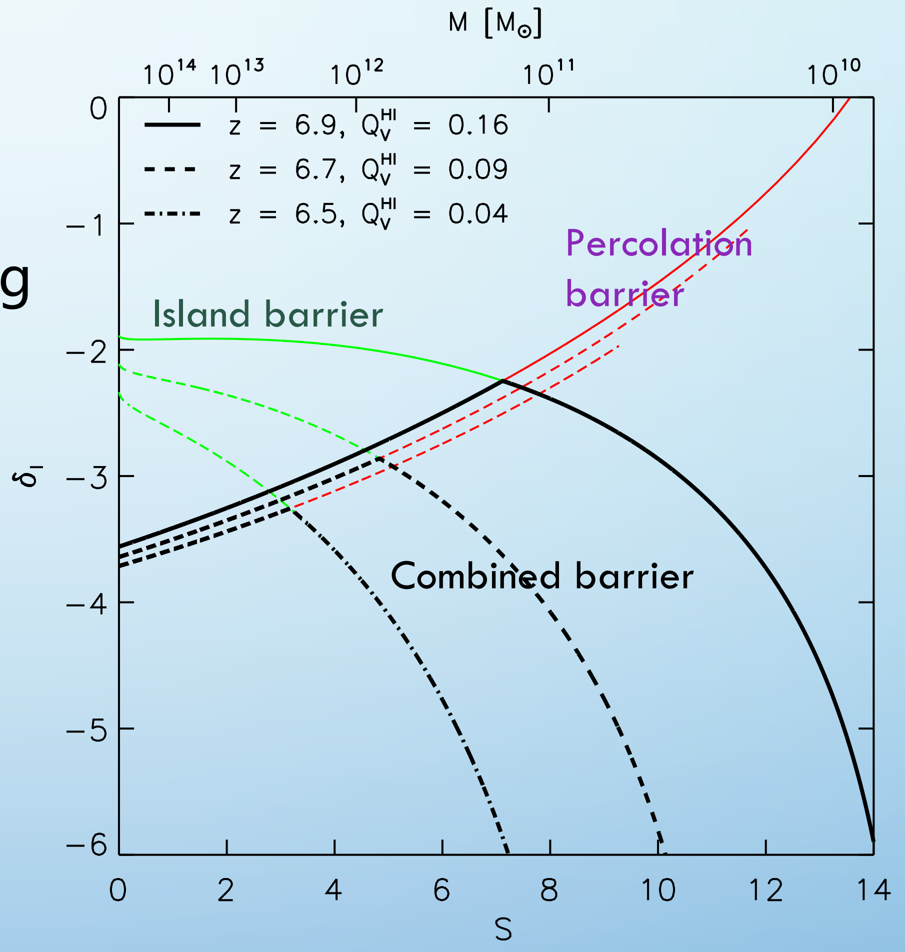
- The bubble model regime: $z > z_{Bp}$ ($x_{HII} < p_c$)
- The island model regime: $z < z_{Ip}$ ($x_{HI} < p_c$)
- The background onset redshift: $z_{Bp} > z_{back} > z_{Ip}$
- The definition of bona fide neutral islands: $q_B < p_c$

# The percolation criterion

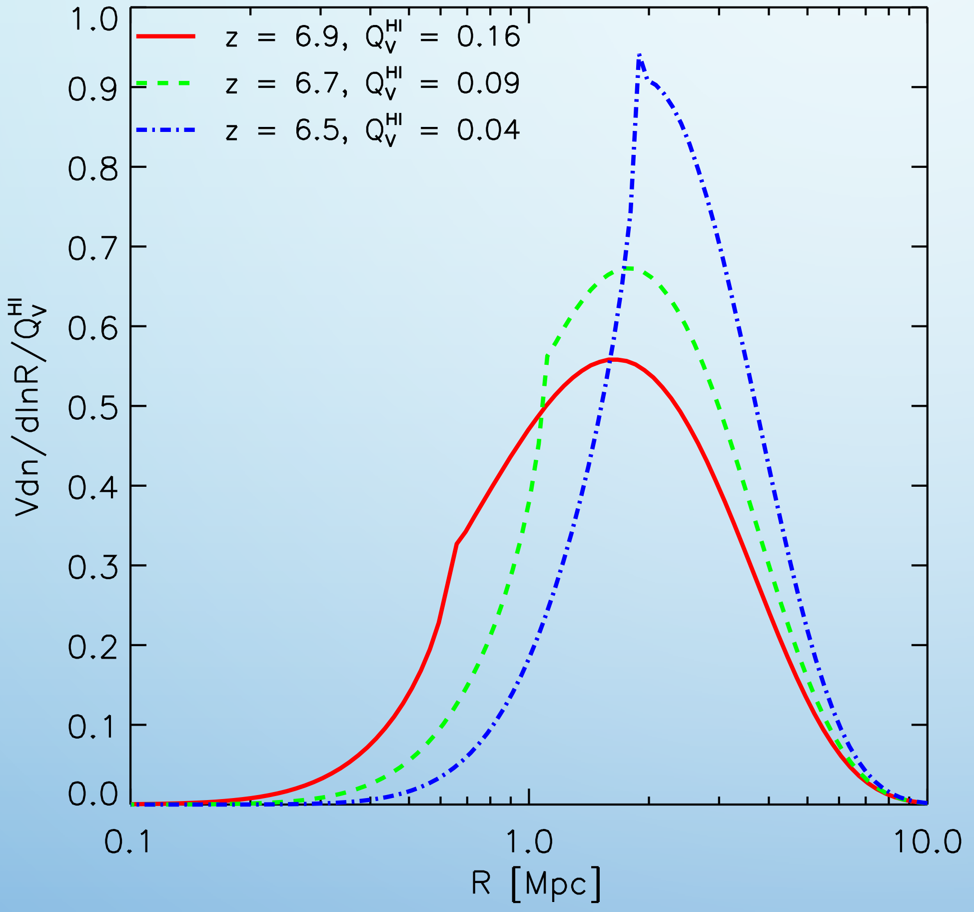
The additional barrier is obtained by solving

$q_B(S_I, \delta_I; z) < p_c$

$p_c = 0.16$ for Gaussian random fields



# Results – the size distribution with $p_c$ cutoff




***A characteristic scale that does not change much with redshift!***

(Xu et al. 2014)

# Semi-numerical simulation – islandFAST

(Xu et al. 2017)

Initial ionization field at z >~ $z_{\rm back}$ generated by the 21cmFAST

A ***two-step*** filtering algorithm

1 – Based on the excursion set theory, we filter the evolved density field and ***find host islands*** with the island barrier including an ionizing background.

2 – ***Find bubbles in islands*** with the bubble barrier without an ionizing background.

A self-consistent treatment for the ionizing background taking into account the effect of absorption systems

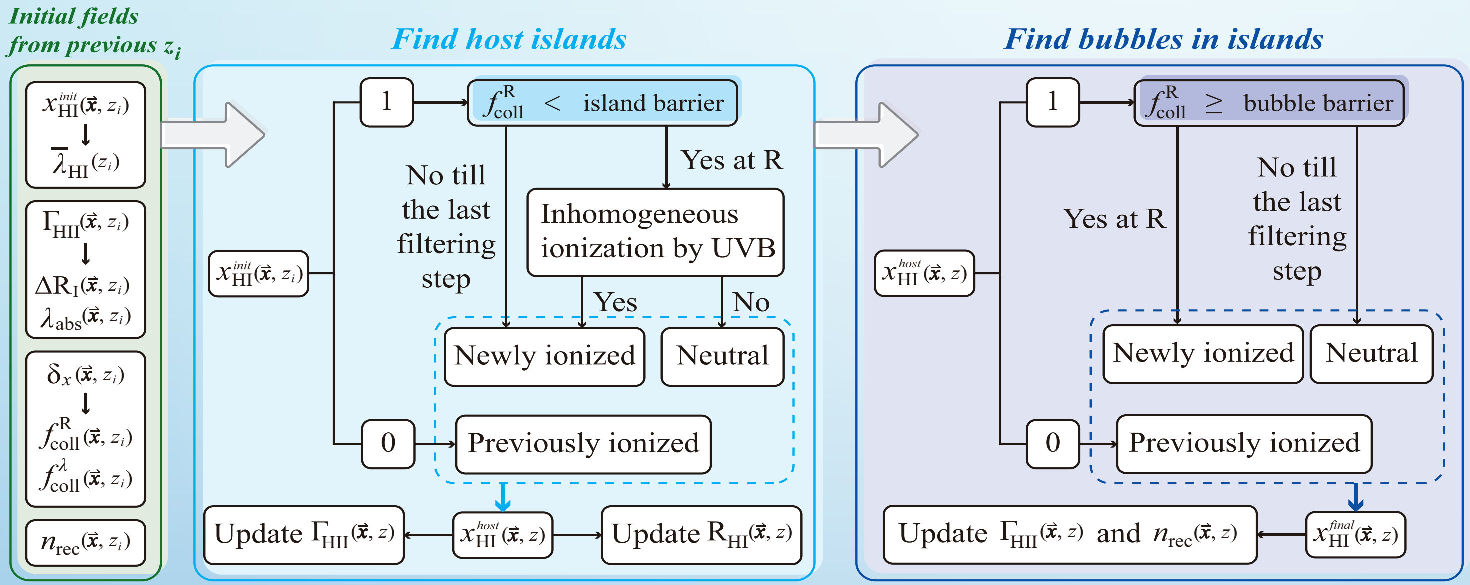
$$n_\gamma(z) = \int_z dz',\, \bar{n}_{\rm H} \left| \frac{df_{\rm coll}(z')}{dz'} \right| f_\star N_{\gamma/H} f_{\rm esc} \times \exp\left[ -\frac{l(z,z')}{\lambda_{\rm mfp}(z)} \right] (1 - f_{\rm H\,I}^{\rm host}),$$

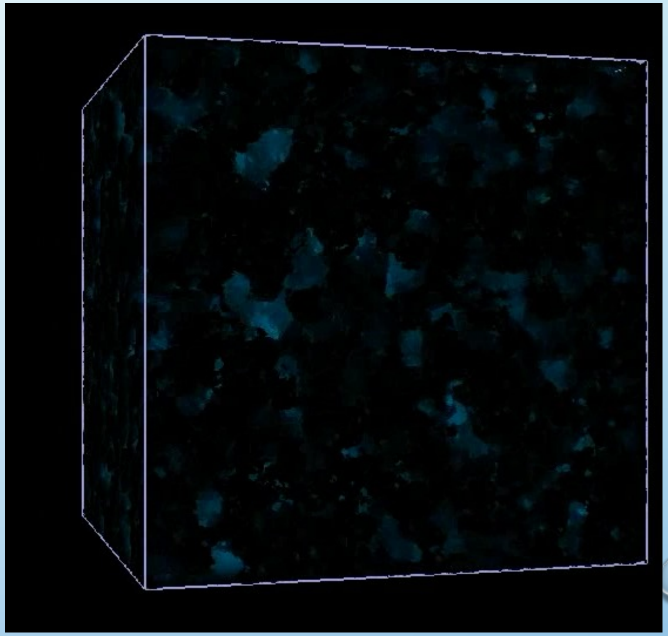
$$\lambda_{\rm mfp}^{-1}(z) = \lambda_{\rm I}^{-1}(z) + \lambda_{\rm abs}^{-1}(z).$$

An ***iterative*** procedure and ***adaptive*** redshift steps.

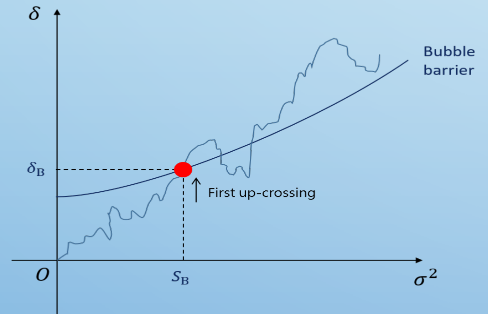
# Semi-numerical simulation – islandFAST

(Xu et al. 2017)



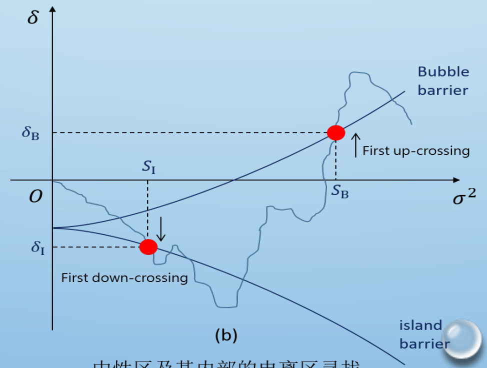



Credit: Xu et al. (NAOC) & Yang Gao (CNIC)




(a)

电离区的寻找（再电离早期）



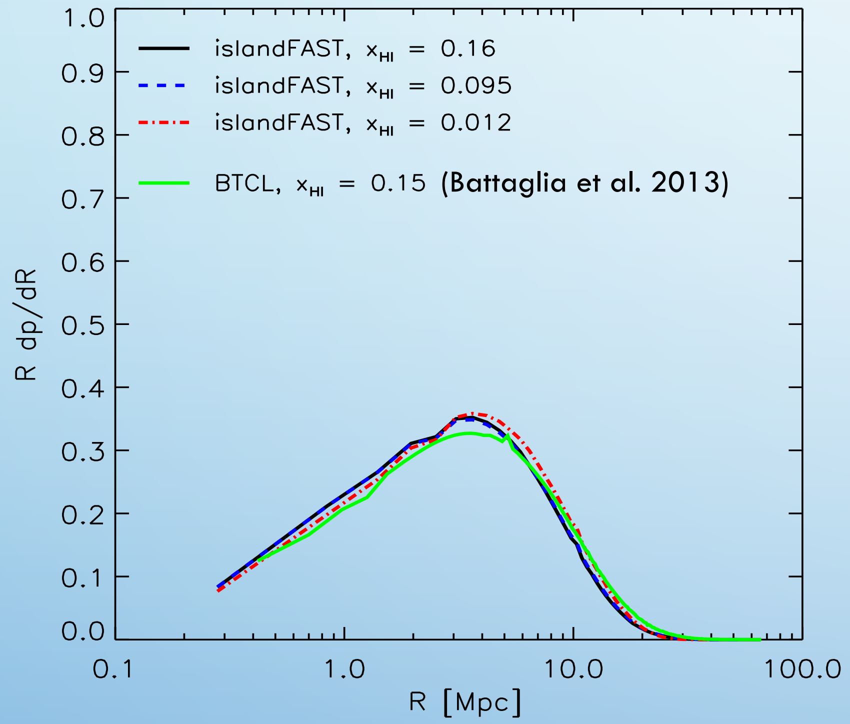

(b)

中性区及其内部的电离区寻找
（再电离晚期）

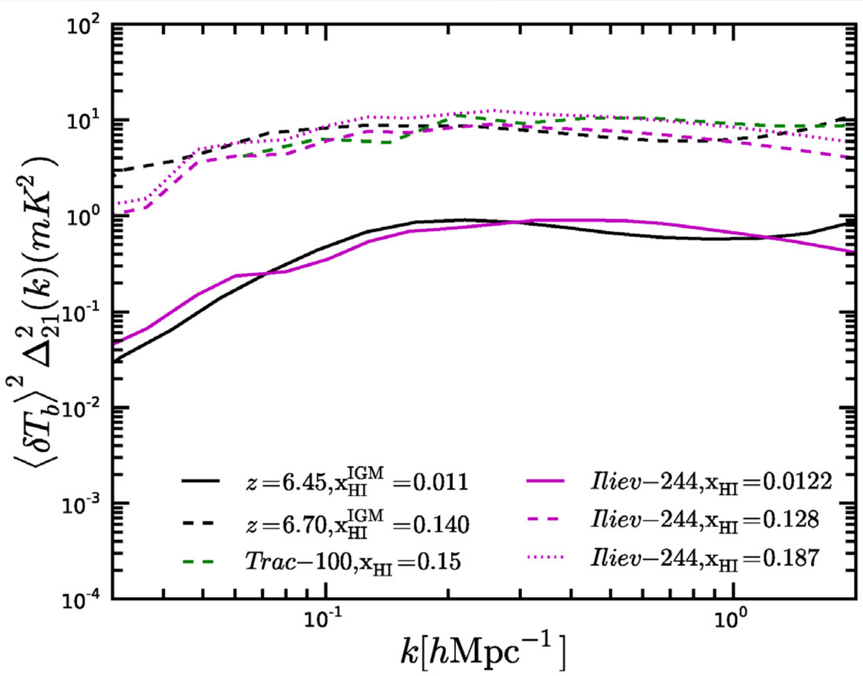
Zhu, **YX** et al. 2023, RAA

# islandFAST vs. radiative transfer simulations

- The size distribution of islands

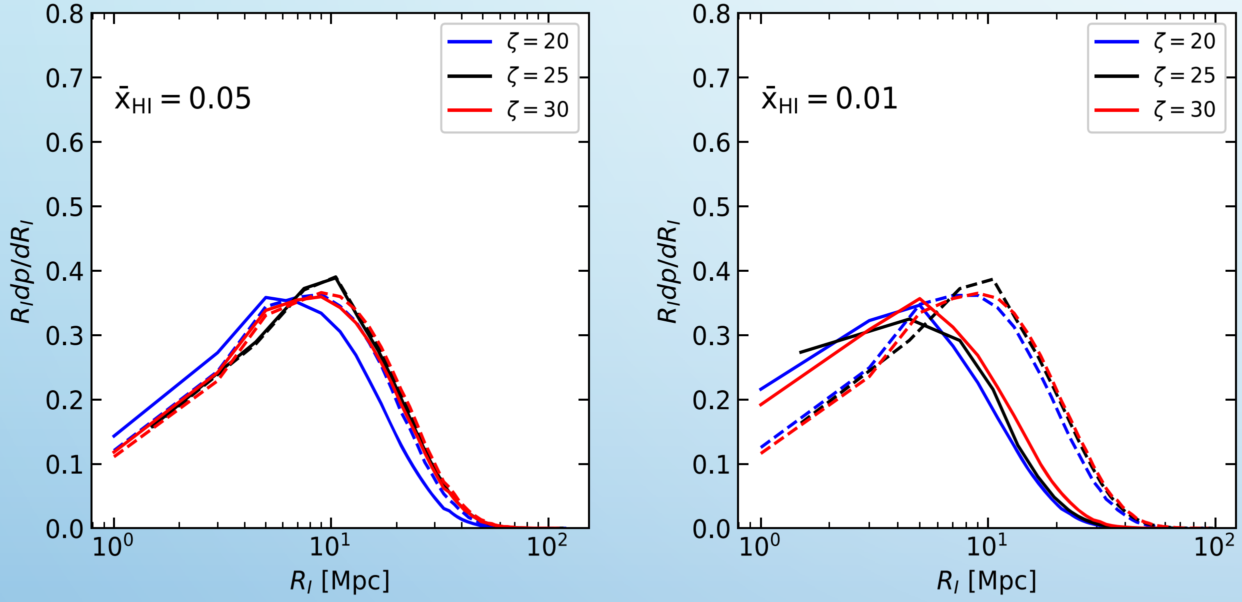



- THE 21 CM POWER SPECTRUM

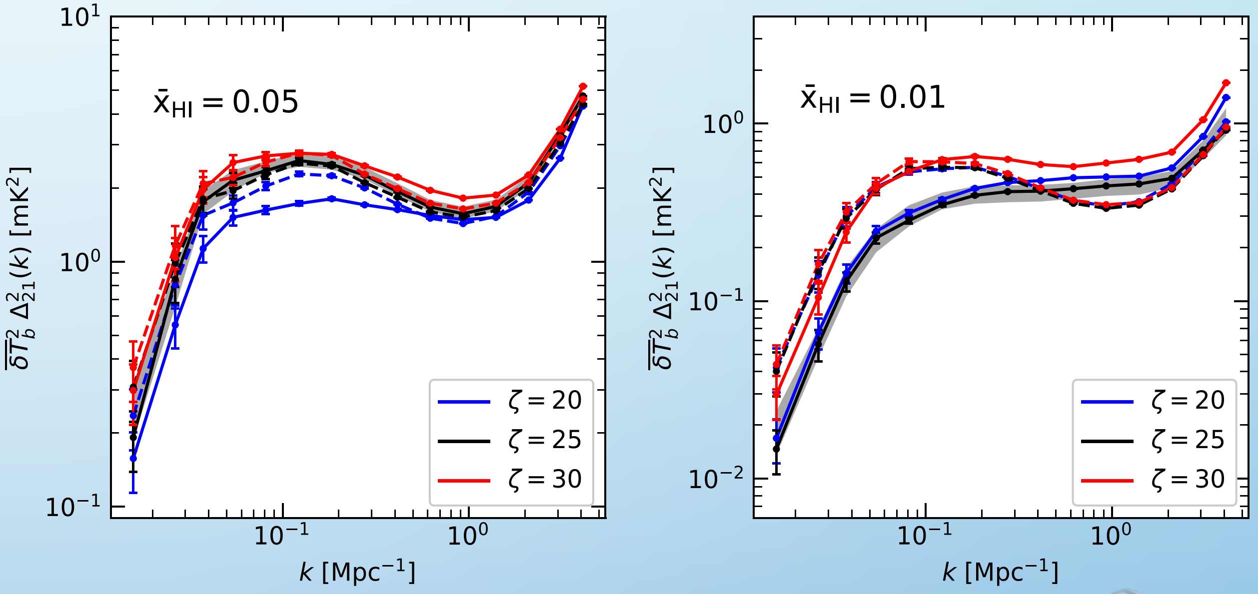



Xu, **YX** et al. 2019 MNRAS

# islandFAST vs. 21cmFAST




Island size distribution

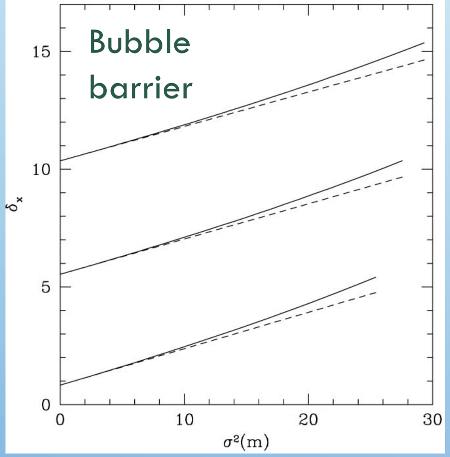



21 cm power spectrum

# BUBBLE MODEL VS. ISLAND MODEL

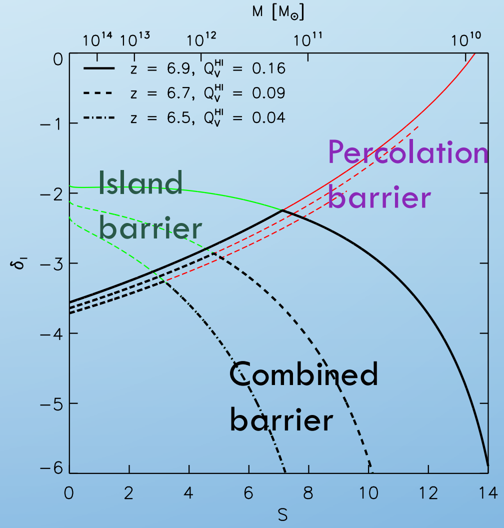
## BEFORE PERCOLATION

- Early EoR
- Growing ionized bubbles
- No UVB in model
- First-up-crossing distribution
- Linear-fitted barrier with analytical solution




## AFTER PERCOLATION

- Late EoR
- Shrinking neutral islands
- With UVB
- First-down-crossing distribution
- Arbitrary shaped barriers with numerical solution
- Bubbles-in-island effect

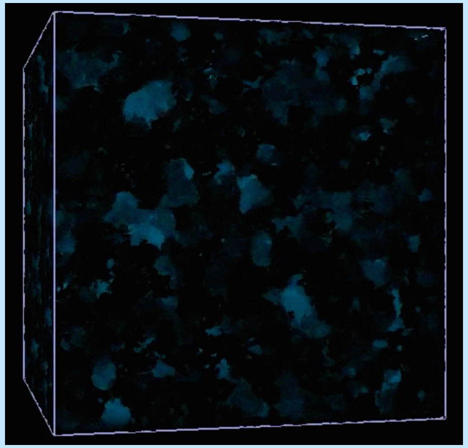


# The Excursion Set Theory of Reionization

**Bubble Model for the early EoR**

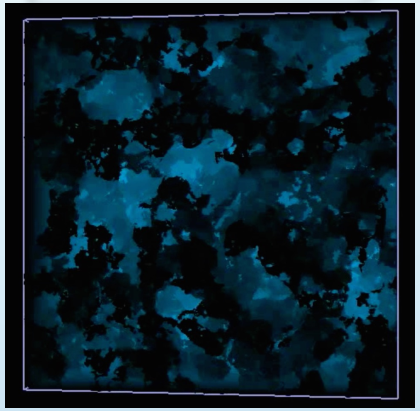
(Furlanetto et al. 2004)

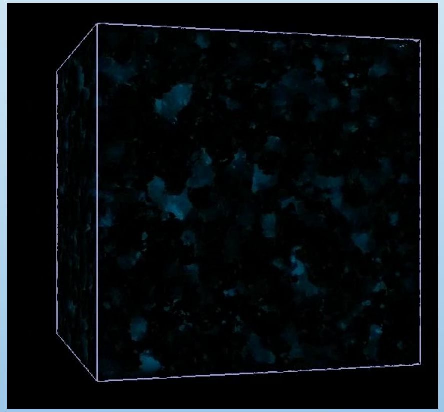
- ✓ Growing isolated ionized bubbles



**21cmFAST**

*percolation*

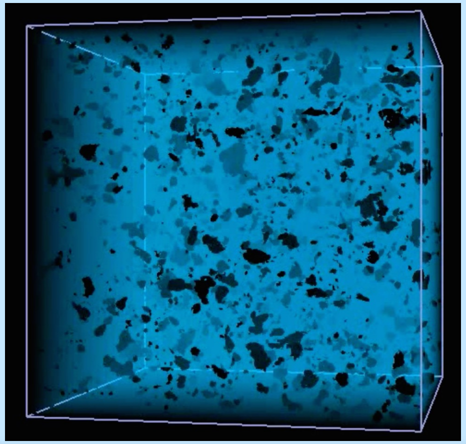




**Island Model for the late EoR**

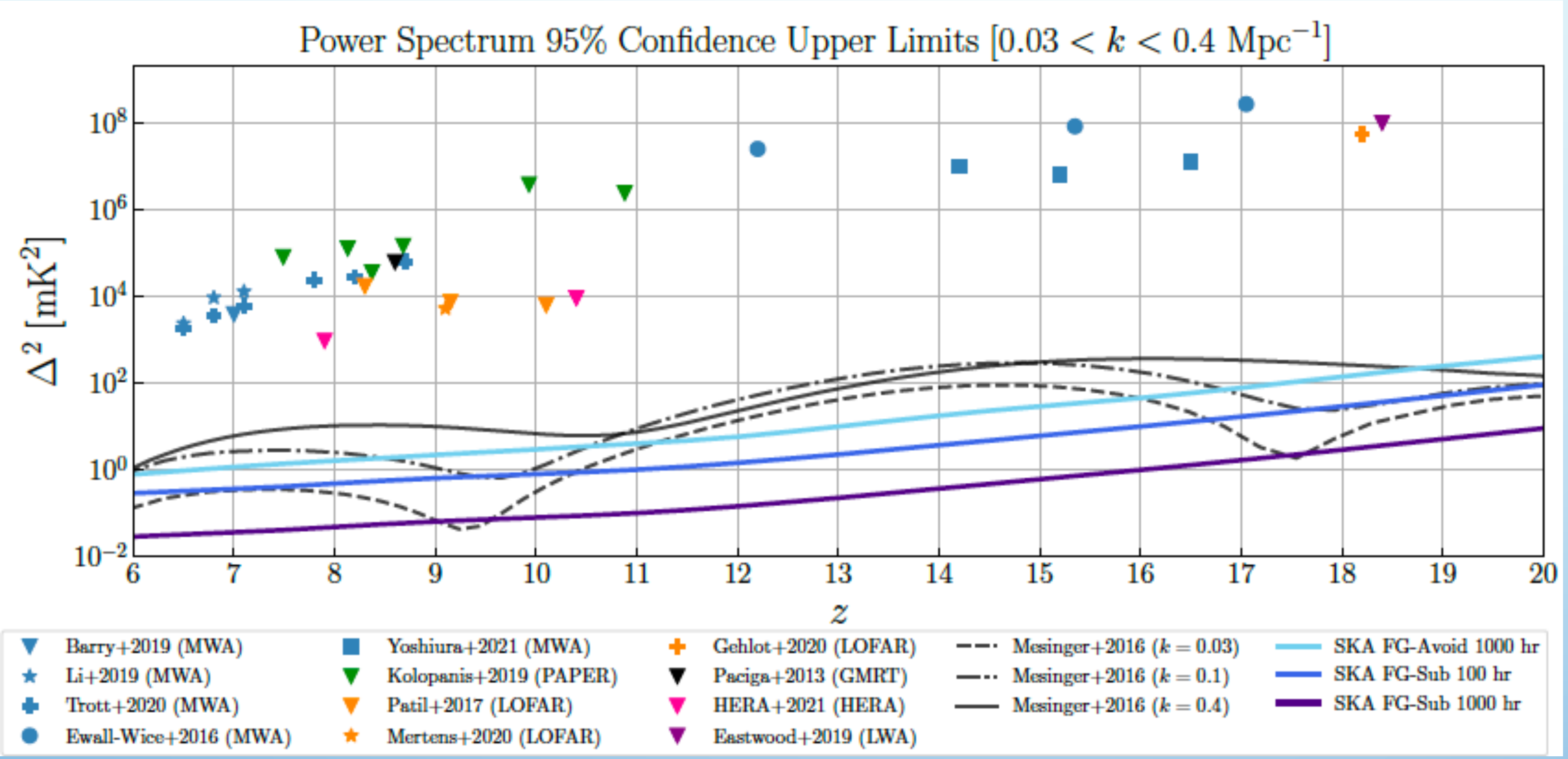
(Xu et al. 2014, 2017)

- ✓ Shrinking isolated neutral islands
- ✓ UVB & small absorbers



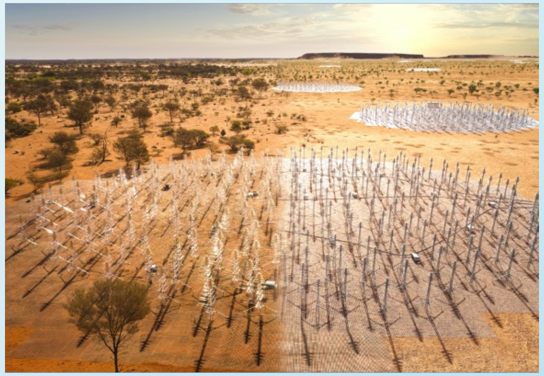
**islandFAST**

# 21 cm Tomography – power spectrum upper limits




Barry et al. arXiv:2110.06173

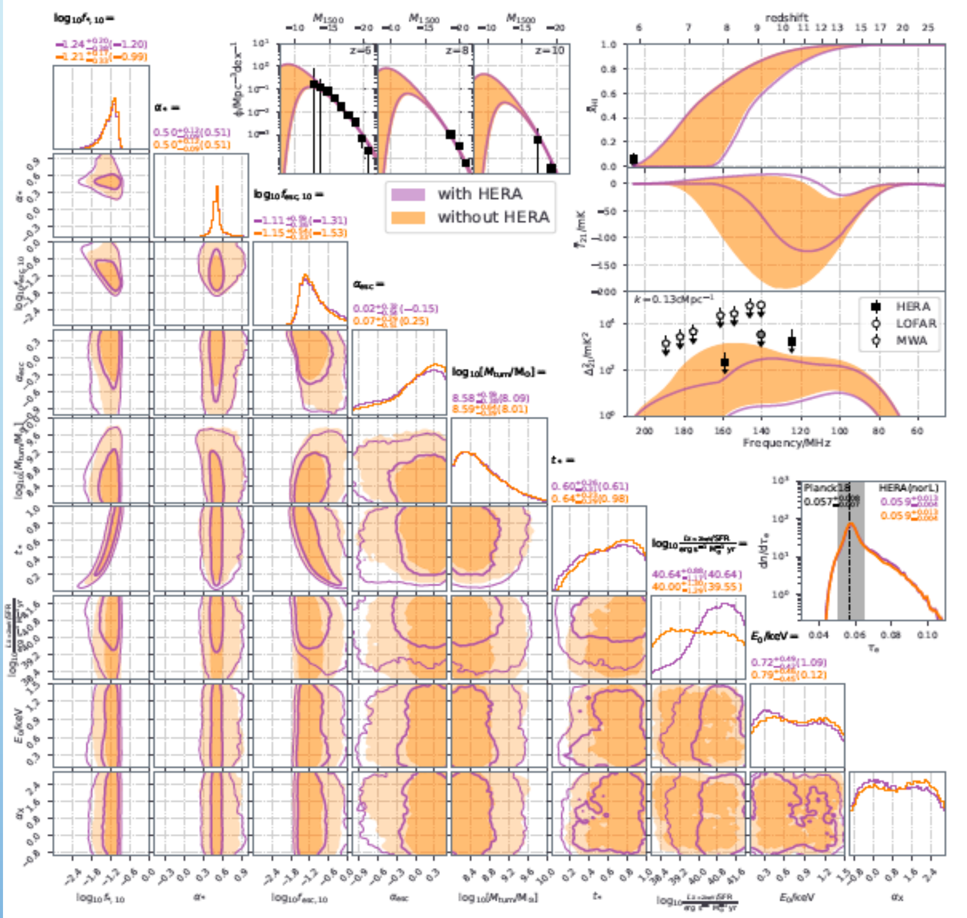
SKAO



LOFAR


HERA

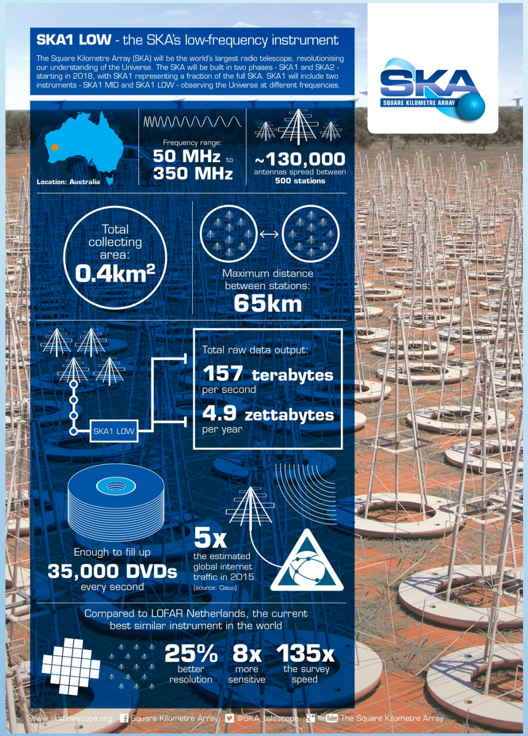
# INFERRING THE EOR PHYSICS



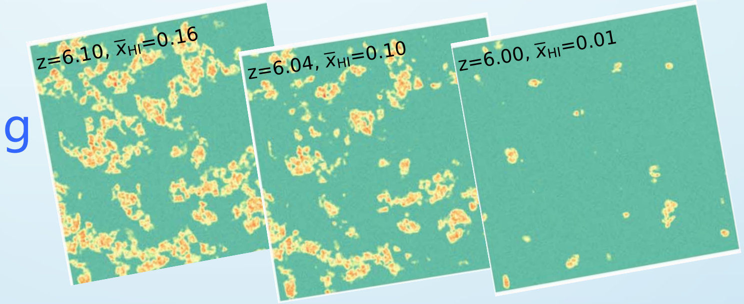

The HERA Collaboration, 2022, ApJ, 924, 51

- **COMPLEX** PHYSICAL PROCESSES INTERPLAYING
- **FAST** REALIZATION OF 3-D LIGHTCONE → PARAMETER INFERENCE
  - 21CMMC (GREIG+15, 18)
  - 21CMDELFI-PS (ZHAO ET AL. 2022)
- MODEL-DEPENDENT → POSSIBLE BIAS → **ACCURATE** MODELING REQUIRED

# 21 cm Tomography – imaging

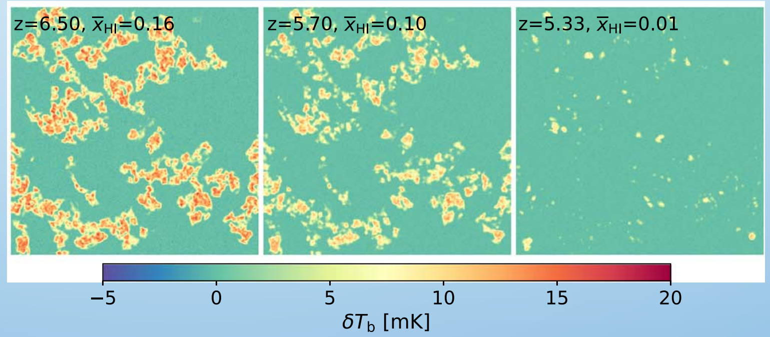



21 cm imaging




21 cm statistics (power spectrum, bispectrum, skewness, ...)

High z → Low z




B = 1 MHz

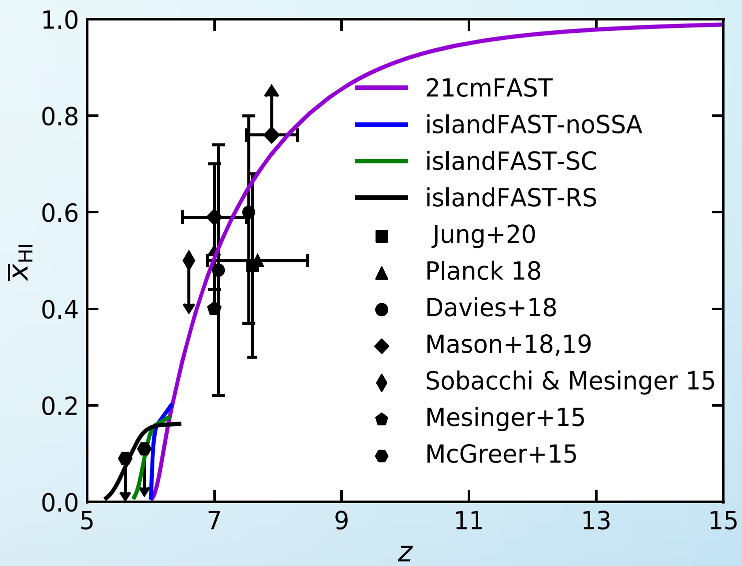
Wu, **YX** et al. 2022, ApJ

# The small-scale absorbers (SSA) Models

- THE IONIZING BACKGROUND (UVB) REGULATED BY SSAS & ISLANDS

$$n_\gamma(z) = \int_z \bar{n}_{\rm H} \left| \frac{{\rm d}f_{\rm coll}(z')}{{\rm d}z'} \right| f_\star N_{\gamma/{\rm H}} f_{\rm esc} \times \exp\left[ -\frac{l(z,z')}{\lambda_{\rm mfp}(z)} \right] (1 - f^{\rm host}_{\rm H\,I}) {\rm d}z',$$

$$\lambda_{\rm mfp}^{-1} = \lambda_{\rm I}^{-1} + \lambda_{\rm abs}^{-1},$$




## 1. islandFAST–noSSA: extremely high UVB

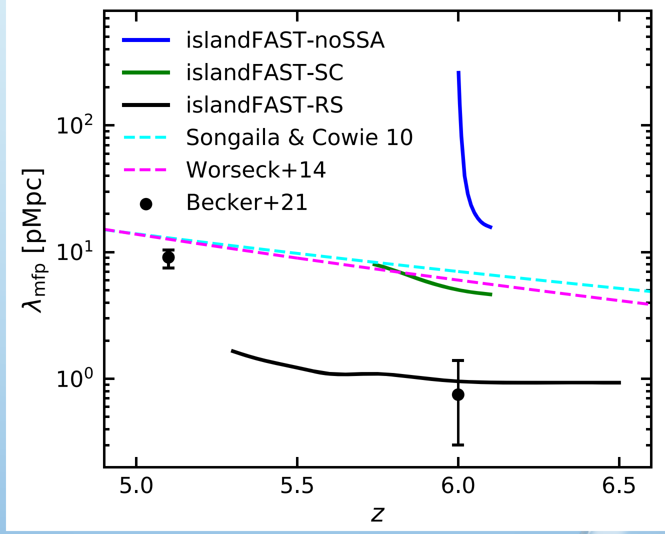
$\lambda_{\rm mfp} = \lambda_{\rm I}$, with $\{\xi = 10,\ T_{\rm min}^{\rm vir} = 10^4\ {\rm K}\}$

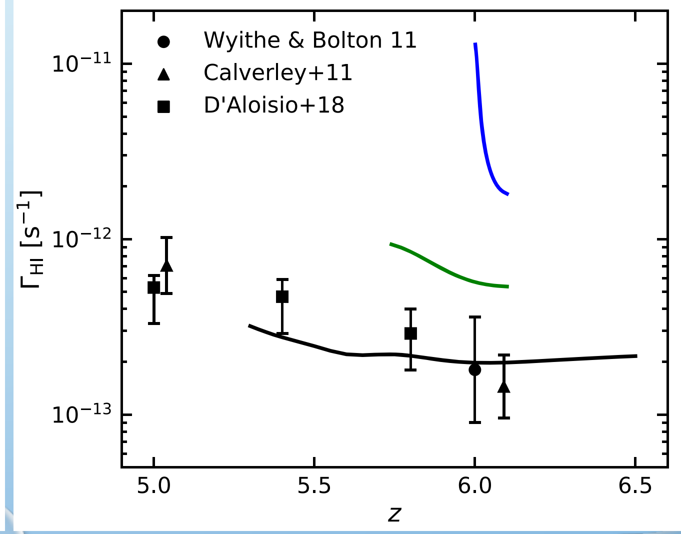
## 2. islandFAST–SC: moderate UVB

$$\lambda_{\rm abs} = 50\left(\frac{1+z}{4.5}\right)^{-4.44}\ [\text{physical Mpc}]$$

Songaila & Cowie (2010)

with $\{\xi = 10,\ T_{\rm min}^{\rm vir} = 10^4\ {\rm K}\}$




## 3. islandFAST–RS: low UVB

Short MFP consistent with Becker et al. (2021)

high-resolution Aurora radiation-hydrodynamical simulations

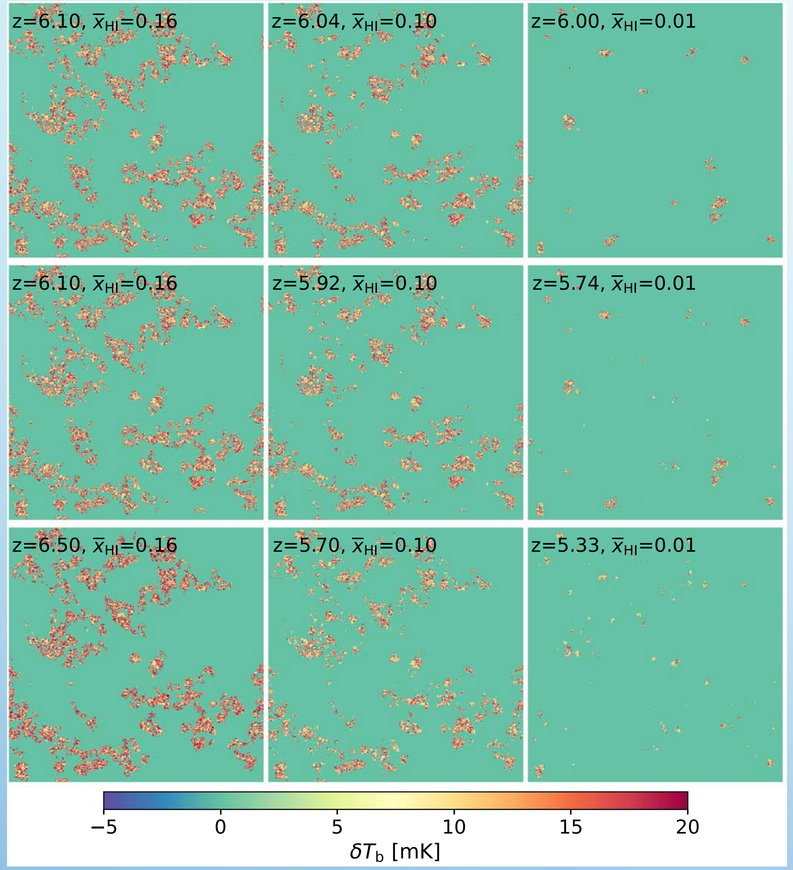
Rahmati & Schaye (2018)

with $\{\xi = 30,\ T_{\rm min}^{\rm vir} = 5 \times 10^4\ {\rm K}\}$

# The intrinsic δTb slices

Thin slices

~ B = 0.1 MHz



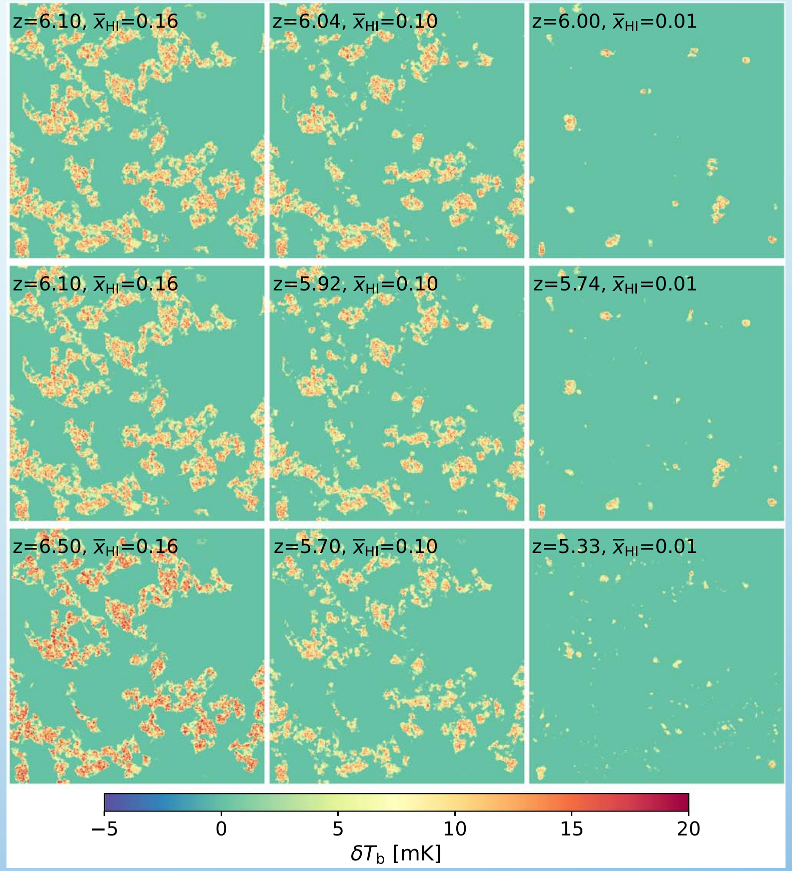

islandFAST-noSSA

islandFAST-SC

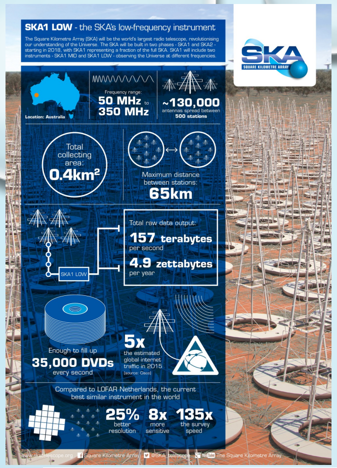
islandFAST-RS

Thick averaged slices

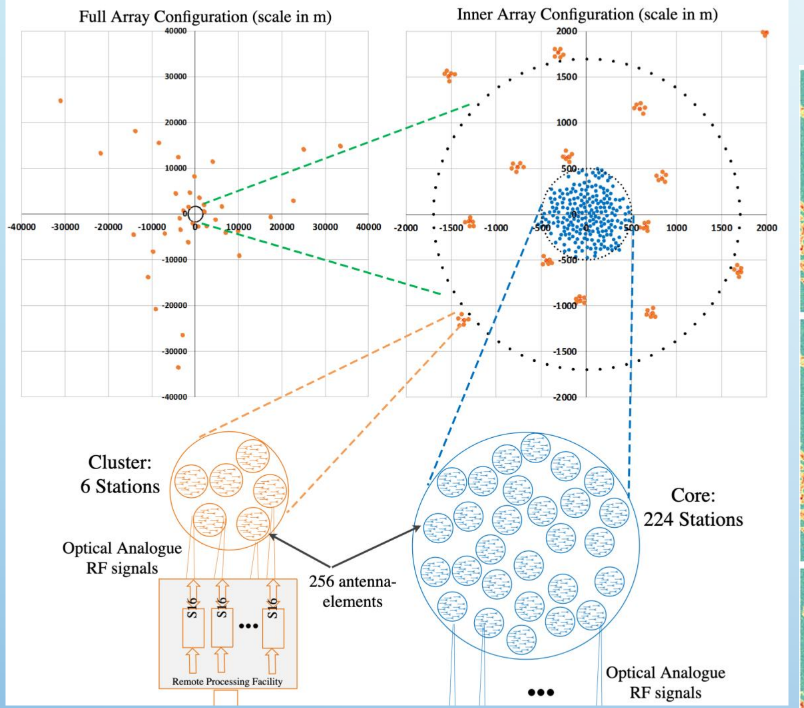
9 adjacent slices ~ B = 1 MHz



# Effects of small-scale absorbers

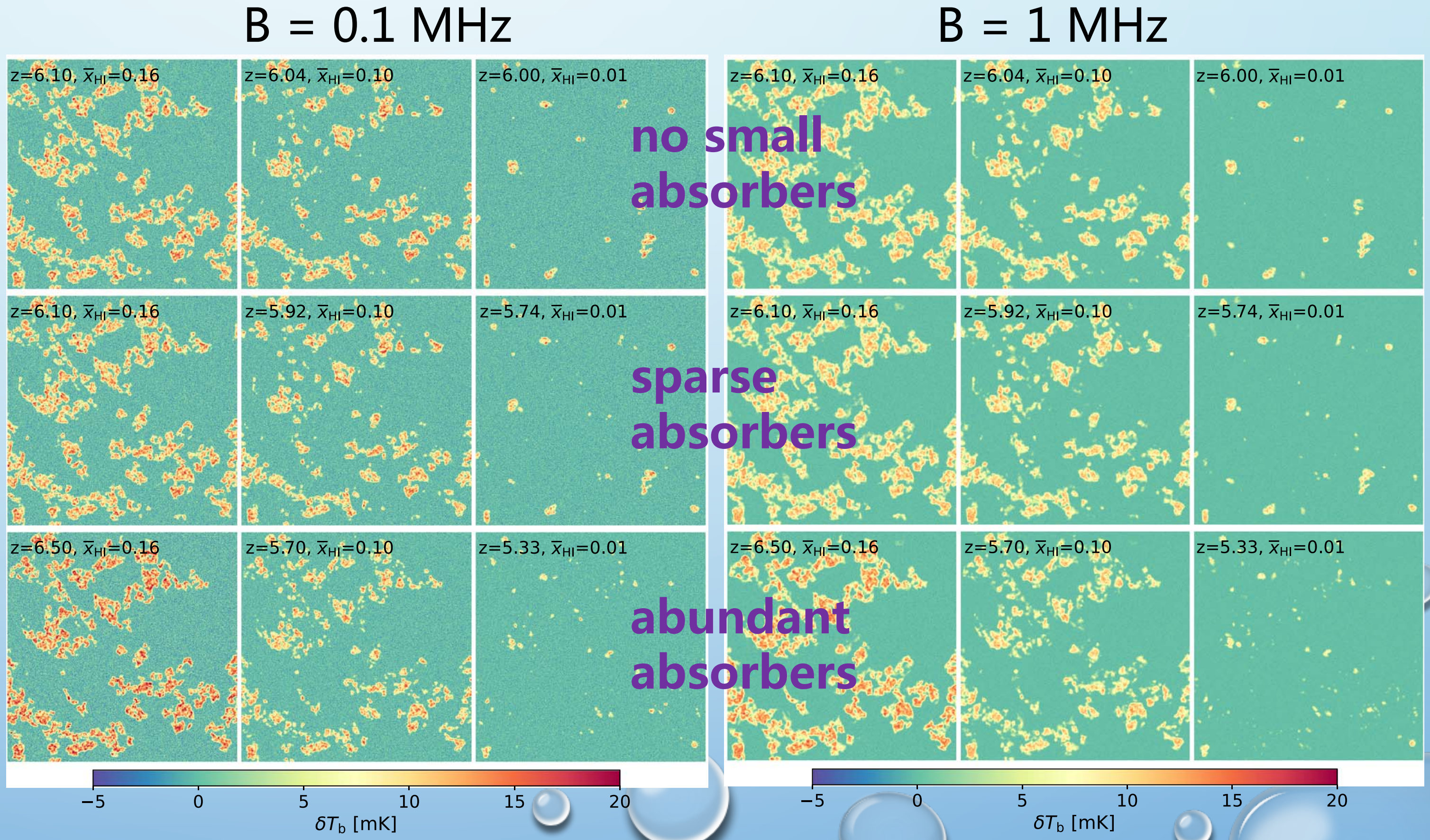



The 21 cm images as observed by the SKA1-Low core array




Credit: SKAO (SKA-TEL-SKO-0001075)

$$\sigma_T = \frac{k_\perp}{2\pi}(D_c^2 \times \Omega_{\rm FoV})^{1/2} \frac{T_{\rm sys}}{\sqrt{B\, t_{\rm int}}} \sqrt{\frac{A_{\rm core} A_{\rm eff}}{A_{\rm coll}^2}}$$

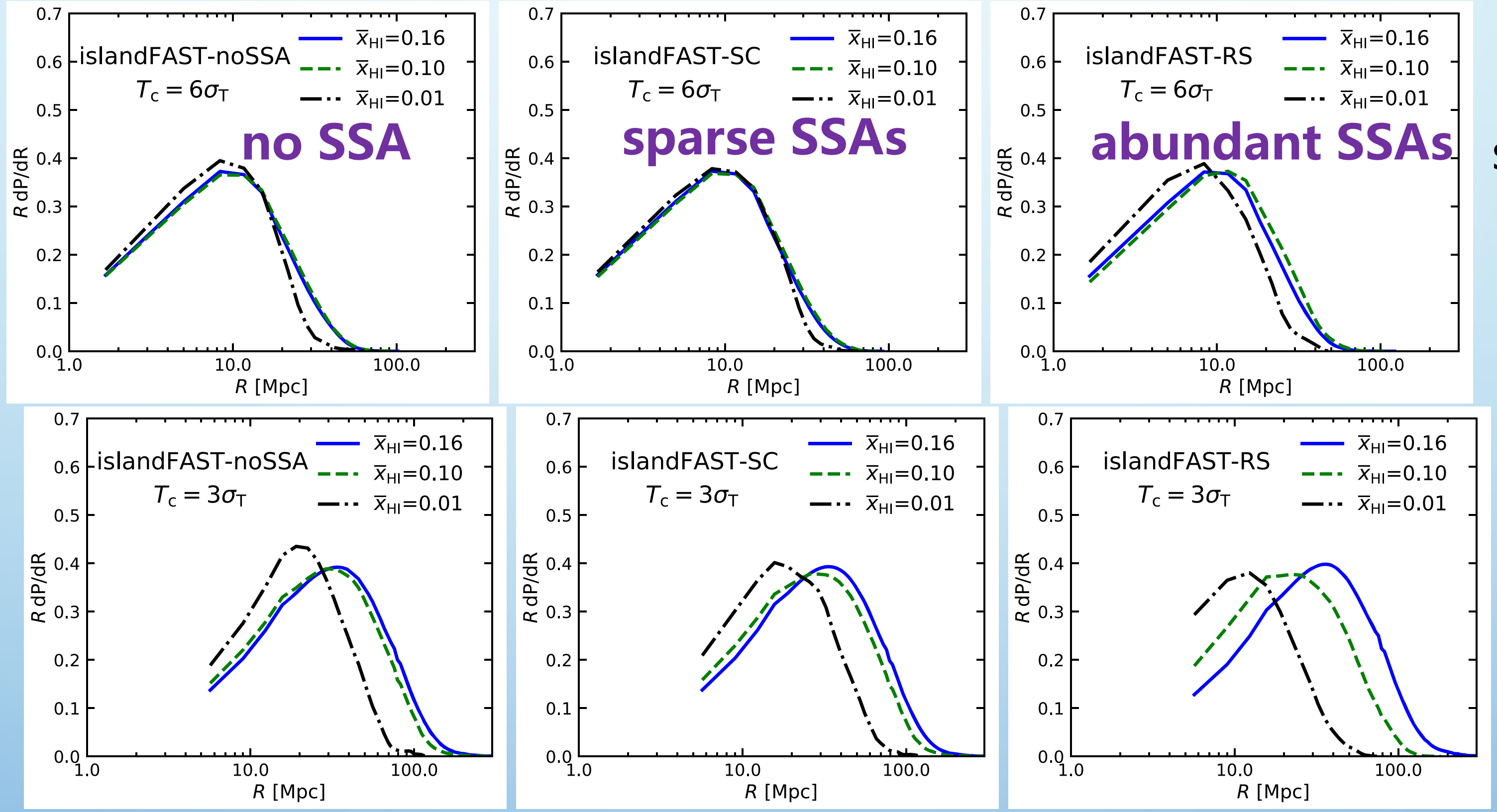



Wu, **YX** et al. 2022, ApJ

# The extracted size distributions from mock $\delta T_b$ images as observed by the SKA1-Low core array

B = 0.1 MHz

B = 1 MHz




smoothed + noise

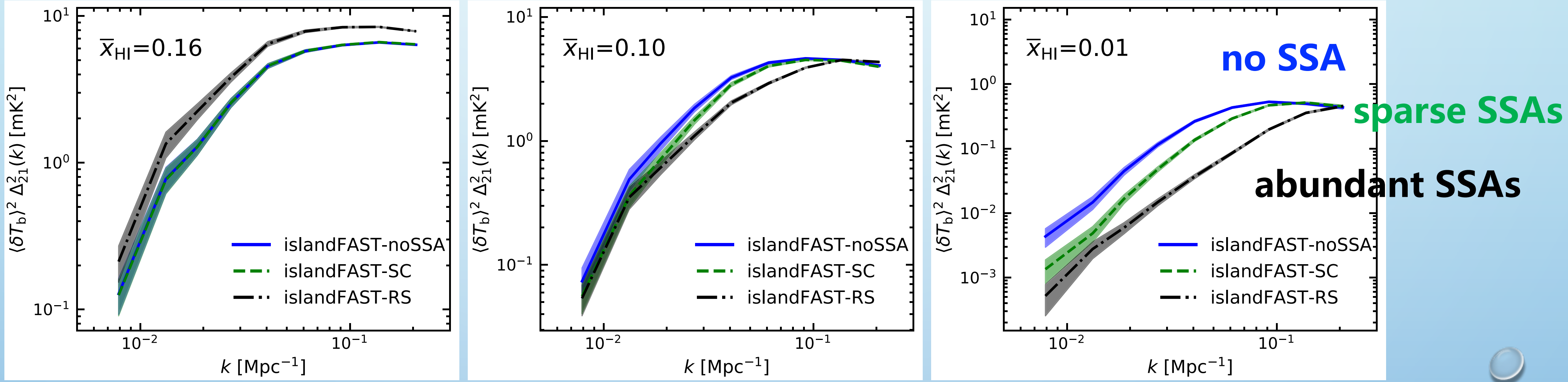
smoothed
+projected
+noise

Only the RS model with **abundant SSAs** shows an **obvious evolution** in the measured island scale.

# The 21 cm power spectrum with expected SKA1-Low core array errors

$$\Delta^2_{\rm Noise} = \frac{2}{\pi} k^{3/2} (D_c^2\, l_z\, \Omega_{\rm FoV}/N_b)^{1/2} \times \left(\frac{T_{\rm sys}}{\sqrt{B\, t_{\rm int}}}\right)^2 \frac{A_{\rm core} A_{\rm eff}}{A^2_{\rm coll}},$$

$N_b = 1,\ B = 1\,{\rm MHz},\ {\rm and}\ t_{\rm int} = 1000\,{\rm hr}.$




- THE ABUNDANCE OF THE SSAS AND THE LEVEL OF THE IONIZING BACKGROUND CAN BE DISTINGUISHED OR CONSTRAINED.

Wu, **YX** et al. 2022, ApJ

# Take-away messages for large-scale modeling for reionization

- Large-scale structure evolution of reionization can be modeled with the Excursion Set Theory of Reionization

**Bubble Model & 21cmFAST for the early EoR**

(Furlanetto et al. 2004)

✓ Growing isolated ionized bubbles

**Island Model & islandFAST for the late EoR**

(Xu et al. 2014, 2017)

✓ Shrinking isolated neutral islands

✓ UVB & small absorbers

- Predictions for 21-cm tomography observations
- Incorporating instrumental effects
- put constraints on the source properties, SSA abundance, the MFP, and the level of the ionizing background.

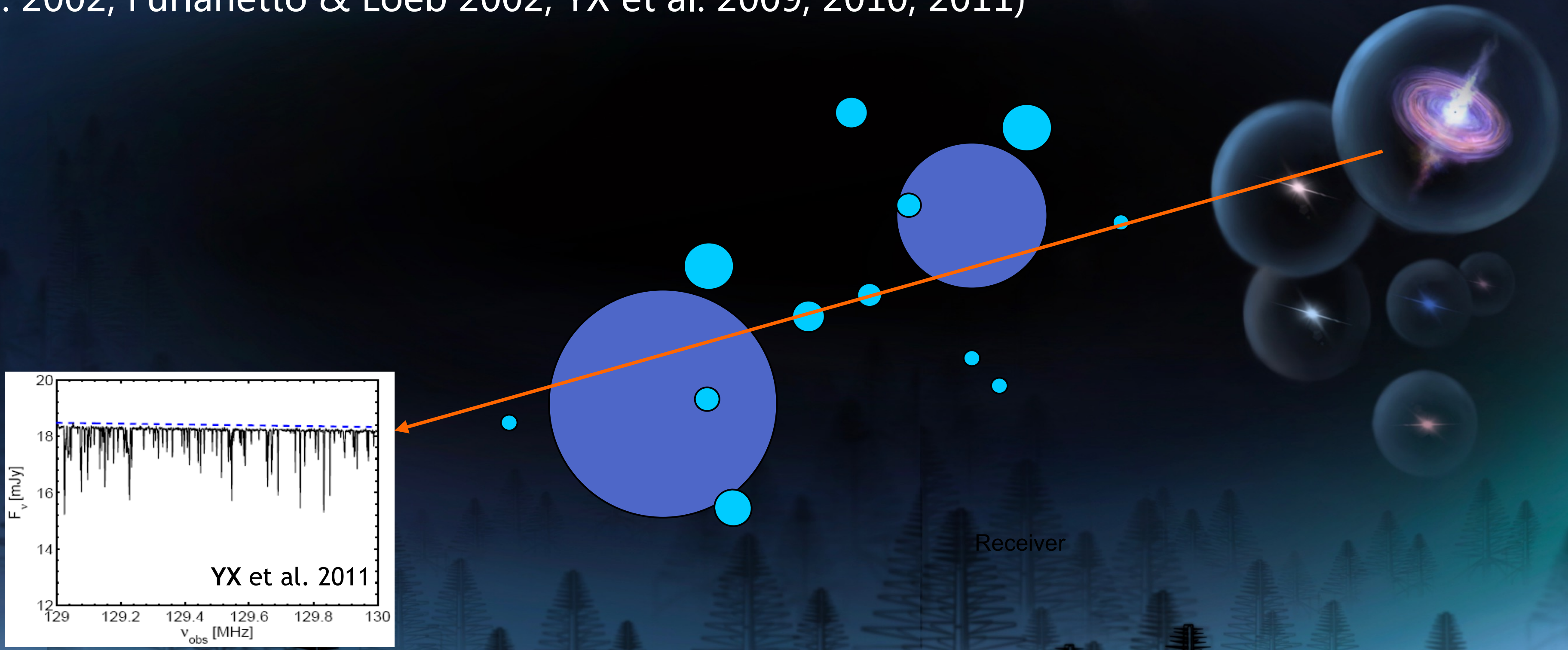
Modeling the small-scale structures of the IGM during EoR

→ 21-cm forest

# 21 cm Forest

## -- absorption lines against high-z radio point sources

(e.g. Carilli et al. 2002; Furlanetto & Loeb 2002; YX et al. 2009, 2010, 2011)



# The 21 cm optical depth

- The 21 cm optical depth of a cloud of hydrogen

$$\tau_\nu = \int \mathrm{d}s\,\sigma_{01}(1 - e^{-E_{10}/k_B T_S})\phi(\nu) n_0$$
$$\approx \sigma_{01}\left(\frac{h\nu}{k_B T_S}\right)\left(\frac{N_{HI}}{4}\right)\phi(\nu),$$

$$\sigma_{01} \equiv \frac{3c^2 A_{10}}{8\pi\nu^2},$$

- The 21 cm optical depth of **non-linear objects**

$$\tau(\nu) = \frac{3\, h_P\, c^3 A_{10}}{32\, \pi^{3/2} k_B}\, \frac{1}{\nu^2} \int_{-\infty}^{+\infty} \frac{n_{HI}(r)}{b(r) T_S(r)} \exp\left[-\frac{(u(\nu) - \bar{v}(r))^2}{b^2(r)}\right] dx$$

$$b(r) = \sqrt{2\, k_B T_K(r)/m_H},$$
$$u(\nu) \equiv c\,(\nu - \nu_{10})/\nu_{10}$$
$$r^2 = (\alpha\, r_{vir})^2 + x^2$$

- The 21 cm optical depth of **a uniform medium** with proper velocity $v_{||}$ (Hubble flow + peculiar velocity)

$$\tau_{\nu_{10}}(z) = \frac{3}{32\pi}\, \frac{h_P\, c^3 A_{10}}{k_B \nu_{10}^2}\, \frac{x_{HI}\, n_H(z)}{T_S\,(1+z)\,(dv_\parallel/dr_\parallel)}$$
$$\approx 0.009\,(1+\delta)\,(1+z)^{3/2}\, \frac{x_{HI}}{T_S}\left[\frac{H(z)/(1+z)}{dv_\parallel/dr_\parallel}\right],$$

# The 21 cm optical depth

- The 21 cm optical depth of **a uniform medium** with proper velocity $v_{||}$ (Hubble flow + peculiar velocity)

$$\tau_{\nu_{10}}(z) = \frac{3}{32\pi}\frac{h_P\, c^3 A_{10}}{k_B \nu_{10}^2}\frac{x_{HI}\, n_H(z)}{T_S\,(1+z)\,(dv_\parallel/dr_\parallel)}$$
$$\approx 0.009\,(1+\boxed{\delta})\,(1+z)^{3/2}\frac{\boxed{x_{HI}}}{\boxed{T_S}}\left[\frac{H(z)/(1+z)}{dv_\parallel/dr_\parallel}\right],$$

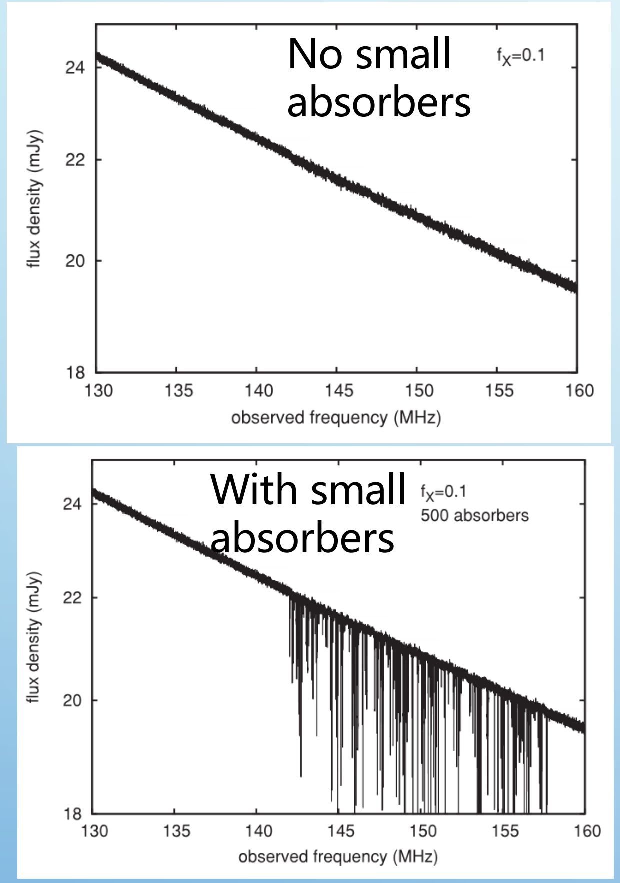
- The 21 cm optical depth of **non-linear objects**

$$\tau(\nu) = \frac{3\,h_P\, c^3 A_{10}}{32\,\pi^{3/2} k_B}\frac{1}{\nu^2}\int_{-\infty}^{+\infty}\frac{\boxed{n_{HI}(r)}}{\boxed{b(r)}\boxed{T_S(r)}}\exp\left[-\frac{(u(\nu)-\bar{v}(r))^2}{b^2(r)}\right]dx$$

$$b(r) = \sqrt{2\,k_B T_K(r)/m_H},$$
$$u(\nu) \equiv c\,(\nu - \nu_{10})/\nu_{10}$$
$$r^2 = (\alpha\, r_{vir})^2 + x^2$$

→ We need detailed profiles of: $n_H$ $\boxed{x_i}$ $\boxed{T_K}$ $\boxed{J_\alpha}$

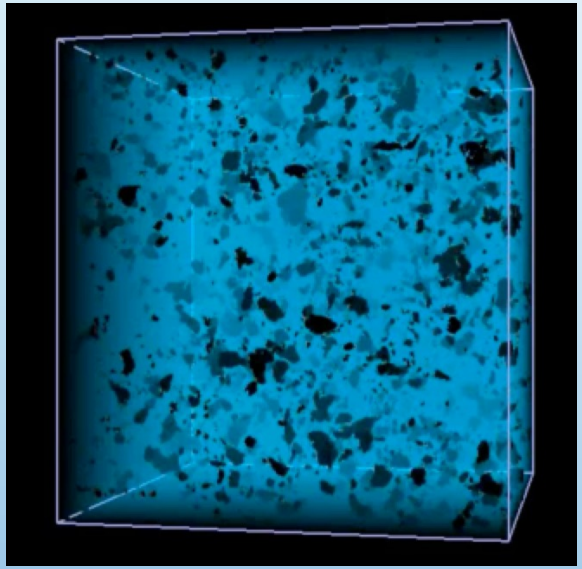
# 21-cm Forest: theoretical challenges




Mack & Wythe, 2012

- Large-scale environment: $x_i(\vec{x})$, $T(\vec{x})$.

1 Gpc



**islandFAST, XuYD** et al. 2017

$2\times10^5$

- Main contributors:

minihalos & ambient IGM

6 kpc @ z=9

$10^6$ M$_\odot$

$\rho$ & $T_K$ profiles,
Local $x_i$ & $v$,
$T_S$ coupling
(collisional, Ly$\alpha$,
CMB)
……

$$\tau_{\nu_0}(\hat{\mathbf{s}}, z) \approx 0.0085\,[1+\delta(\hat{\mathbf{s}}, z)]\,(1+z)^{3/2}\left[\frac{x_{\rm HI}(\hat{\mathbf{s}}, z)}{T_{\rm S}(\hat{\mathbf{s}}, z)}\right]\left[\frac{H(z)/(1+z)}{{\rm d}v_\parallel/{\rm d}r_\parallel}\right]\left(\frac{\Omega_{\rm b}h^2}{0.022}\right)\left(\frac{0.14}{\Omega_{\rm m}h^2}\right)$$

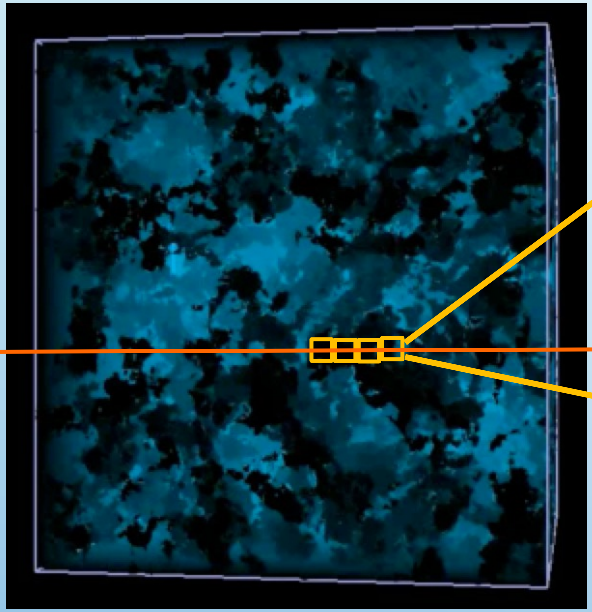
# Theoretical challenge → multi-scale hybrid modeling

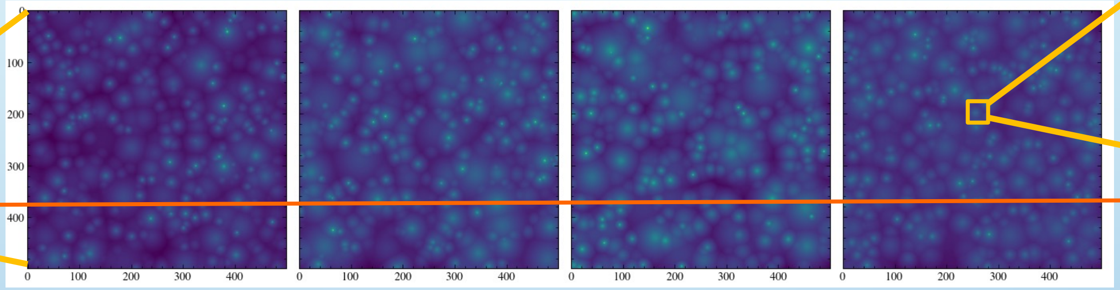
- LARGE SCALES: SEMI-NUMERICAL SIMULATION
- Small scales: analytic modeling

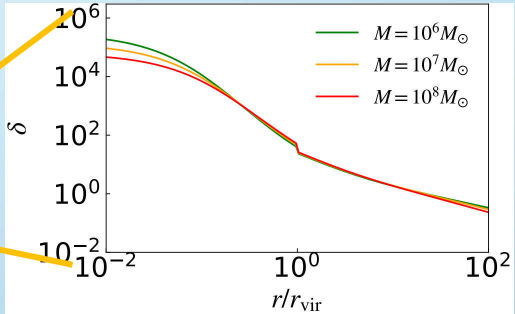
Box 1 Gpc → $500^3$ Grids

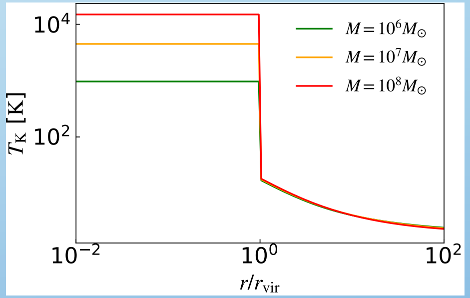
2 Mpc → $500^3$ Voxels

~ kpc






$\rho, T_K, x_i, v, T_S$

21cmFAST/islandFAST

XuYD et al. 2017

# Modeling the *small-scale* structures during the EoR

- THE MODEL
  - The halos – Sheth-Tormen mass function (Sheth & Tormen 1999) & NFW density profile (Navarro, Frenk & White 1997)
  - Gas density profile
    - Inside – hydrostatic equilibrium
    - Outside – "Infall Model" (Barkana 2004)
  - Star Formation Criterion

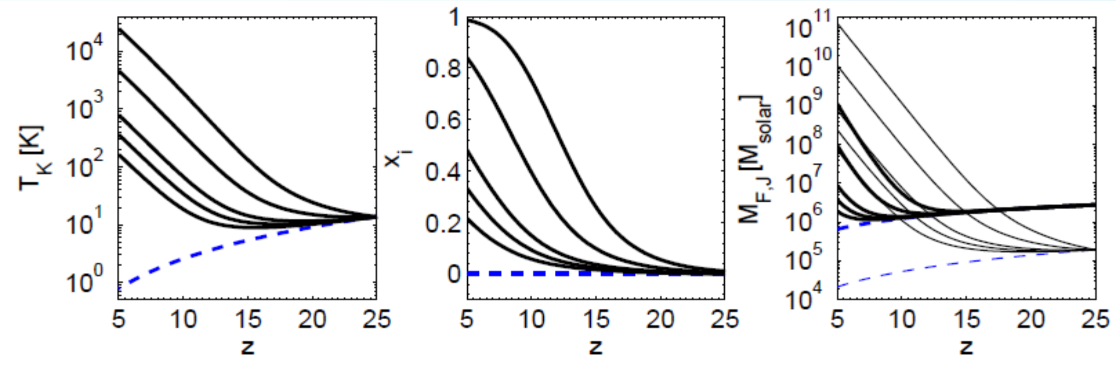
$$t_s = t_F + t_{SB}$$

where

$$t_{SB} = \max\{ t_{ff}, t_{cool} \}$$

$$\begin{cases} t_s < t_H \longrightarrow \text{a star burst occurs at time } t_s \longrightarrow \text{DGs} \\ t_s > t_H \longrightarrow \text{not able to form stars} \longrightarrow \text{MHs} \end{cases}$$

# Modeling the *small-scale* structures during the EoR




- *The X-ray background*
  - Ionization & heating

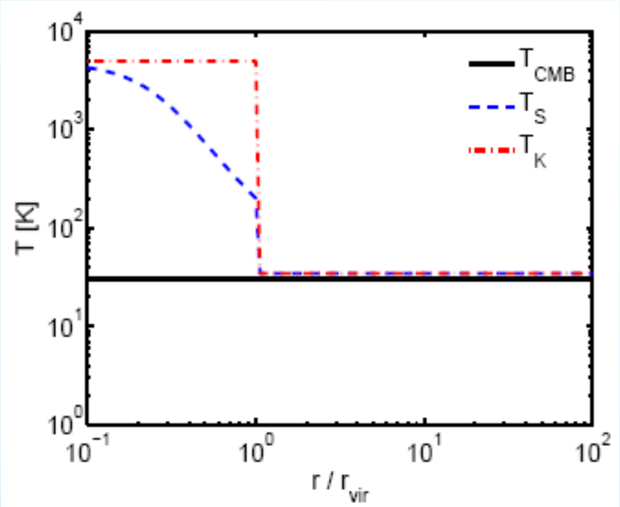
  ($f_X$ = 0, 0.05, 0.1, 0.2, 1, 5)

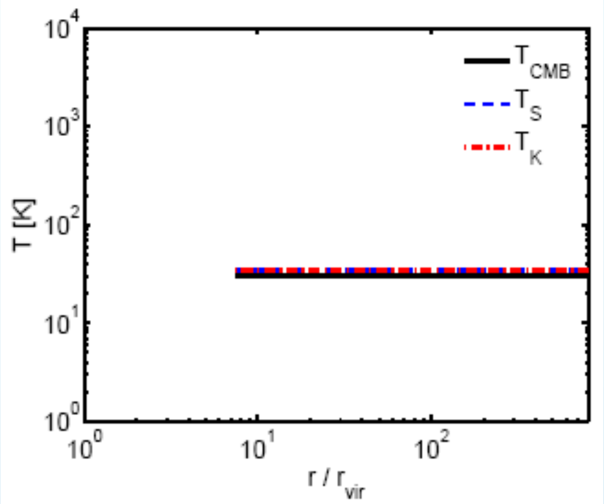
- *Modeling the minihalos*
  - $x_i$ : collisional ionization equilibrium (CIE)
  - $T_K$ : $T_{vir}$ (inside) + $T_{IGM}$ (outside)
  - $J_\alpha$ : recombination + Lyα background (outside)

- *Modeling the dwarf galaxies*
  - $x_i$ : photonionization
  - $T_K$ : Hubble expansion + background X-rays + local soft X-rays
  - $J_\alpha$ : recombination + Lyα background + soft X-ray cascading

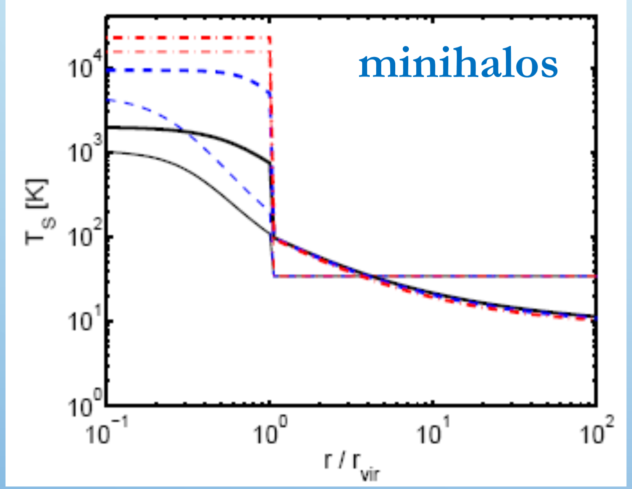
See Xu, Ferrara, & Chen (2011) for details

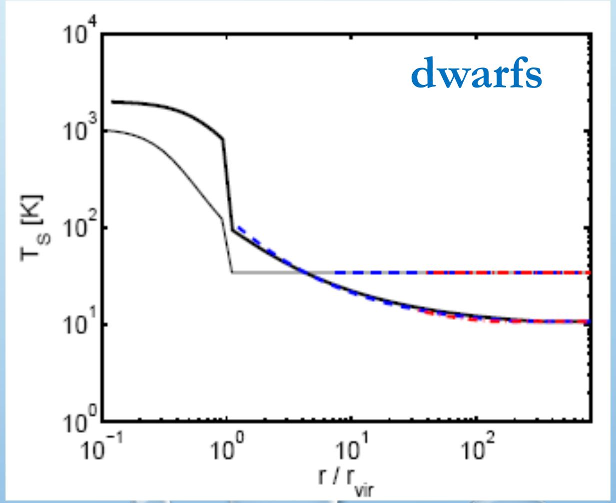
# The Coupling Effects and Spin Temperature




$M = 10^7\, \mathrm{M}_\odot$ at redshift 10.

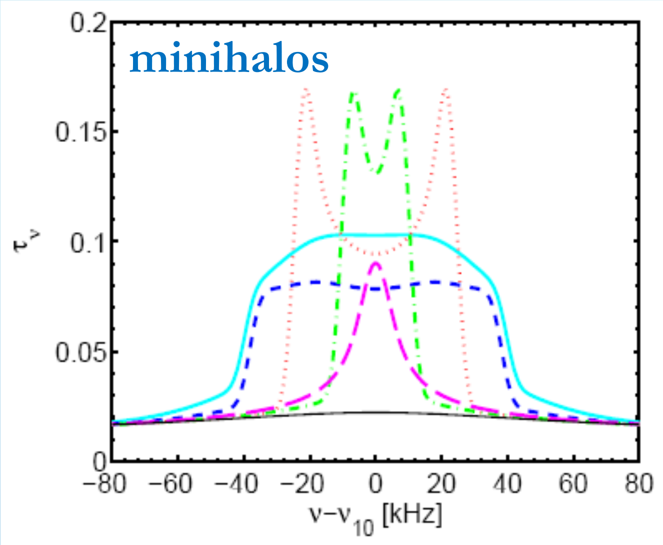


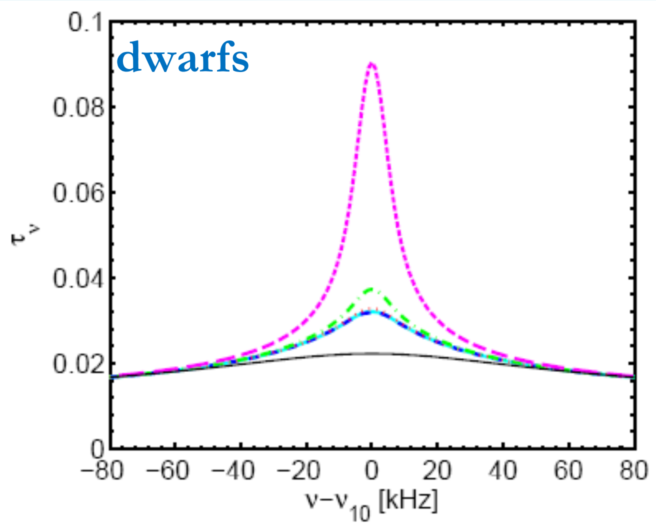




$M = 10^6\, \mathrm{M}_\odot$ (solid curves), $M = 10^7\, \mathrm{M}_\odot$ (dashed curves) and $M = 10^8\, \mathrm{M}_\odot$ (dot–dashed curves)

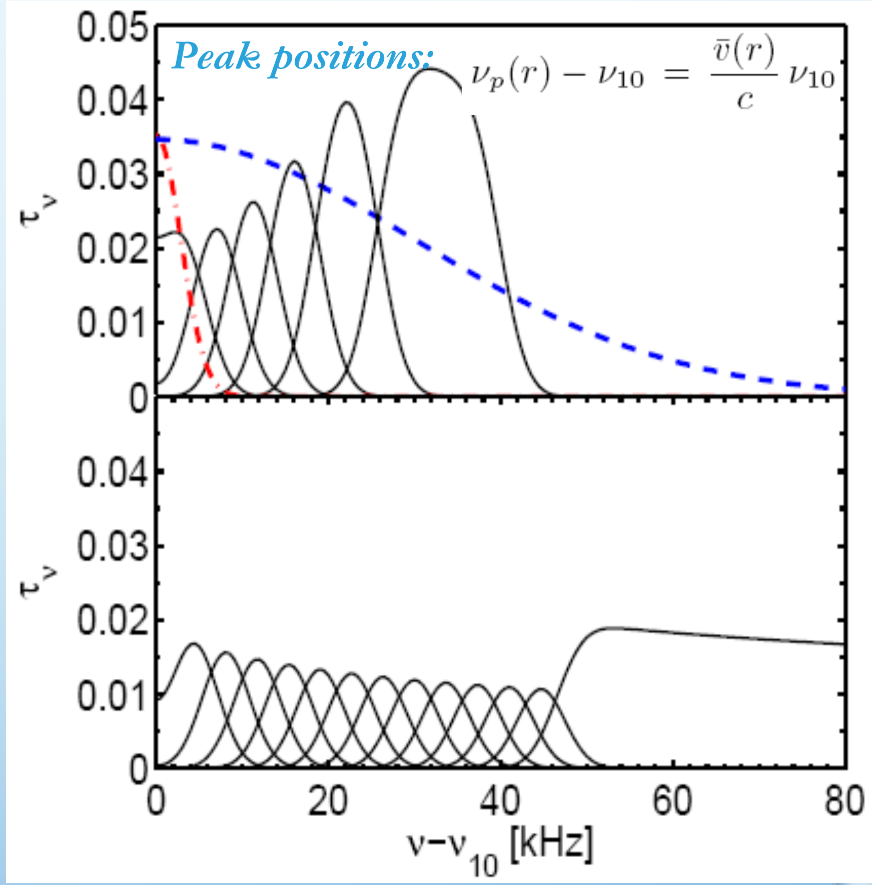
# The 21 cm Absorption Line Profiles

$M = 10^7\ \mathrm{M}_\odot$ at redshift 10.

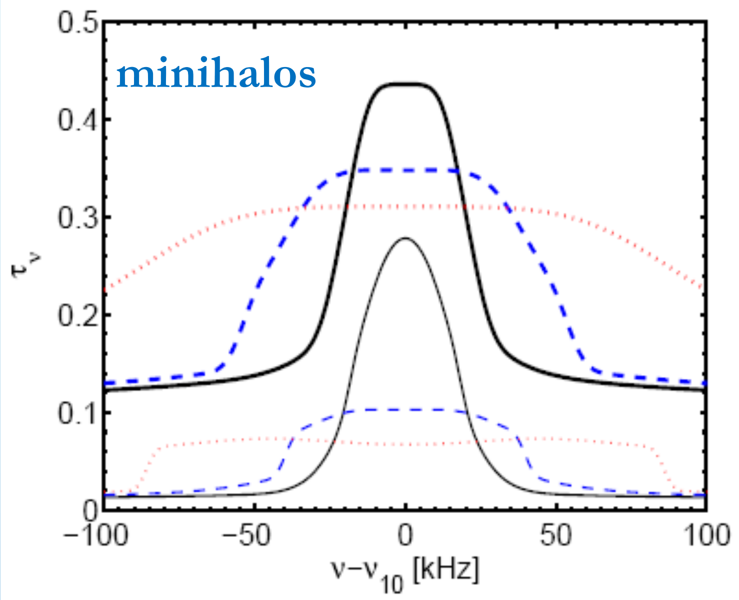


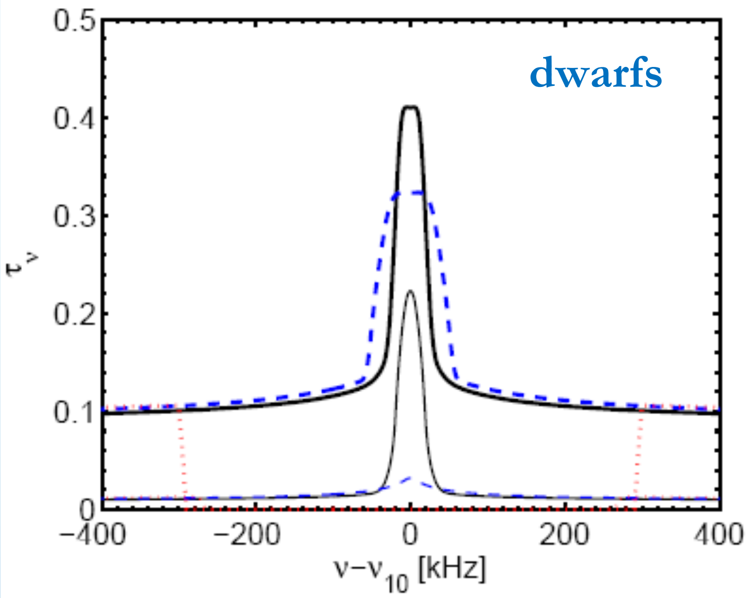




$\alpha = 0$ (solid cyan), 0.3 (short-dashed blue), 1 (dotted red), 3 (dot–dashed green) and 10 (long-dashed magenta),



# The 21 cm Absorption Line Profiles

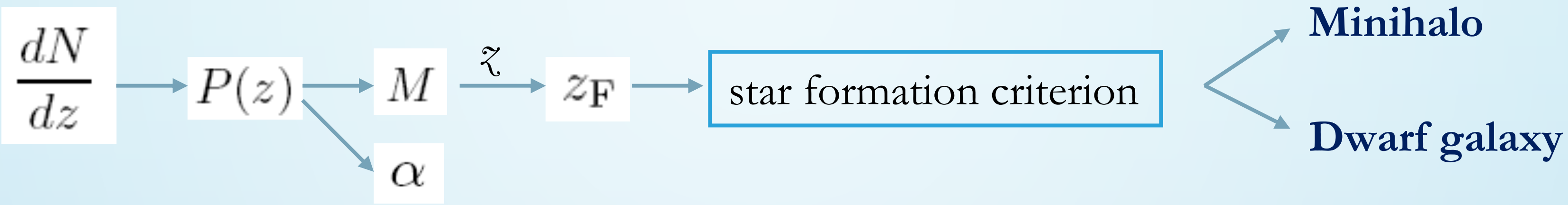


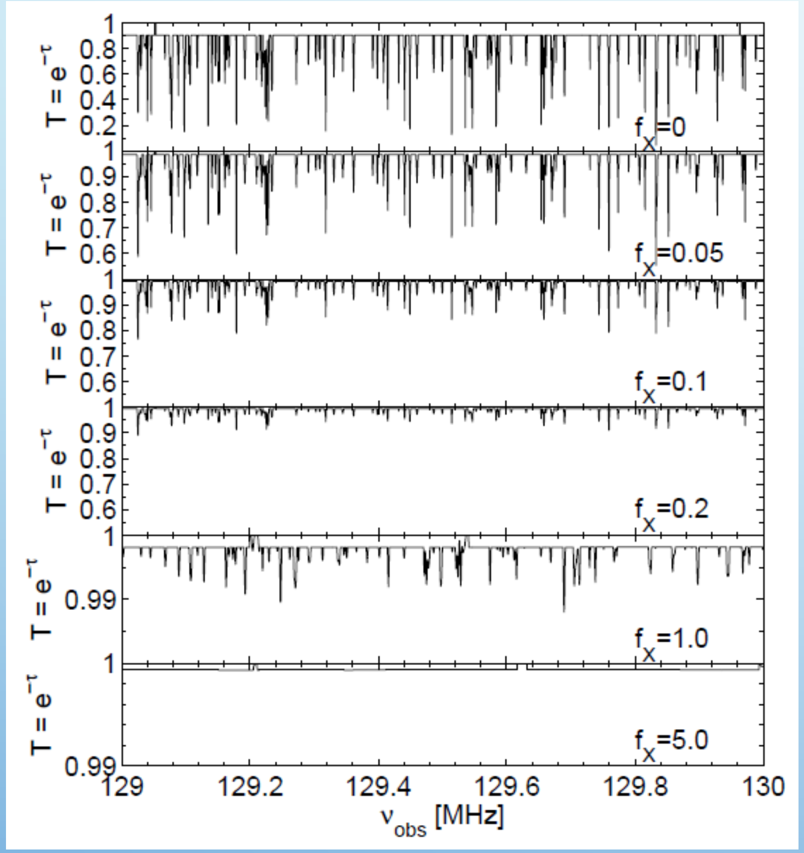




**z = 20**
z = 10

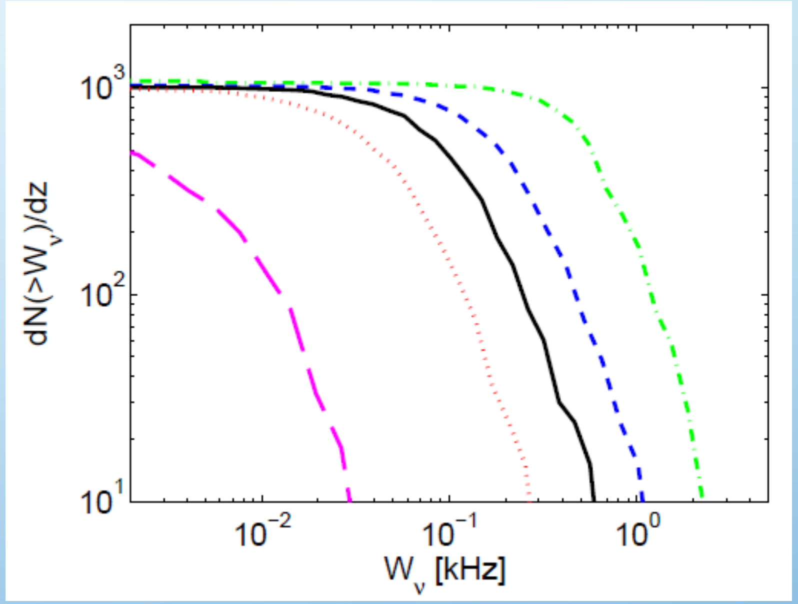
$M = 10^6\,\mathrm{M}_\odot$ (solid curves), $M = 10^7\,\mathrm{M}_\odot$ (dashed curves) and $M = 10^8\,\mathrm{M}_\odot$ (dotted curves)

# Theoretical Spectrum



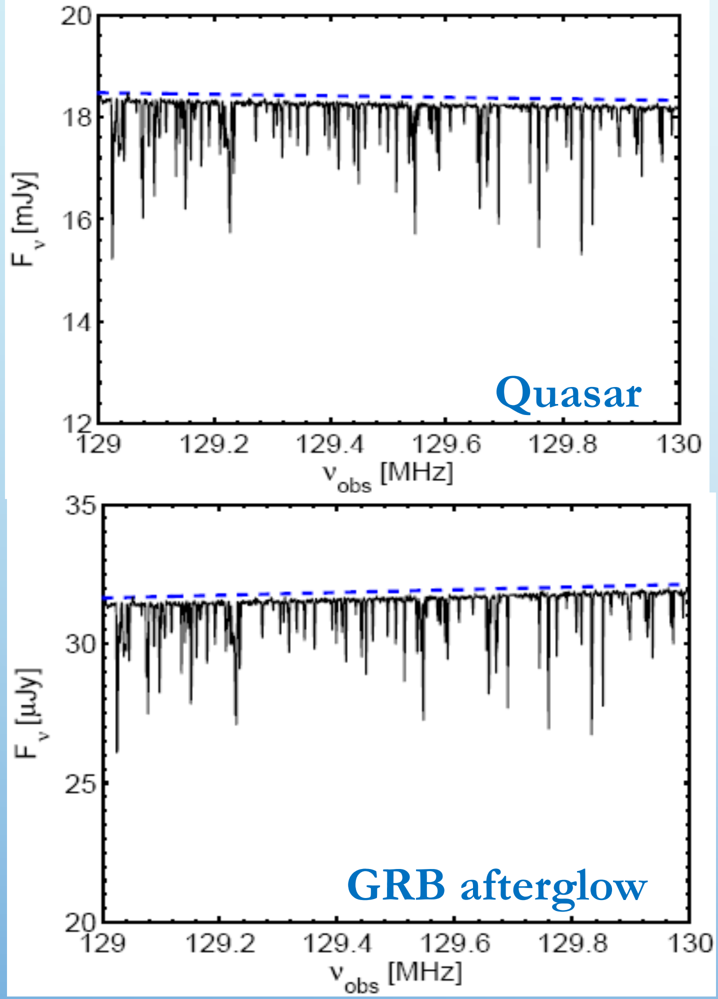




- The equivalent width distribution



# Observational Spectrum

- High-resolution (1 kHz): $F_{\min} = 542\mu\text{Jy}$ ($\tau \sim 0.1$)

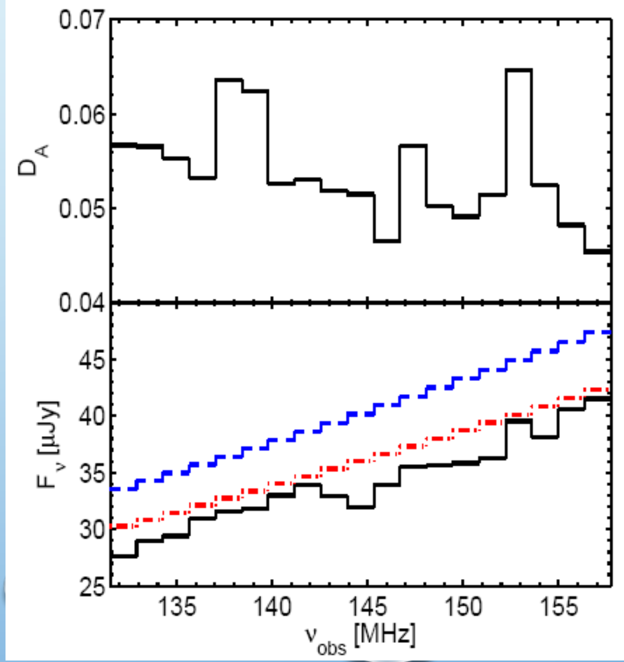



- Low-resolution:

$$D_{\rm A} = \langle \frac{f_{\rm IGM} - f_{\rm obs}}{f_{\rm IGM}} \rangle = \langle 1 - e^{\tau_{\rm IGM} - \tau} \rangle = 1 - e^{\tau_{\rm IGM} - \tau_{\rm eff}}$$

$\rightarrow$ $F_{\min} = 77.4\mu\text{Jy}$

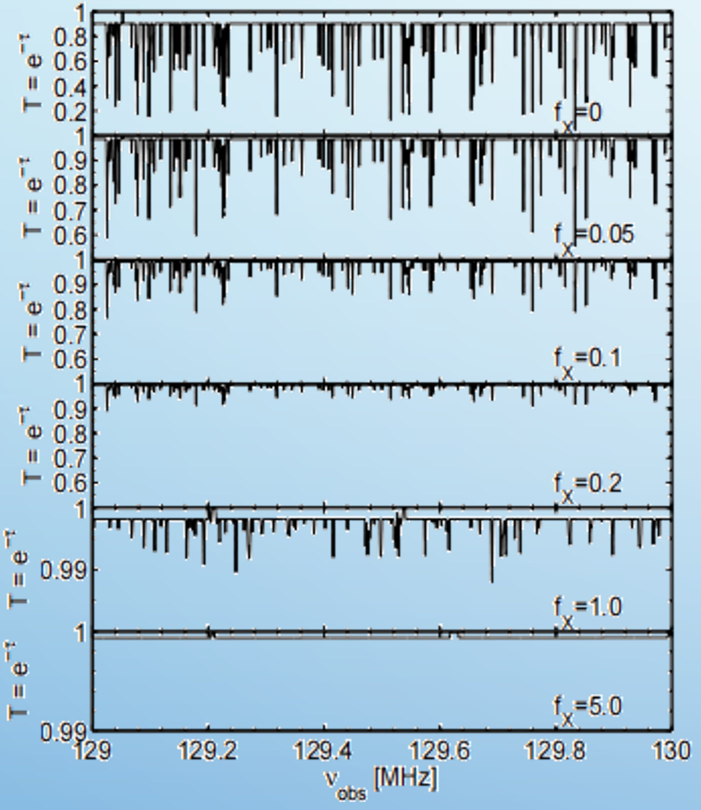
- Optimal $z_{GRB}$: to maximize the number of pixels
  - $z_{GRB} = 9.8$; $\Delta\nu_{ch} = 1.38$ MHz; $N_{pixel} = 19$ (if there is no X-ray background)



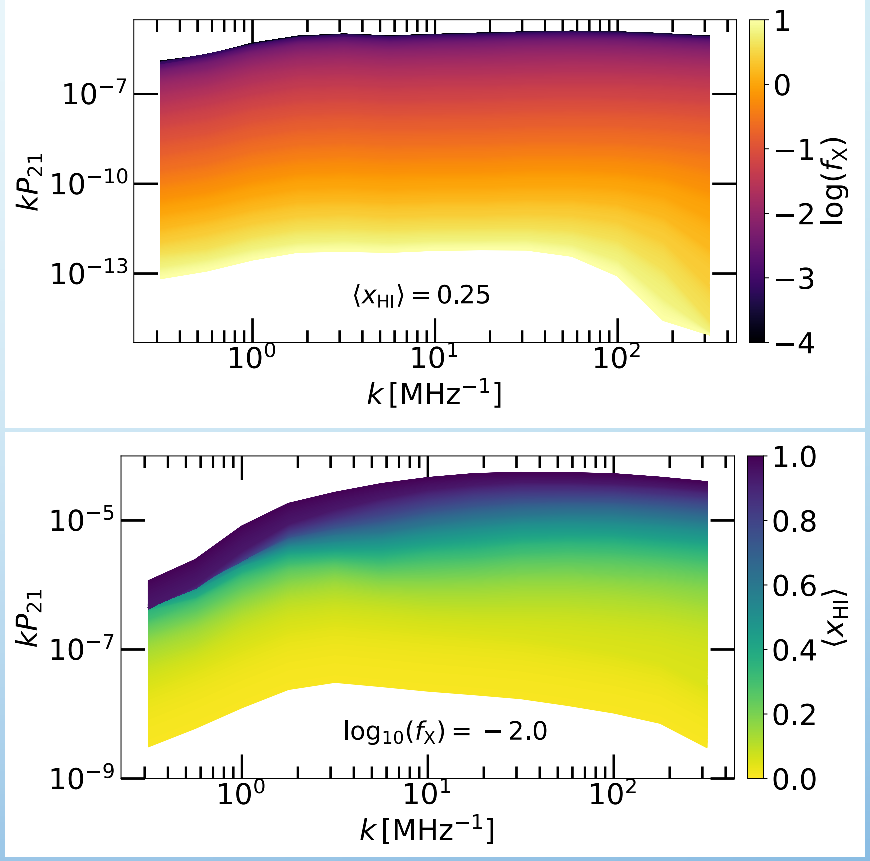
# Physics with 21-cm Forest

Sensitive probe to $T_{IGM}$ and xHI → properties of the first galaxies & black holes

$$\tau \propto \frac{1}{T_S}$$



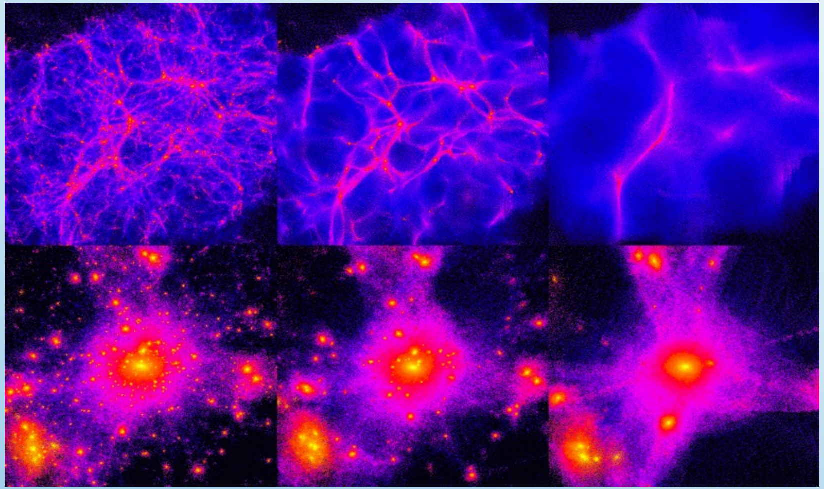

Xu YD et al. 2009, 2010, 2011




Šoltinský et al. 2025

# Physics with 21-cm Forest

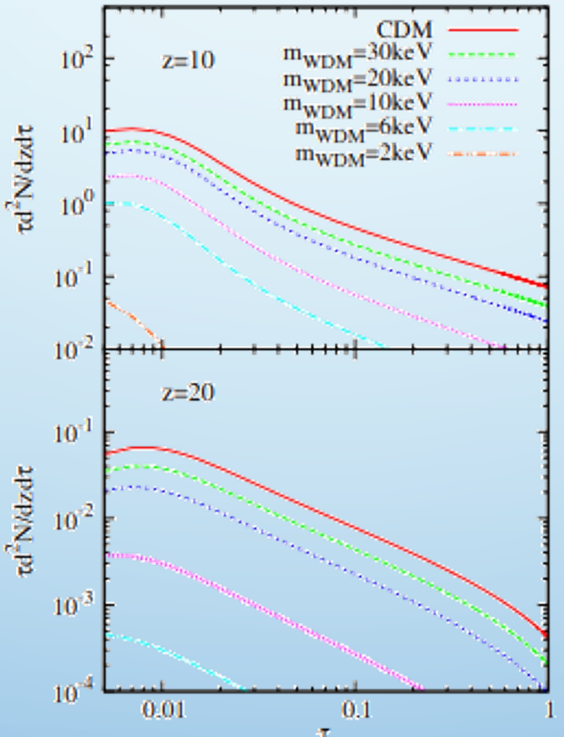
- Unique probe to **small–scale structures** at cosmic dawn (CD) → Dark Matter properties at CD



CDM
WIMP/AXION

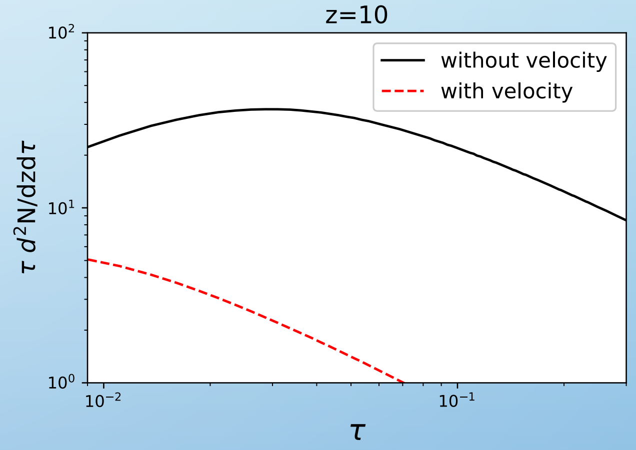
WDM
Sterile
Neutrino

HDW
3 Neutrinos



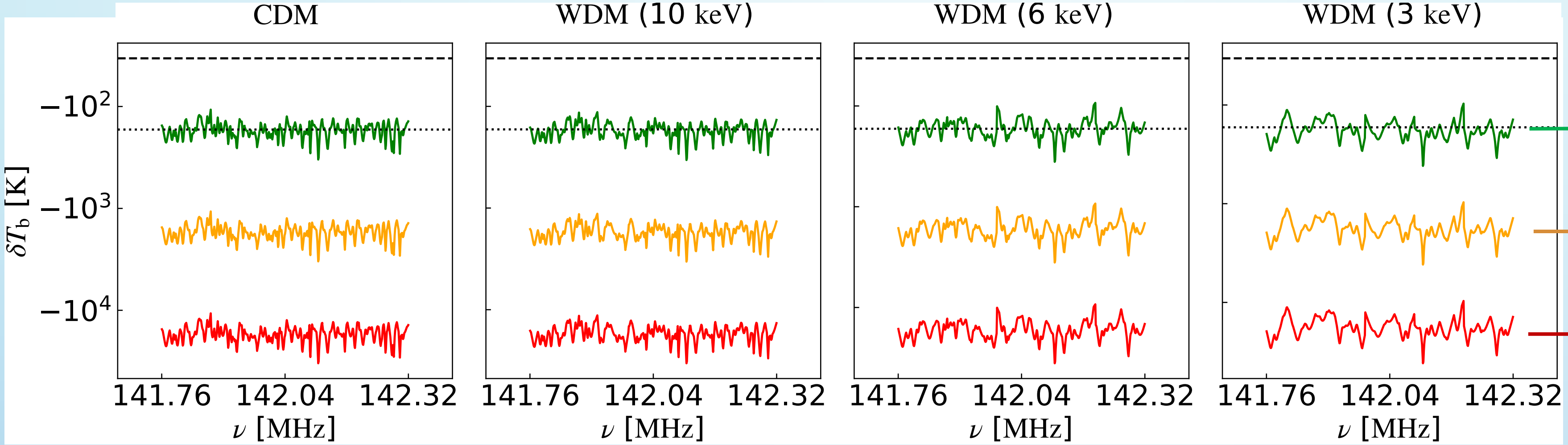

Shimabukuro et al. 2014

→ the neutrino mass,
→ the running spectral index,
→ the relative velocity between dark matter and baryons




Shimabukuro et al. 2023

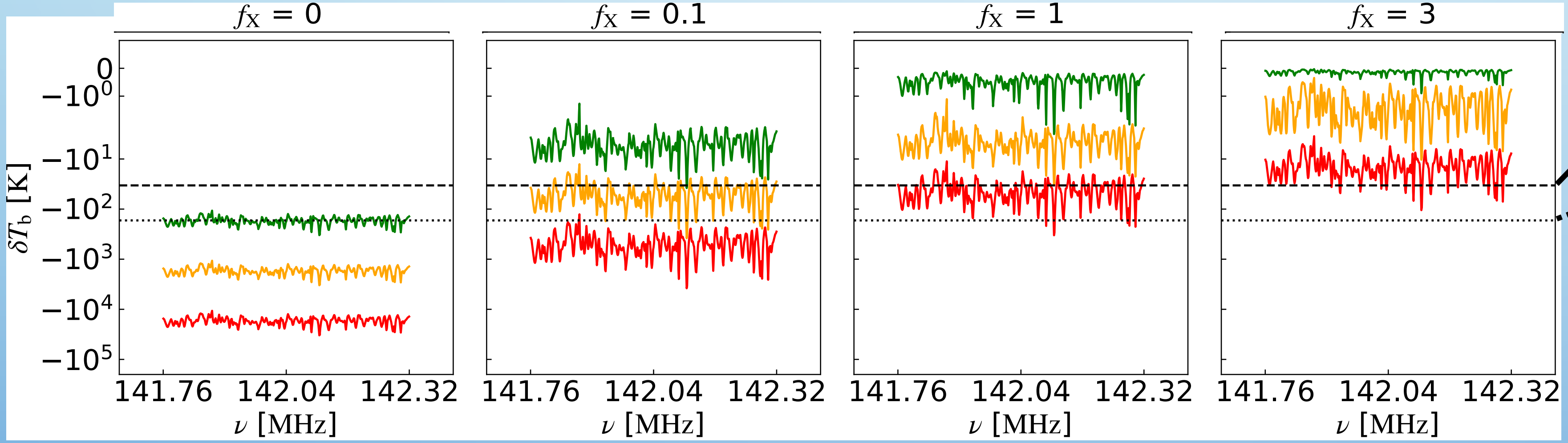
# The mock 21-cm signals




$S_{150}$ = 1 mJy

$S_{150}$ = 10 mJy

$S_{150}$ = 100 mJy




$A_{eff}/T_{sys}$ =

SKA2-Low 4000 m² K⁻¹

SKA1-Low 800 m² K⁻¹

100 hr, 1 kHz

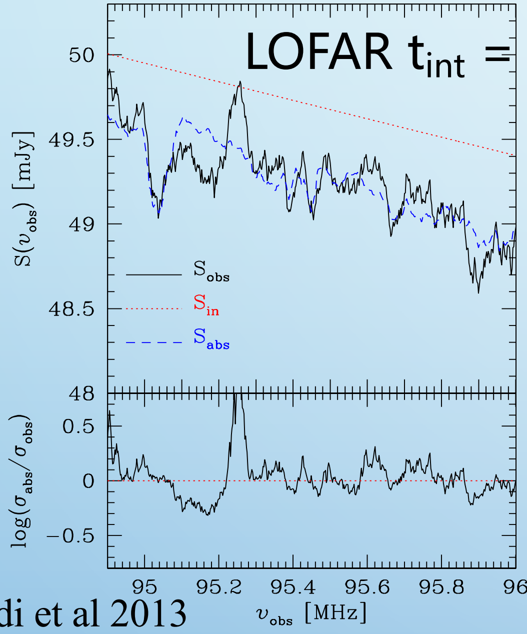
$$\delta T^{\rm N} = \delta S^{\rm N}\left(\frac{\lambda_z^2}{2k_{\rm B}\Omega}\right) \approx \frac{\lambda_z^2 T_{\rm sys}}{A_{\rm eff}\Omega\sqrt{2\delta\nu\delta t}},$$

# 21-cm Forest: observational challenges

- Probing thermal history ⟺ easily suppressed (**weak**)

$$\tau \propto \frac{1}{T_S}$$

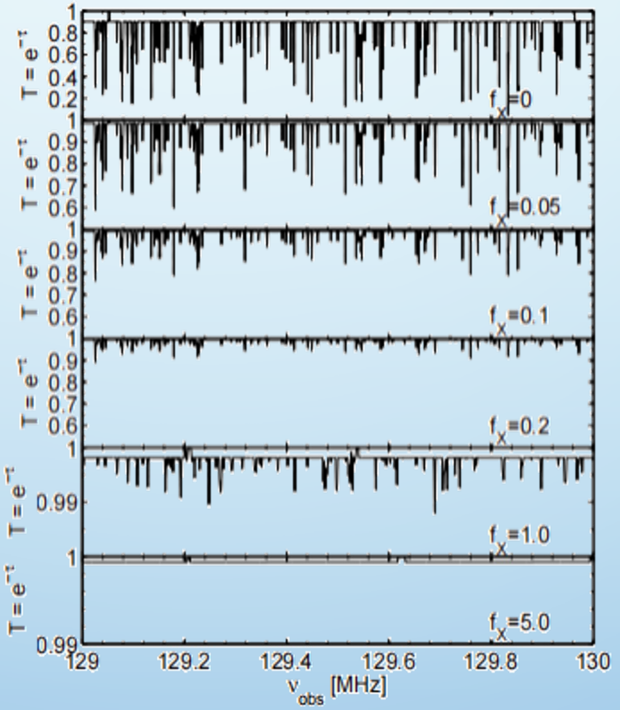
LOFAR $t_{int}$ = 1000 hr



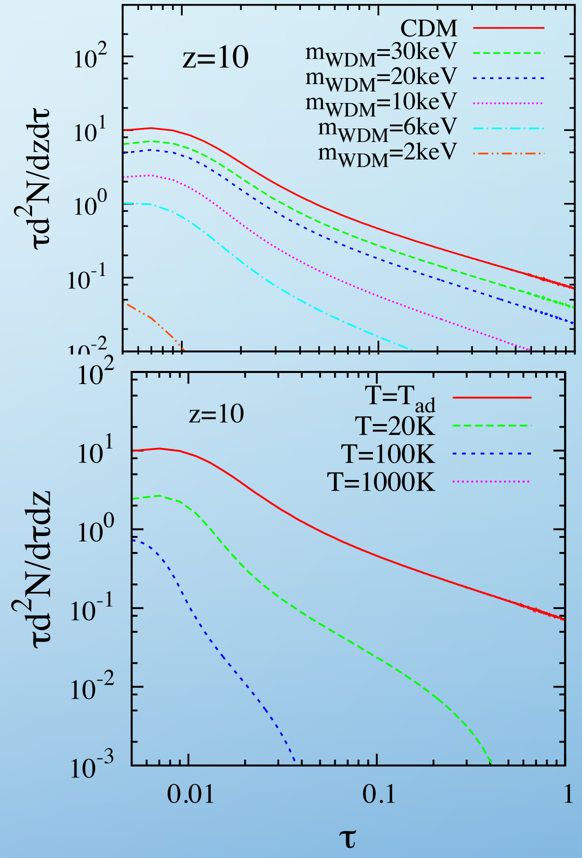

Ciardi et al 2013

**Figure 13.** *Upper panel:* Spectrum of a source positioned at $z = 14$ (i.e. $\nu \sim 95$ MHz), with an index of the power-law $\alpha = 1.05$ and a flux density $S_{in}(z_s) = 50$ mJy. The lines are the same as those in Figure 10. Here we have assumed the noise $\sigma_n$ given in eq. 3, a bandwidth $\Delta\nu = 20$ kHz, smoothing over a scale $s = 20$ kHz, and an integration time $t_{int} = 1000$ h. The IGM absorption is calculated from the reference simulation $\mathcal{L}4.39$.

- Constraining DM: **degenerate** with astrophysics




Xu YD et al. 2011




Shimabukuro et al. 2014

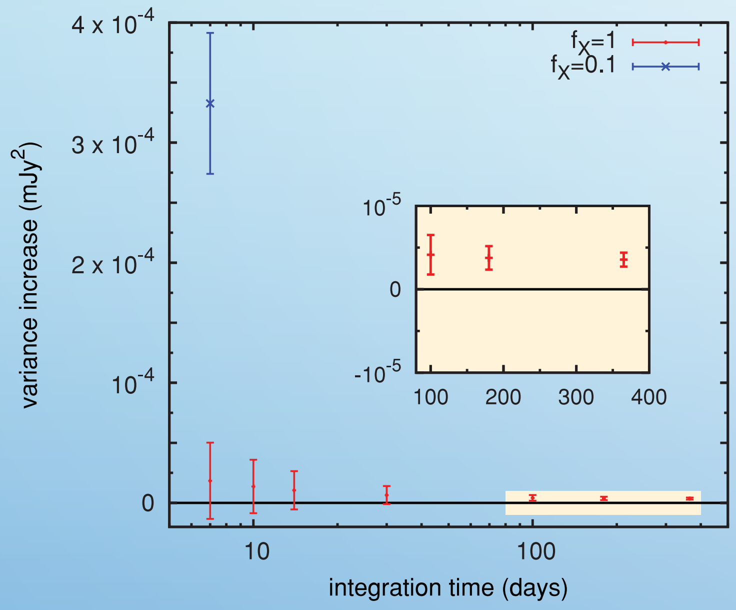
# Sensitivity challenge → statistical observables

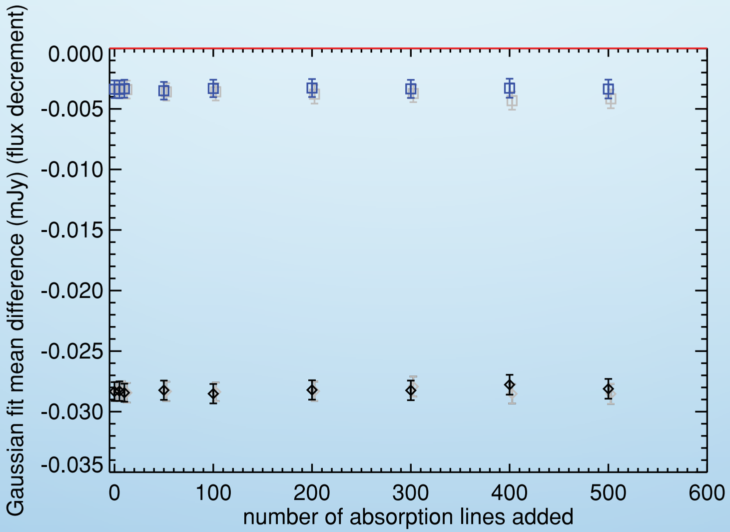
- statistical observables to enhance the sensitivity of measurements with a reasonable observation time

Increasement in mean variance

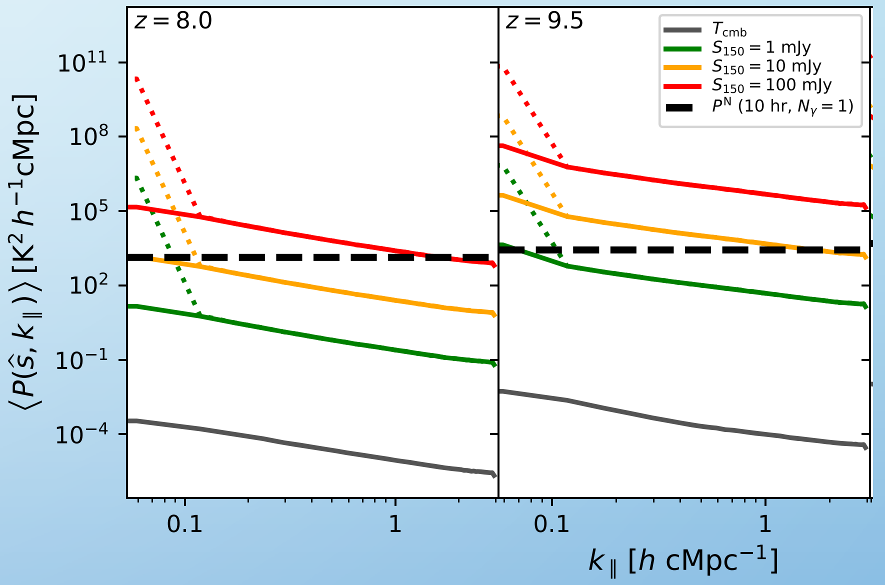
Mean difference in flux decrement

1-D power spectrum




Mack & Wyithe, 2012




Thyagarajan 2020

# Degeneracy challenge → 1-D cross-power spectrum

- Cross-correlate two measurements to suppress the noise

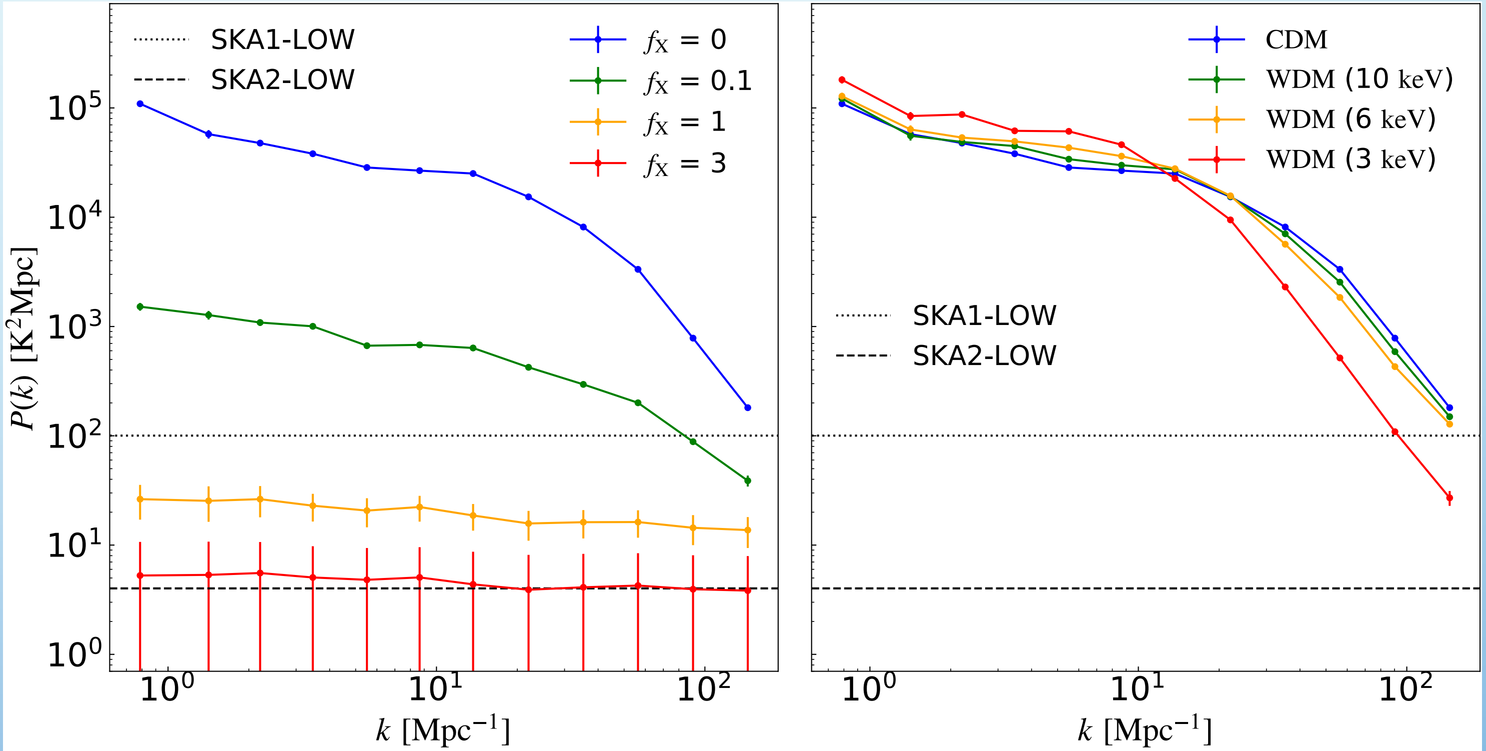
~ 10 sources with $S_{150}$ = 10 mJy at z = 9

$$P\left(\hat{\boldsymbol{s}}, k_{\parallel}\right) = \left|\delta \widetilde{T}'\left(\hat{\boldsymbol{s}}, k_{\parallel}\right)\right|^2 \left(\frac{1}{\Delta r_z}\right)$$

$$P^S = \sigma_P(k)/\sqrt{N_s \cdot N_m}$$

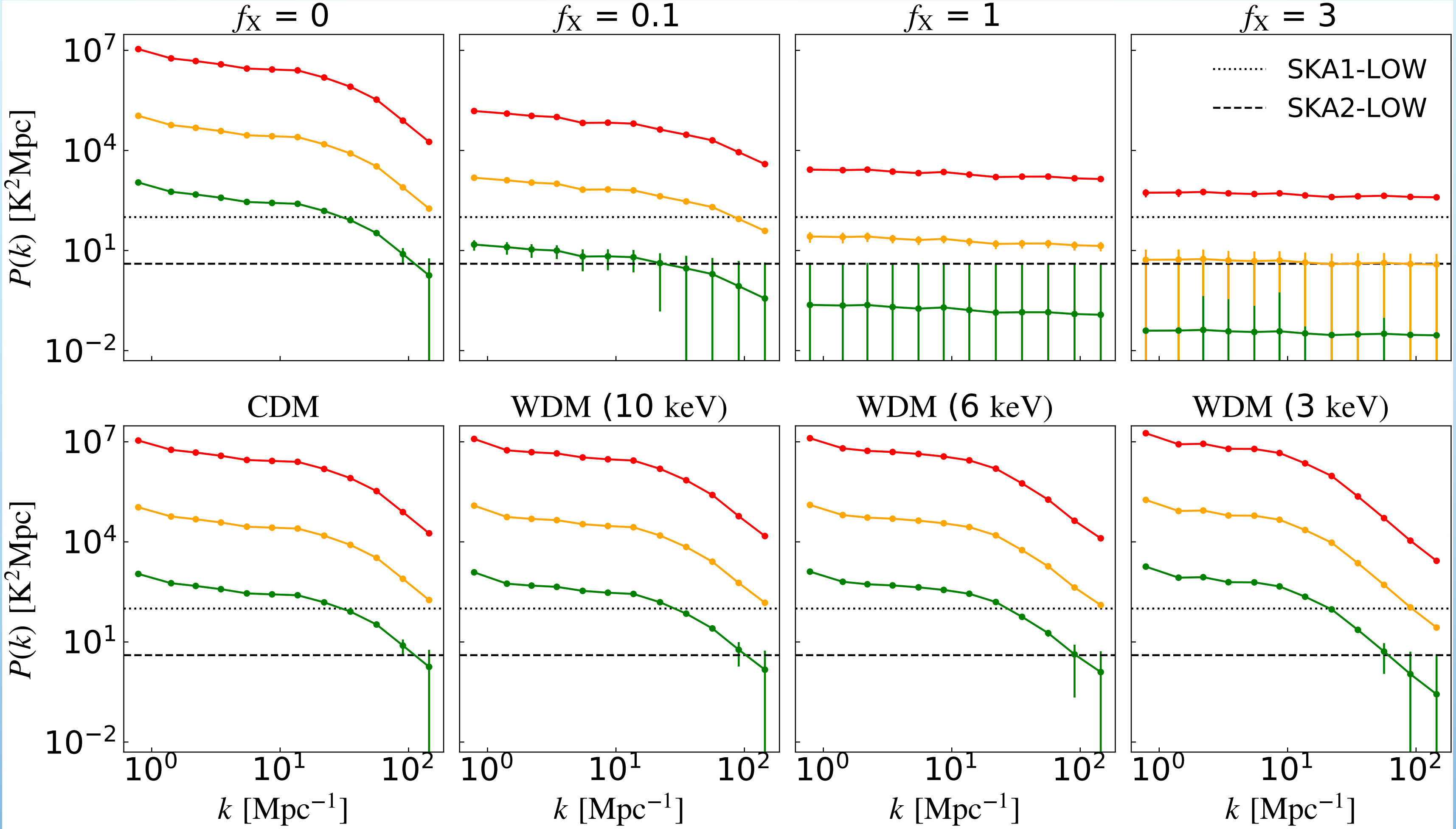
$$P^N = \frac{1}{\sqrt{N_s}}\left(\frac{\lambda_z^2 T_{\rm sys}}{A_{\rm eff}\Omega}\right)^2\left(\frac{\Delta r_z}{2\Delta\nu_z \delta t_{0.5}}\right)$$

$t_{int}$ = 2 * 50 hr




Shao Y., XuYD, et al. 2023 NA

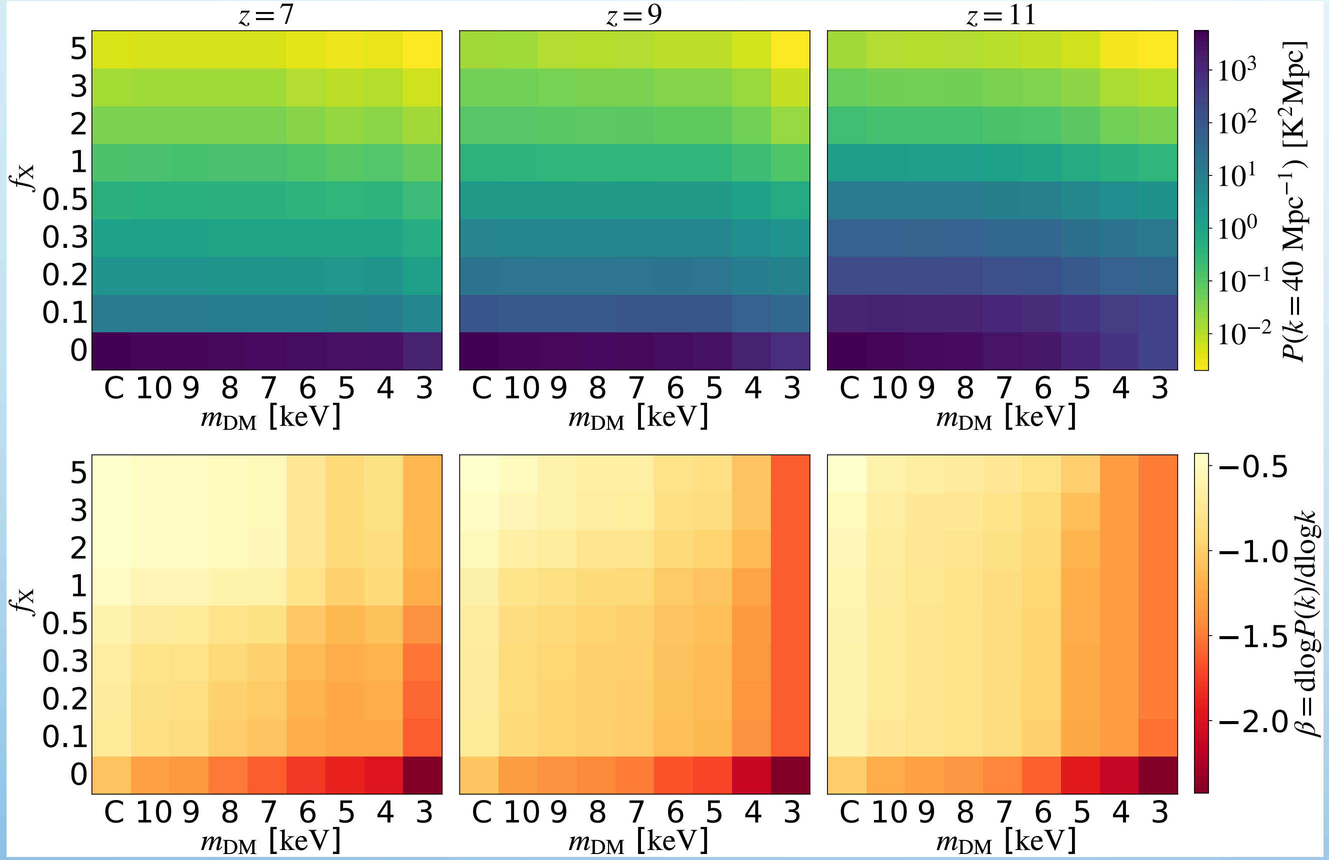
# 1-D cross-power spectrum



# 1-D cross-power spectrum ➔ Two birds with one stone

Amplitude

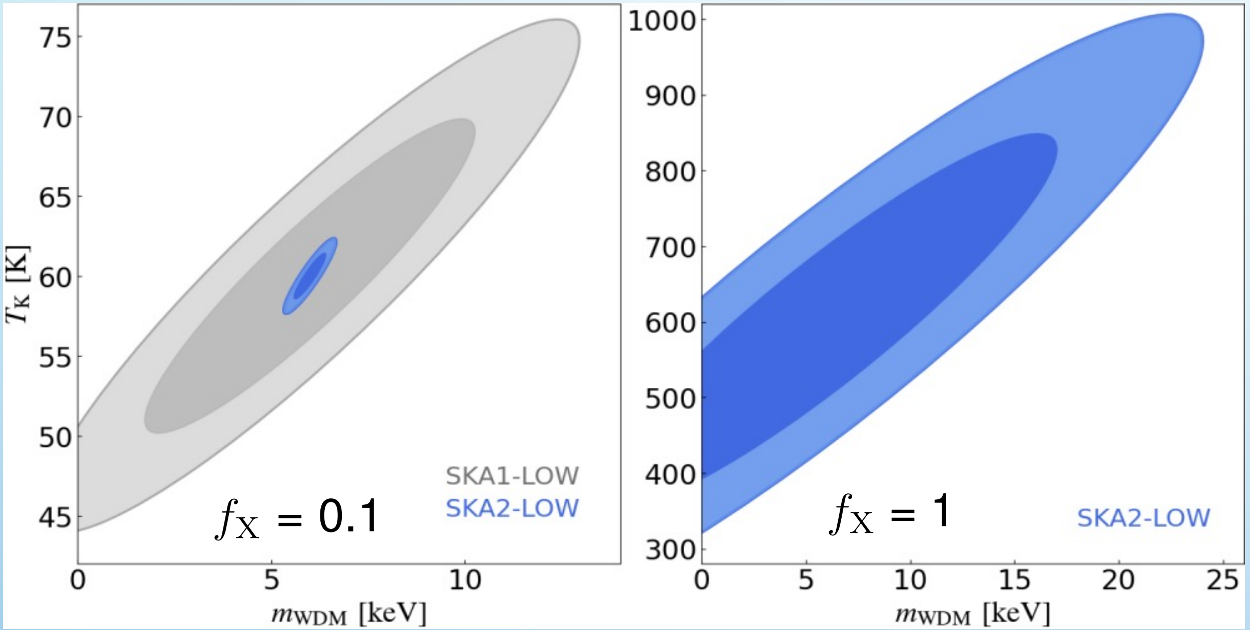
Slope

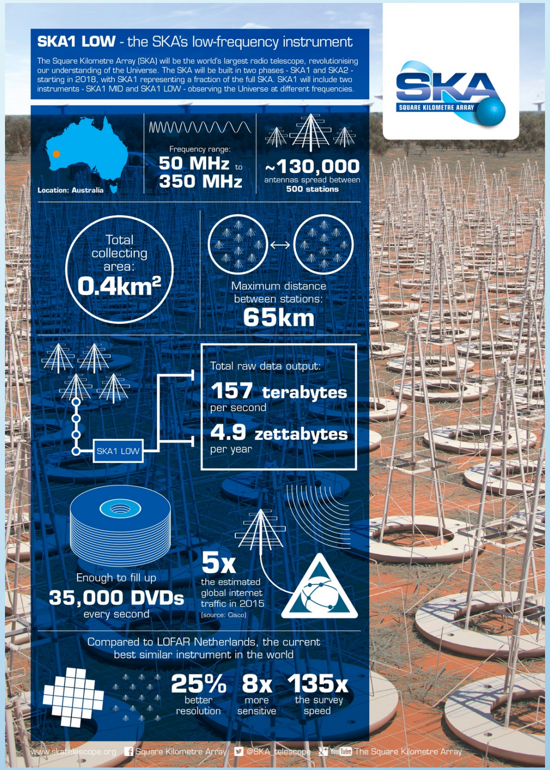



- Scientifically:
  1. DM particle mass
  2. Cosmic thermal history
- Technologically:
  1. Increase the sensitivity → feasible
  2. Breaking the degeneracy → simultaneous constraints

# 21-cm forest: a simultaneous probe of DM & first galaxies

Using ~ 10 sources with $S_{150}$ = 10 mJy at z = 9




- **FOR SKA1-LOW (800 M^2/K ):**

  $$\sigma_{m_{\mathrm{WDM}}} = 2.8\ \mathrm{keV}\ \ \text{and}\ \ \sigma_{T_{\mathrm{IGM}}} = 6.5\ \mathrm{K}$$

- **For SKA2-Low (4000 m^2/K ):**

  $$\sigma_{m_{\mathrm{WDM}}} = 0.3\ \mathrm{keV}\ \ \text{and}\ \ \sigma_{T_{\mathrm{IGM}}} = 1.0\ \mathrm{K}$$

- **For SKA2-Low:**

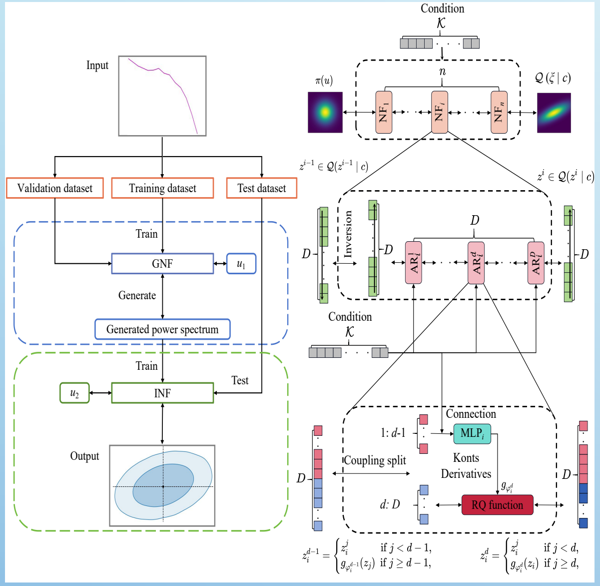
  $$\sigma_{m_{\mathrm{WDM}}} = 7.3\ \mathrm{keV}\ \ \text{and}\ \ \sigma_{T_{\mathrm{IGM}}} = 164\ \mathrm{K}$$

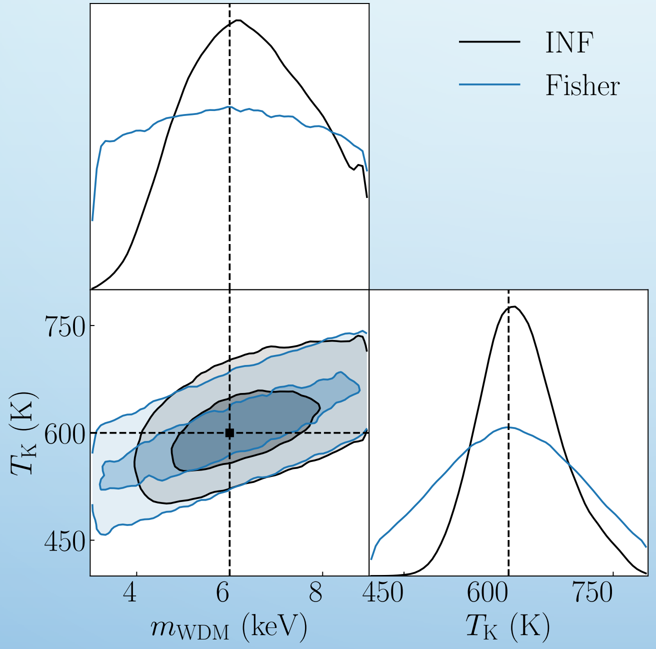
Shao, XuYD, et al. 2023 NA

# Other statistical measurement & inference tools

## Deep learning-driven inference:

Non-Gaussian information



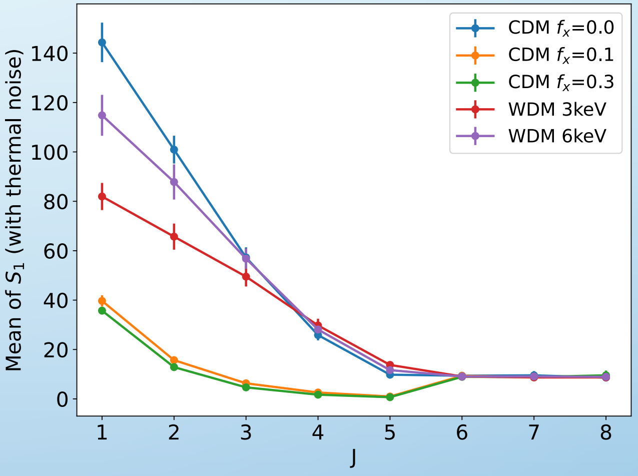



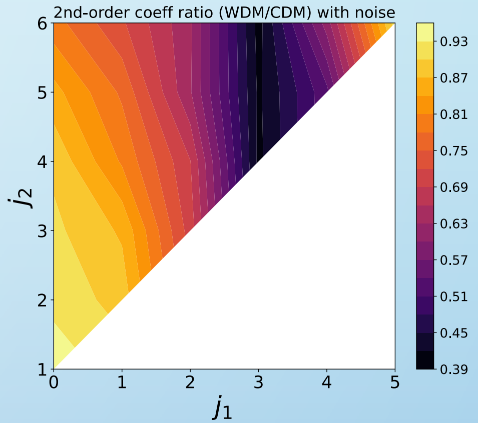

Sun et al. 2025 CP

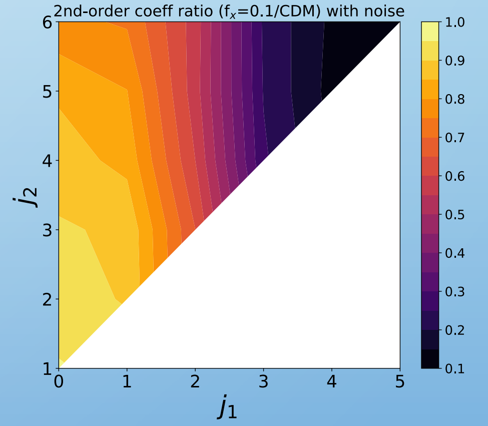
## Wavelet Scattering Transform:

Non-Gaussian information

Suppressing noise




Shimabukuro et al. 2025 submitted

# High-redshift radio sources?? Yes!

- High-z radio-loud quasars

  ~ 317 quasars discovered at redshift z≥6 (>496 at z >5.7)

  (http://www.sarahbosman.co.uk/list_of_all_quasars)

  **~ 15 radio-loud quasars at z > 6**

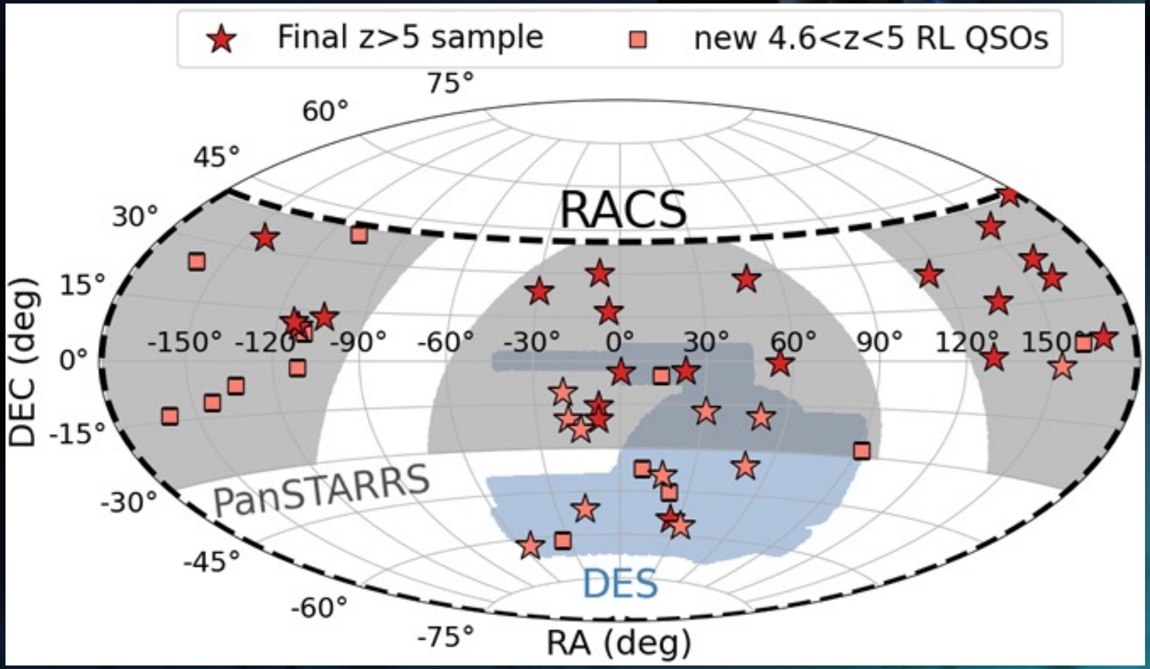
  (https://tomassoltinsky.github.io//eor/)

➔ A few hundred radio quasars with > 8 mJy at z ~ 6 are expected (Gloudemans+2021)

➔ **~ 2000 sources with > 6 mJy at 8 < z < 12** (Haiman+2004)

- Radio afterglows of high-z GRBs
  - GRB090423 at z = 8.1 (Salvaterra+2009)
  - GRB090429B at z = 9.4 (Cucchiara+2011)

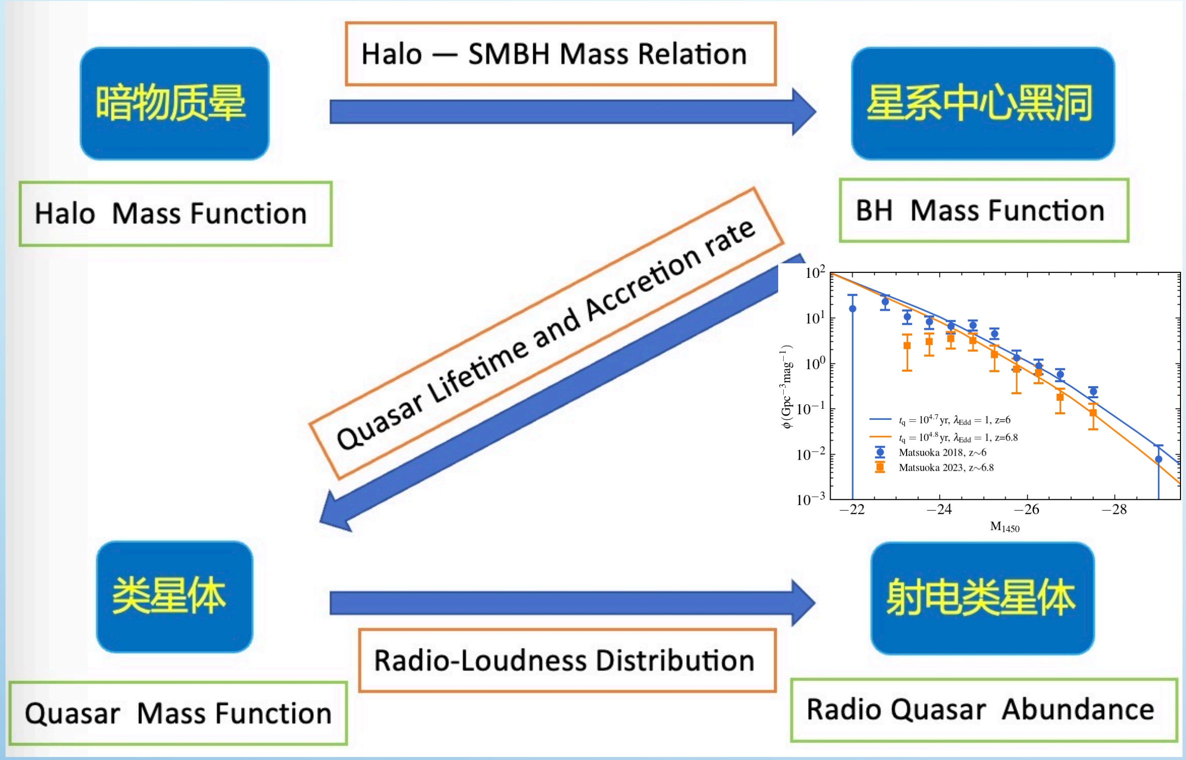
➔ The expected detection rate of luminous GRBs from Population III stars is **3 – 20 $yr^{-1}$ at z > 8** (Kinugawa+2019)

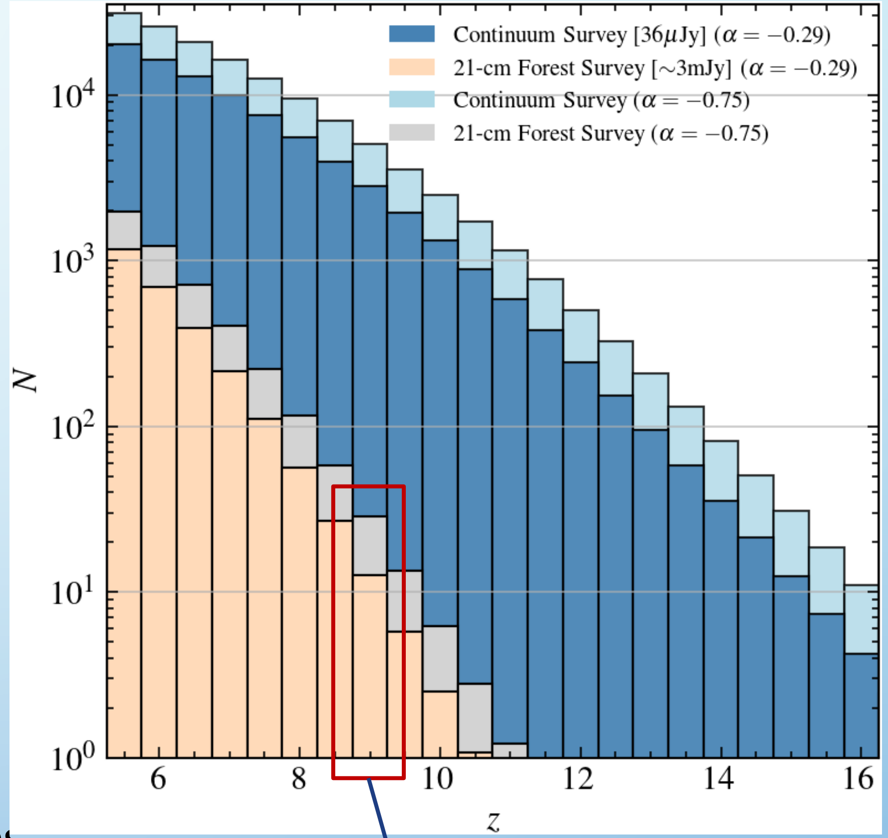



**L. Ighina, et al. 2025, arxiv: 2504.10573**

24 new high-z radio Quasars from the radio Rapid ASKAP Continuum Survey (RACS)

# Abundance of high-z radio-loud quasars



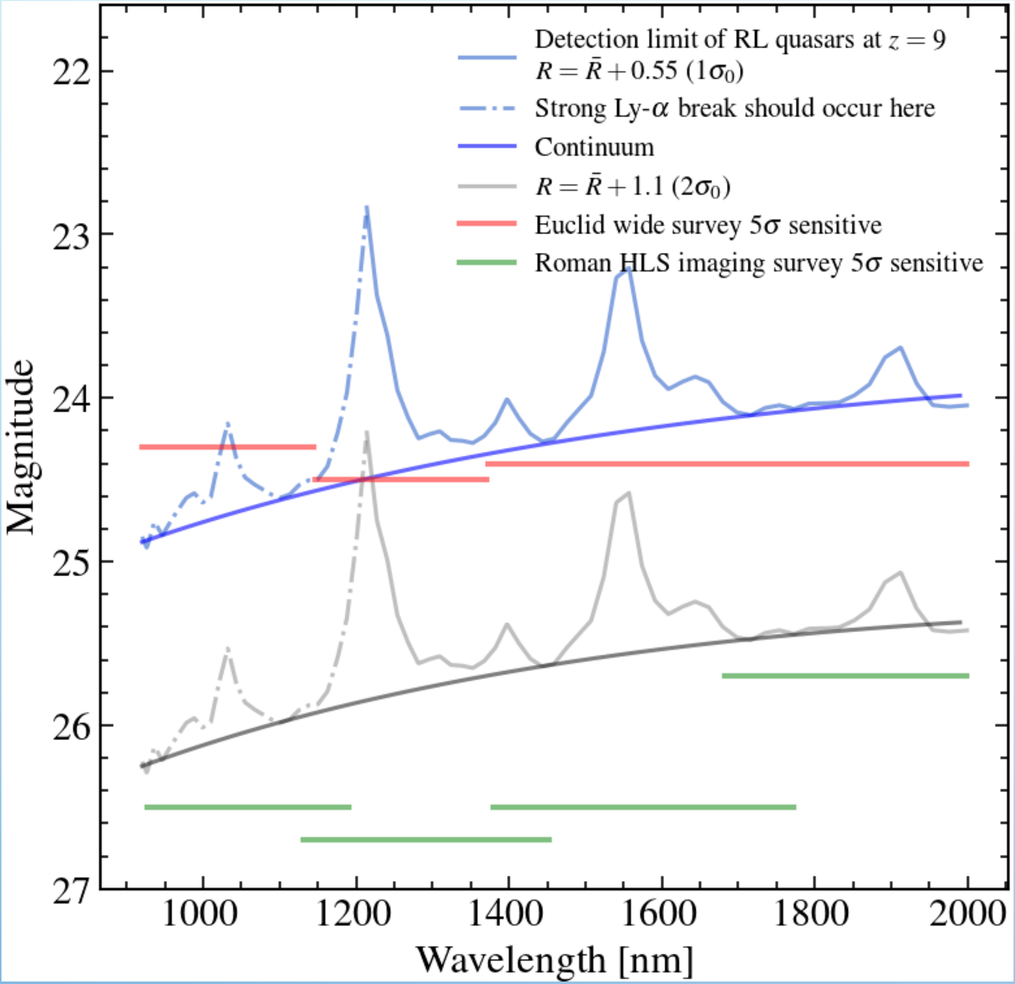

1. Continuum Detection Survey :
   - spec. resolution: 10 MHz
   - survey area : 10313 $\mathrm{deg}^2$
   - total time : 365 × 6 hr

2. Follow up resolving 21-cm lines :
   - spec. resolution: 5 kHz
   - integration time : 100 hr

~ 20 sources @ z ~ 9 for SKA1-Low

Niu Q., et al. 2025, ApJ, 978, 145

# Preparing for 21-cm forest obs. with the SKA-Low




- Why SKA-Low? Bands & sensitivity!
- Searching for high-z radio-loud quasars
  - Require coordination with Euclid or Roman
- Band: 50 - 200MHz
- Freq. resolution: 1kHz/5kHz/10kHz
- Obs. mode: deep tracking (multi-beam)
  - ~ 100 hr per source

# The 21-cm forest: a simultaneous probe of DM & first galaxies

- Physics with 21-cm forest:

1. **$T_{IGM}$ & $x_{HI}$** → Cosmic heating & reionization history → the first galaxies
2. **Small-scale structure** → fundamental physics (DM, neutrino, B-DM velocity, ...)

✓Complementary to global spectrum & 21-cm tomography

- Challenges & strategies

1. Theoretical challenge → hybrid modeling & simulation
2. Observational challenges – statistical measurement (1-D power spectrum, WST, ...)

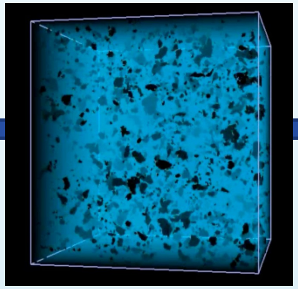
✓ Make the probe actually feasible by increasing sensitivity

✓ Constrain simultaneously DM & thermal history as it breaks the degeneracy

- Preparing for the SKA-Low

# Modeling the various structures during the CD/EoR

Global evolution



1. Global 21-cm spectrum

Large-scale structure

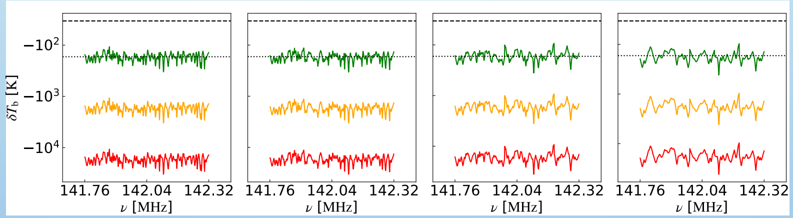
Semi-numerical simulation

2. 21-cm tomography

Small-scale structure

Analytical modeling

3. 21-cm forest




Observational constraints

Fast interpretation

# Thanks! Questions?