

The Large-Scale Structure Measurements

The summer school for 21cm cosmology

July 2nd - 15th, 2025 Xi'an, P. R. China

Yichao LI (李毅超) from Northeastern University



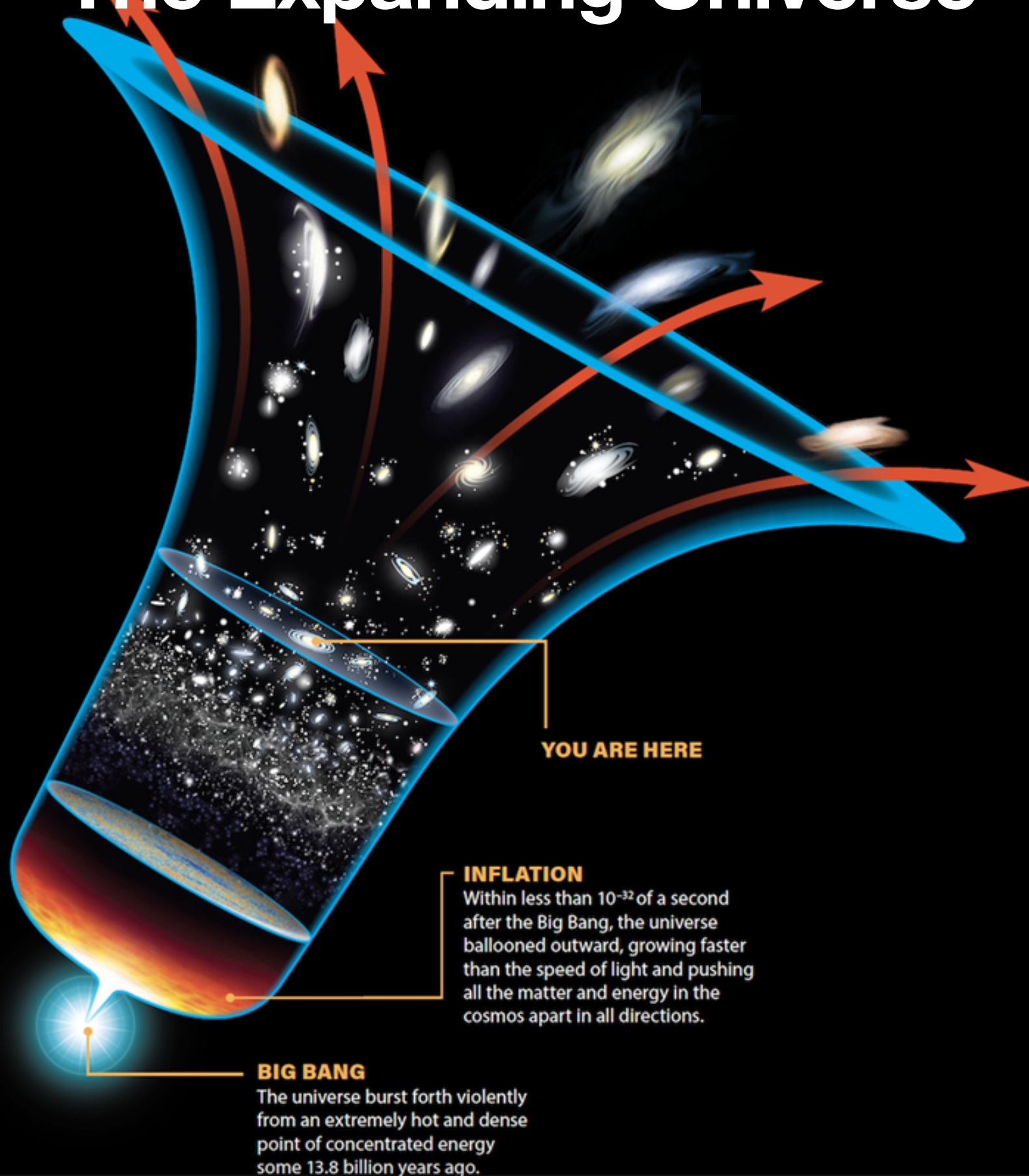
- A Brief Introduction to Large-Scale Structure
 - A pretty biased view from an observational astronomer
- Powerspectrum Estimation Basic
 - Definition of correlation function
 - Definition of power spectrum
 - Relation between correlation function and power spectrum
- Galaxy Survey
 - Galaxy sample
 - Selection function
 - Window function
 - Error estimation
 - Correction of Alias Effects
- HI Intensity Mapping Survey
 - Fisher Information Matrix
 - FFT
 - Cross-correlation
 - Delay power spectrum

The Expanding Universe

四方上下曰宇 古往今来曰宙

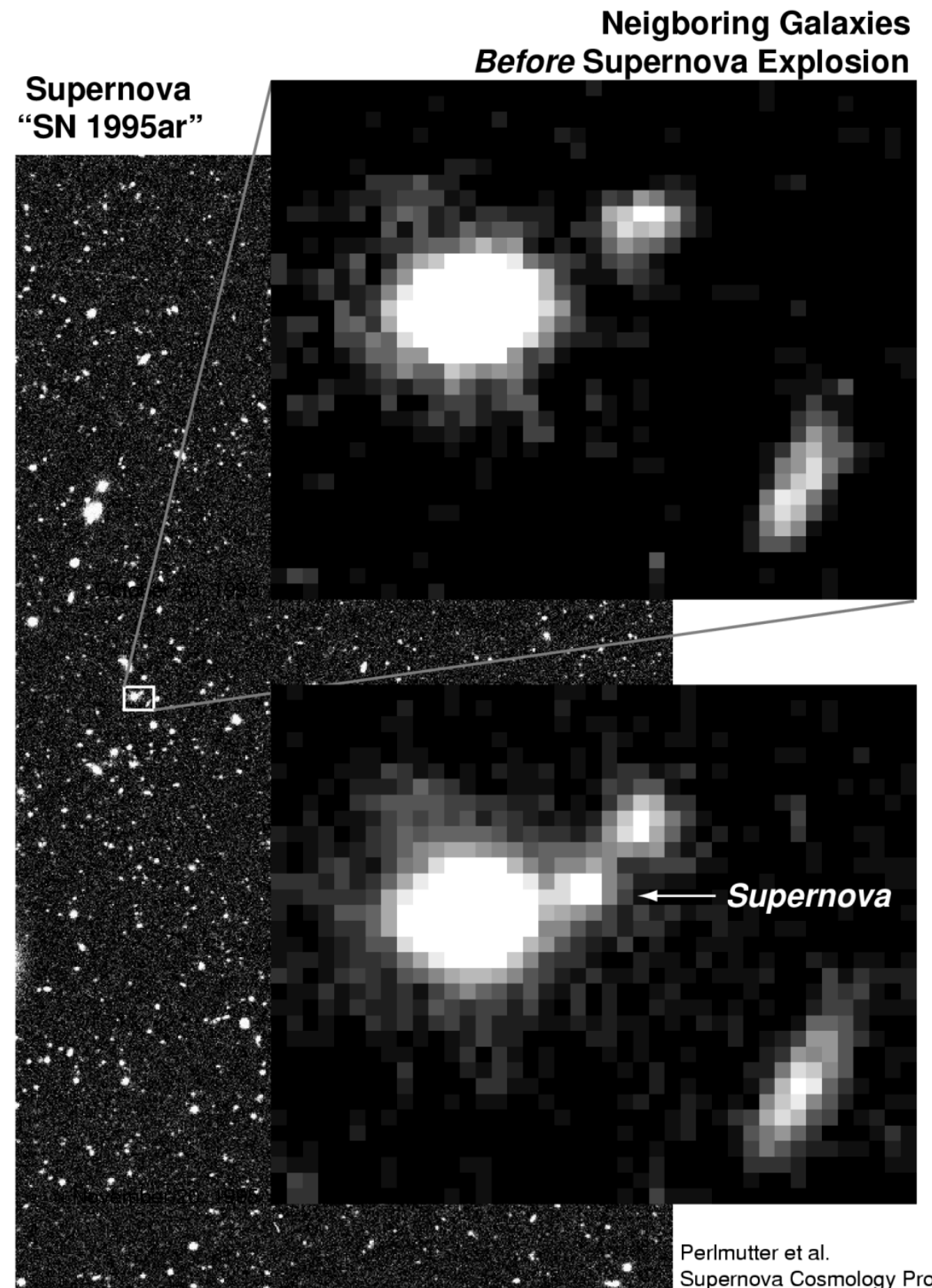
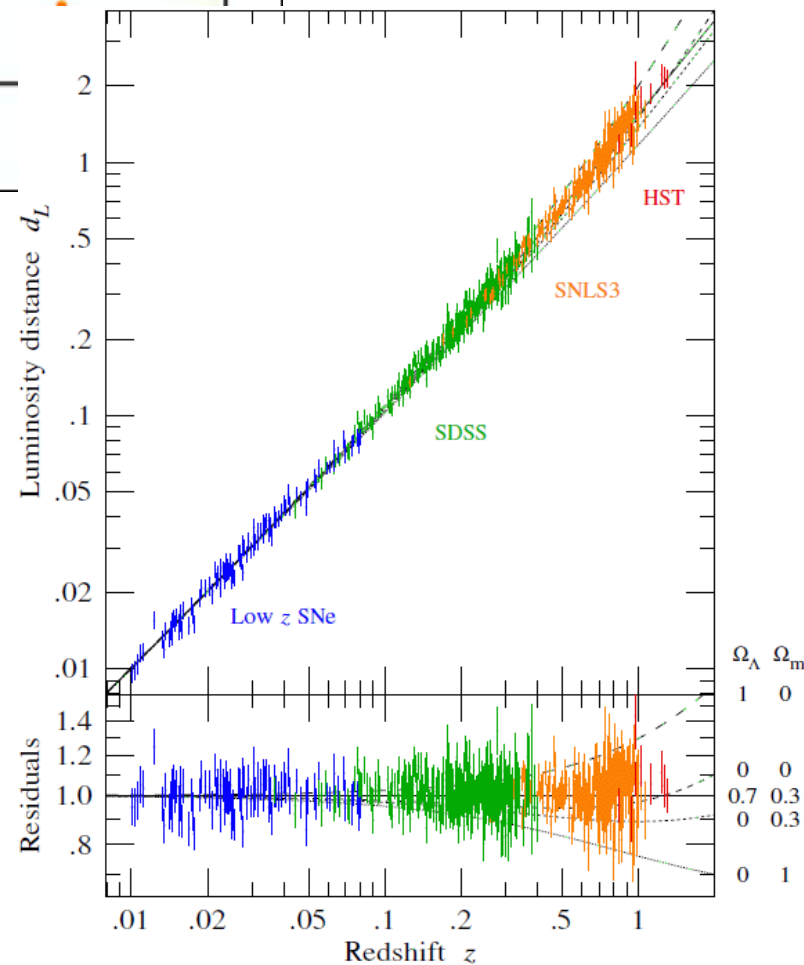
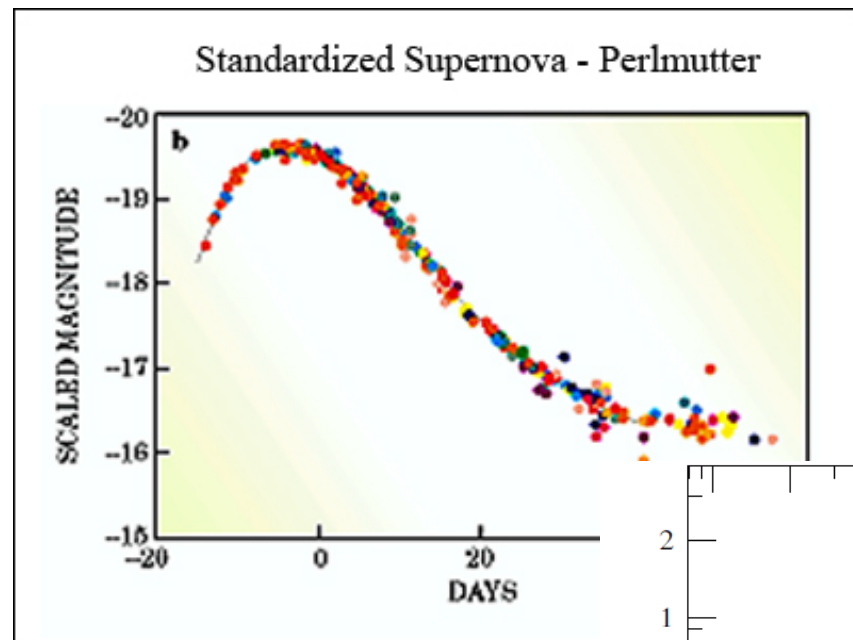
$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$



The Expanding Universe

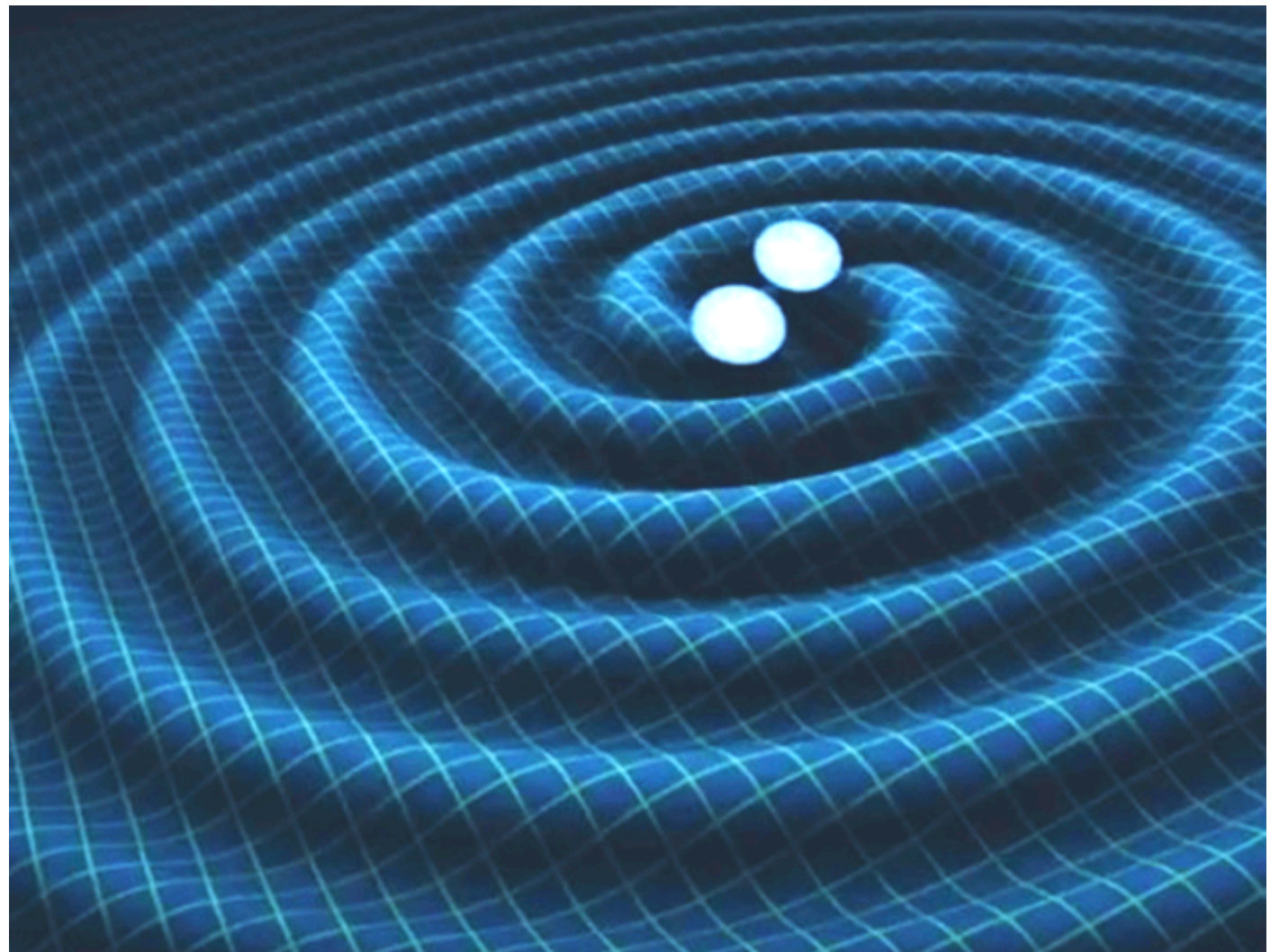
$$\mu = m_B + M_0 + \text{corr.} = 5 \log(d_L/10\text{pc}), \quad d_L = (1+z) \int_0^z \frac{cdz'}{H(z')}$$



The Expanding Universe

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

- The Standard Candle
- The Standard Sirens



The Expanding Universe

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

快速射电暴：宇宙学中的“标准琶音”

原创 张骥国、张鑫 中国科学院国家天文台 2024年07月12日 08:01 北京

- The Standard Candle
- The Standard Sirens
- The Standard Pings



PRL 115, 121301 (2015)

PHYSICAL REVIEW LETTERS

week ending
18 SEPTEMBER 2015

Dispersion Distance and the Matter Distribution of the Universe in Dispersion Space

Kiyoshi Wesley Masui

*Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia V6T 1Z1, Canada
and Canadian Institute for Advanced Research, CIFAR Program in Cosmology and Gravity,
Toronto, Ontario M5G 1Z8, Canada*

Kris Sigurdson

*Department of Physics and Astronomy, University of British Columbia,
Vancouver, British Columbia V6T 1Z1, Canada*

(Received 5 June 2015; revised manuscript received 3 August 2015; published 18 September 2015)

We propose that “standard pings,” brief broadband radio impulses, can be used to study the three-dimensional clustering of matter in the Universe even in the absence of redshift information. The dispersion of radio waves as they travel through the intervening plasma can, like redshift, be used as a cosmological distance measure. Because of inhomogeneities in the electron density along the line of sight, dispersion is an imperfect proxy for radial distance and we show that this leads to calculable dispersion-space distortions in the apparent clustering of sources. Fast radio bursts (FRBs) are a new class of radio transients that are the prototypical standard ping and, due to their high observed dispersion, have been interpreted as originating at cosmological distances. The rate of fast radio bursts has been estimated to be several thousand over the whole sky per day and, if cosmological, the sources of these events should trace the large-scale structure of the Universe. We calculate the dispersion-space power spectra for a simple model where electrons and FRBs are biased tracers of the large-scale structure of the Universe, and we show that the clustering signal could be measured using as few as 10 000 events. Such a survey is in line with what may be achieved with upcoming wide-field radio telescopes.

DOI: [10.1103/PhysRevLett.115.121301](https://doi.org/10.1103/PhysRevLett.115.121301)

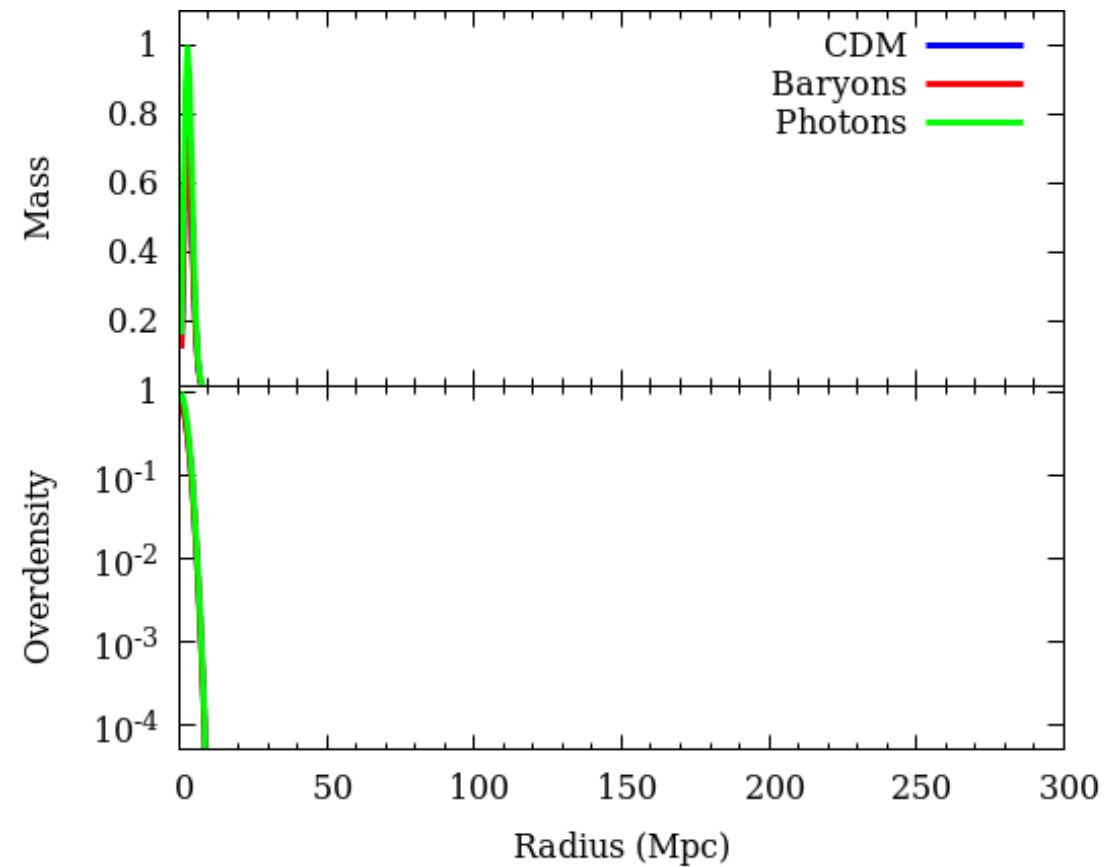
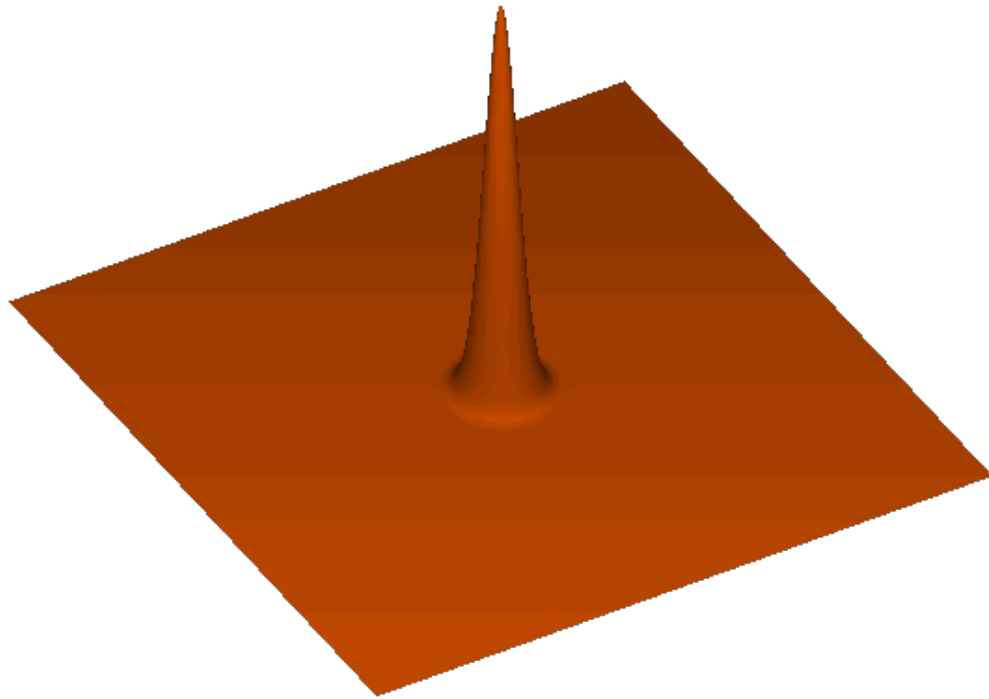
PACS numbers: 98.62.Py, 95.75.Wx, 95.85.Bh, 98.62.Ra

The Expanding Universe

$$H^2 = \left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{a^2}$$

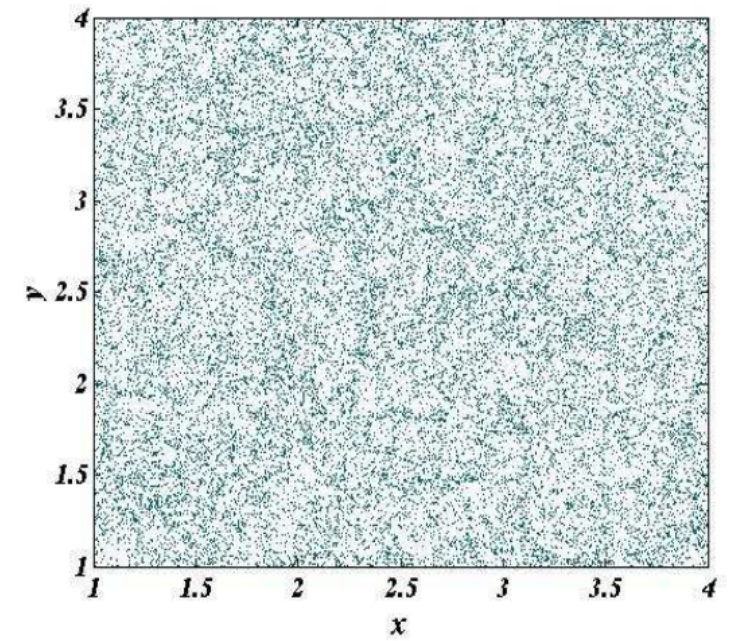
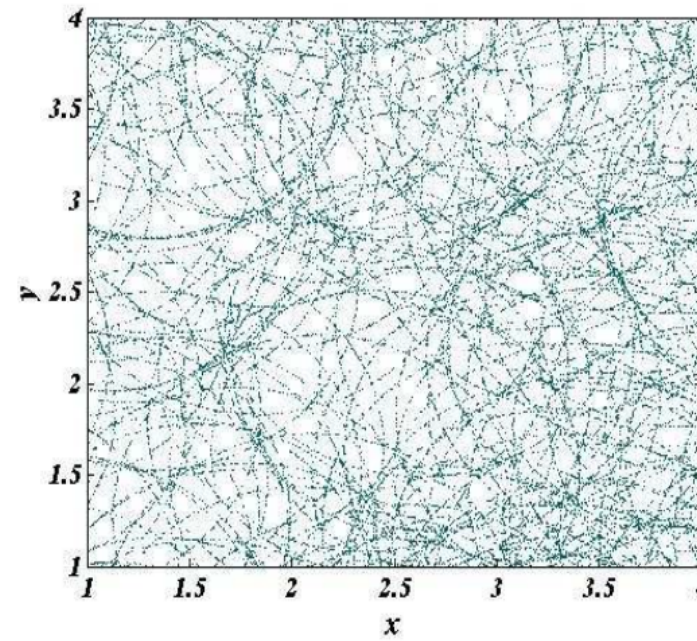
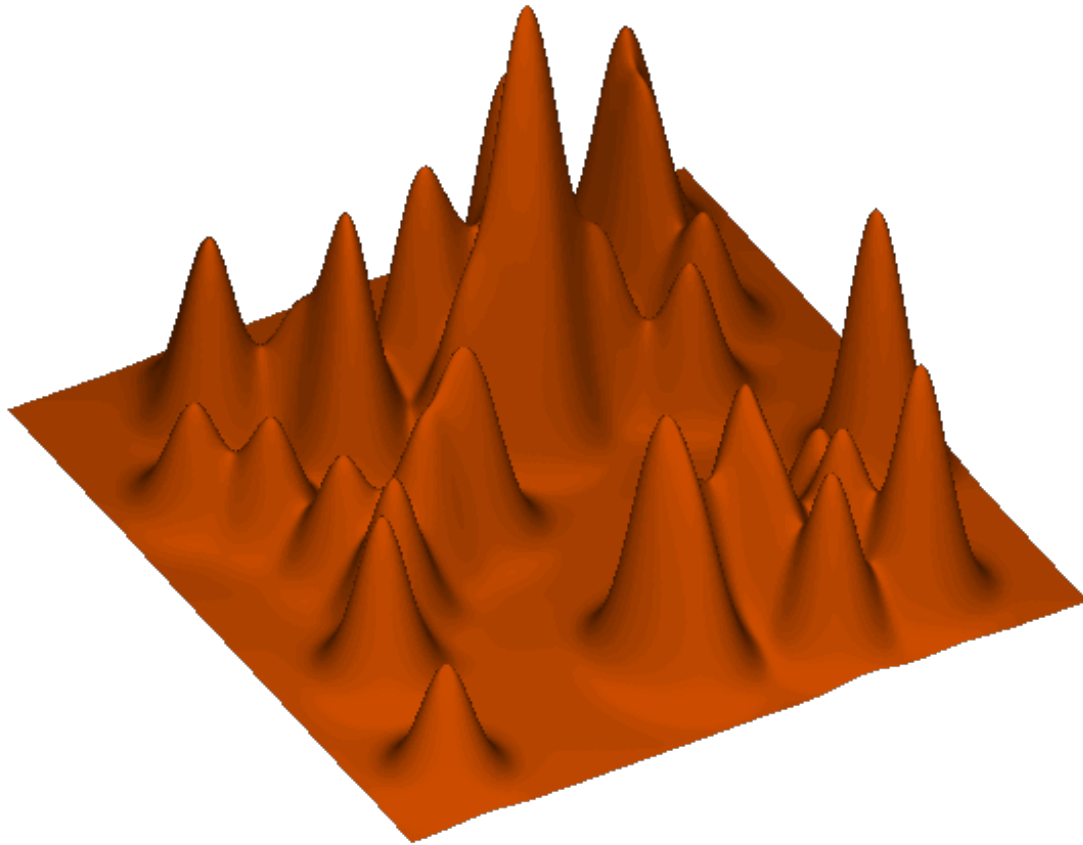
- The Standard Candle
- The Standard Sirens
- The Standard Pings
- The Standard Ruler
 - The Baryon Acoustic Oscillation (BAO)

The Baryon Acoustic Oscillation



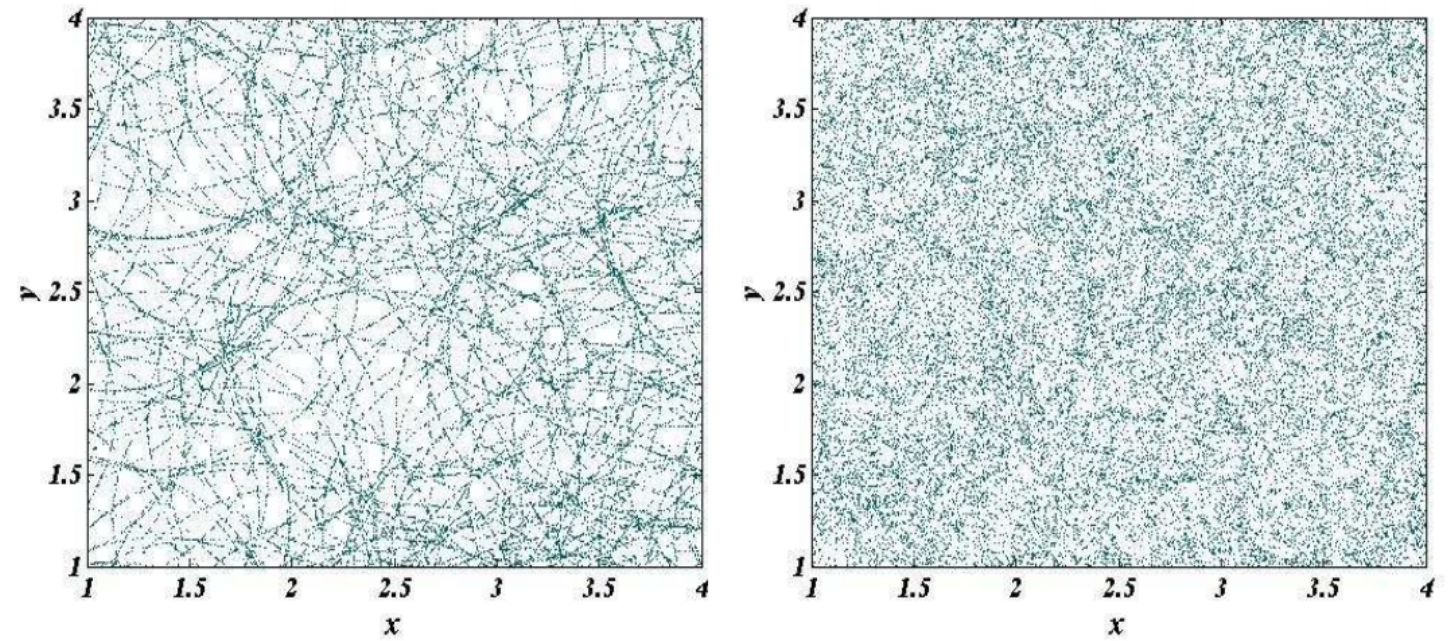
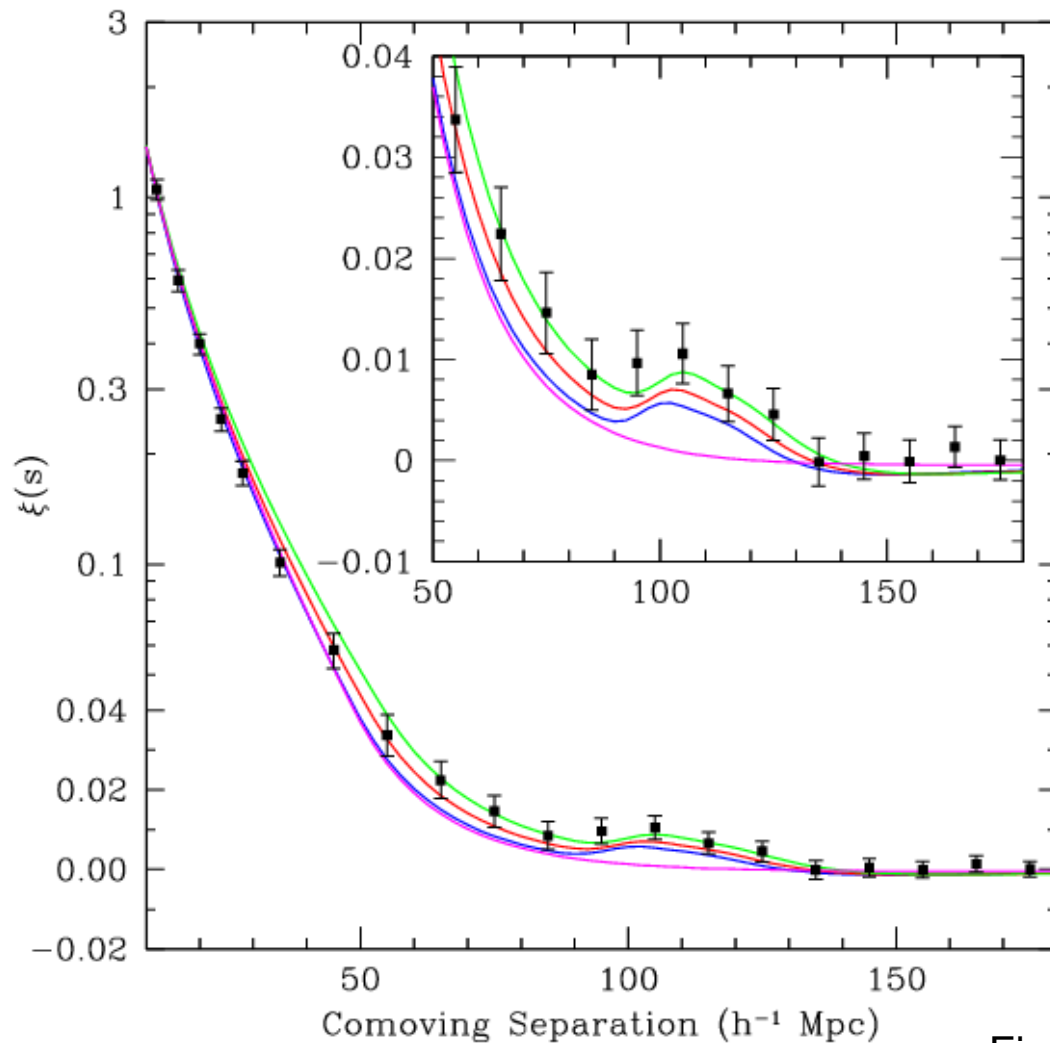
<http://galaxies-cosmology-2015.wikidot.com/baryon-acoustic-oscillations>

The Baryon Acoustic Oscillation



<http://galaxies-cosmology-2015.wikidot.com/baryon-acoustic-oscillations>

The Baryon Acoustic Oscillation



The Cosmic Large-Scale Structure

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

Eisenstein D.~J., et al., 2005, ApJ, 633, 560. doi:10.1086/466512

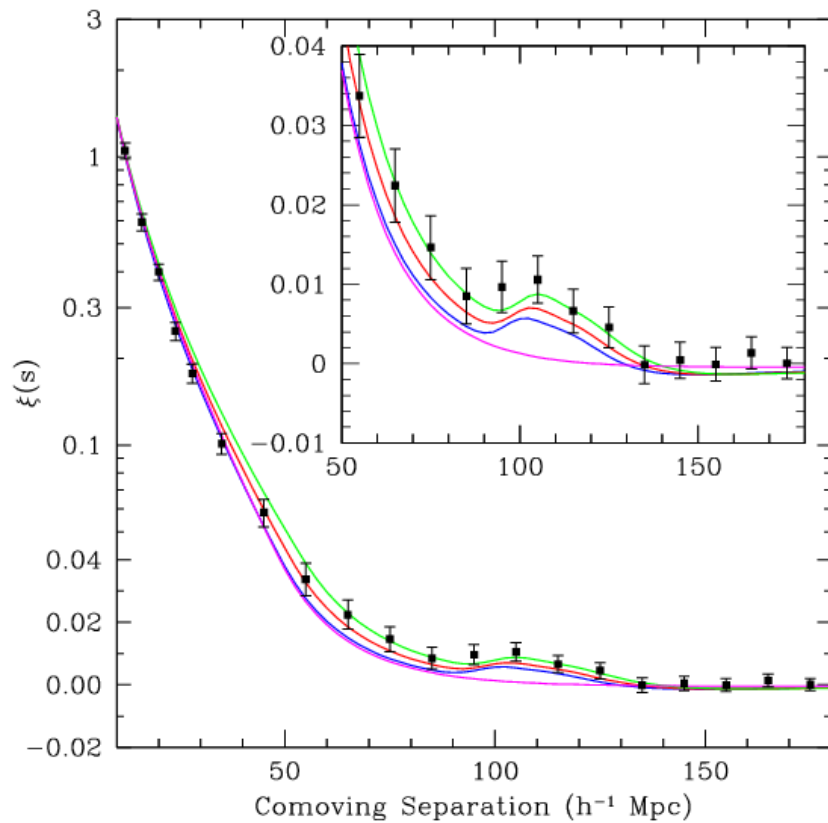
The Cosmic Large-Scale Structure

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

The Cosmic Large-Scale Structure



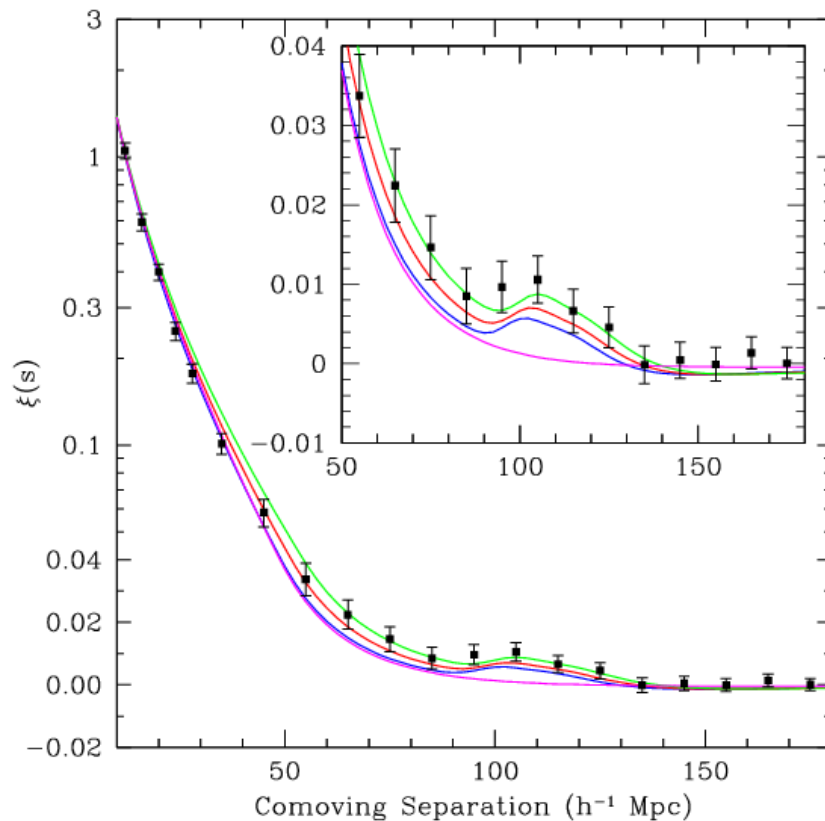
$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

Eisenstein D.~J., et al., 2005,
ApJ, 633, 560. doi:10.1086/466512

The Cosmic Large-Scale Structure



Eisenstein D.~J., et al., 2005,
ApJ, 633, 560. doi:10.1086/466512

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

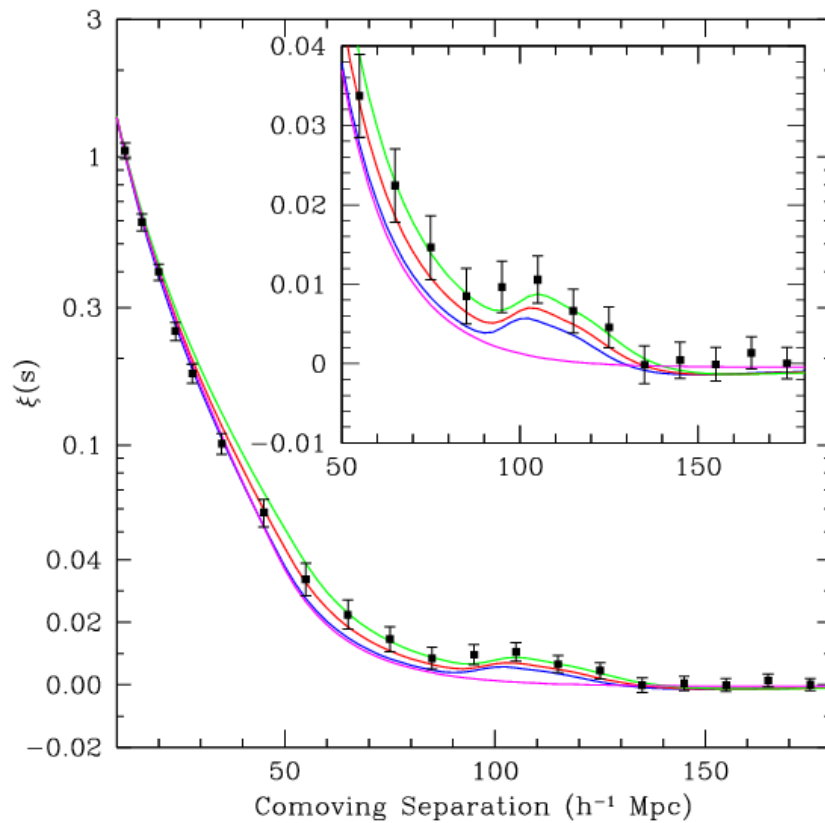
Transfer to Fourier space

$$\delta(k) = \int \frac{d^3x}{(2\pi)^{3/2}} \delta(x) e^{-ik \cdot x}$$

$$\langle \delta(k) \delta(k') \rangle = 2\pi^2 P(k) \delta_D(k + k')$$

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle = \int \frac{dk}{k} P(k) \frac{\sin(kr)}{kr}$$

The Cosmic Large-Scale Structure



Eisenstein D.~J., et al., 2005,
ApJ, 633, 560. doi:10.1086/466512

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

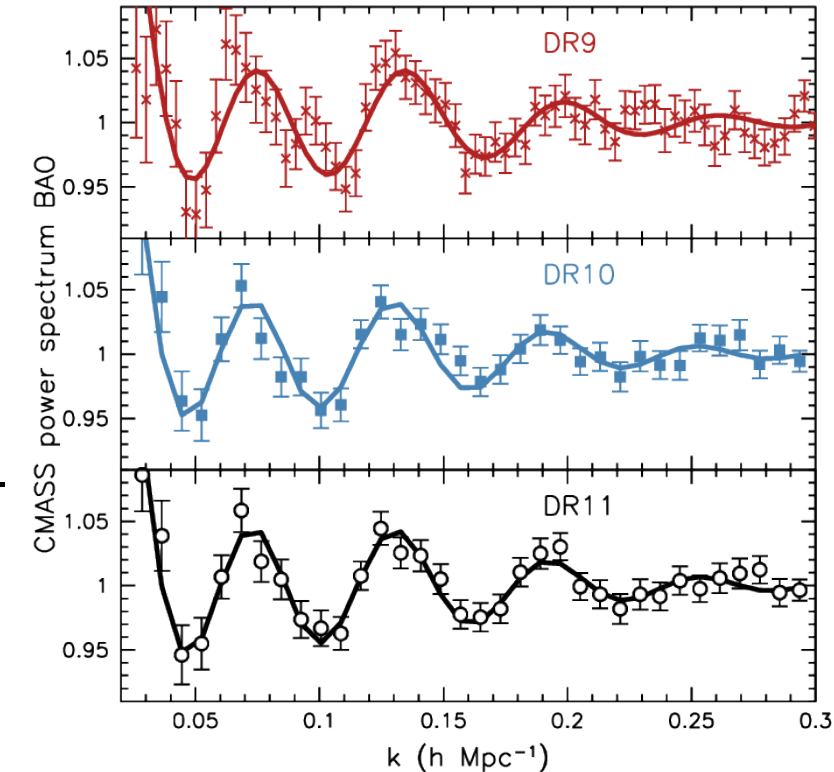
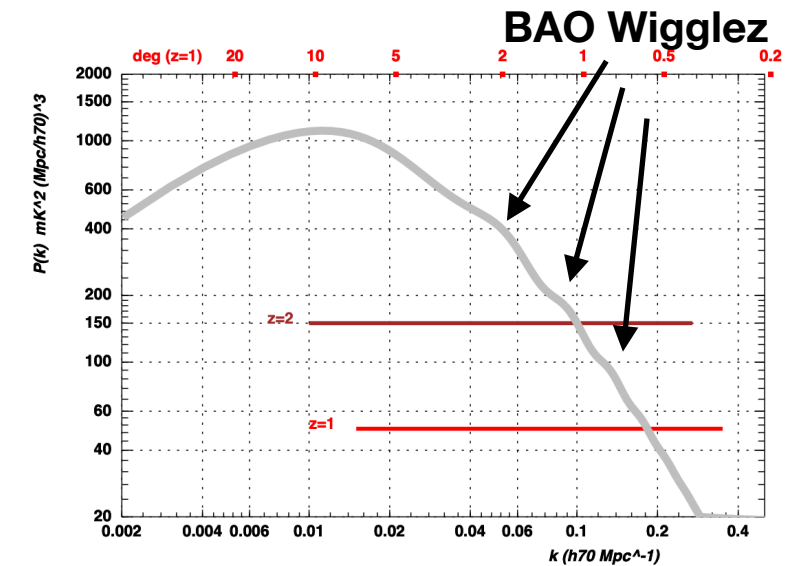
$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle$$

Transfer to Fourier space

$$\delta(k) = \int \frac{d^3x}{(2\pi)^{3/2}} \delta(x) e^{-ik \cdot x}$$

$$\langle \delta(k)\delta(k') \rangle = 2\pi^2 P(k) \delta_D(k+k')$$

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle = \int \frac{dk}{k} P(k) \frac{\sin(kr)}{kr}$$



The Cosmic Large-Scale Structure

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

$$\langle \delta(k) \delta(k') \rangle = 2\pi^2 P(k) \delta_D(k + k')$$

Why is the 2-point function (either the correlation function or the power spectrum) the statistic of choice in characterizing LSS?

The Cosmic Large-Scale Structure

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

$$\langle \delta(k) \delta(k') \rangle = 2\pi^2 P(k) \delta_D(k + k')$$

Why is the 2-point function (either the correlation function or the power spectrum) the statistic of choice in characterizing LSS?

The correlation function or power spectrum is the (co)variance of density, which is the lowest order irreducible moment after the mean. The central limit theorem implies that a density distribution is asymptotically Gaussian in the limit where the density results from the average of many independent random processes. A Gaussian is completely characterized by its mean and variance.

The Cosmic Large-Scale Structure

$$dP_{12} = \bar{n}^2 (1 + \xi_{12}(r)) dV_1 dV_2$$

The Two-point correlation function

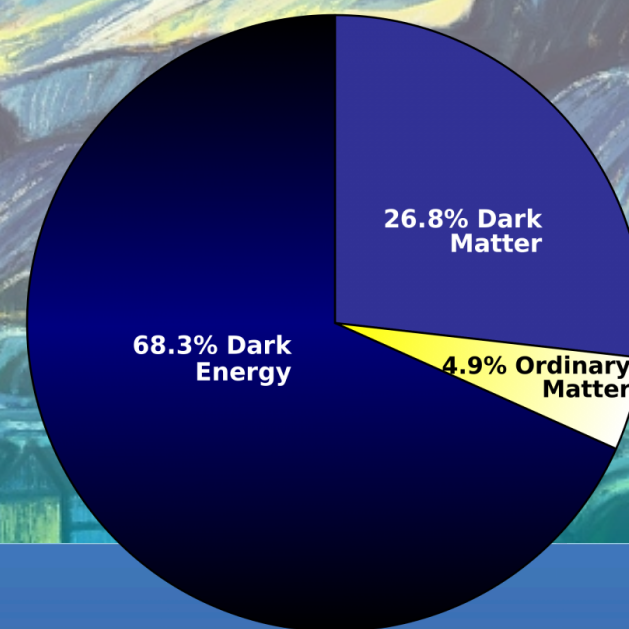
$$\xi(r) = \langle \delta(x) \delta(x + r) \rangle$$

$$\langle \delta(k) \delta(k') \rangle = 2\pi^2 P(k) \delta_D(k + k')$$

Why is the 2-point function (either the correlation function or the power spectrum) the statistic of choice in characterizing LSS?

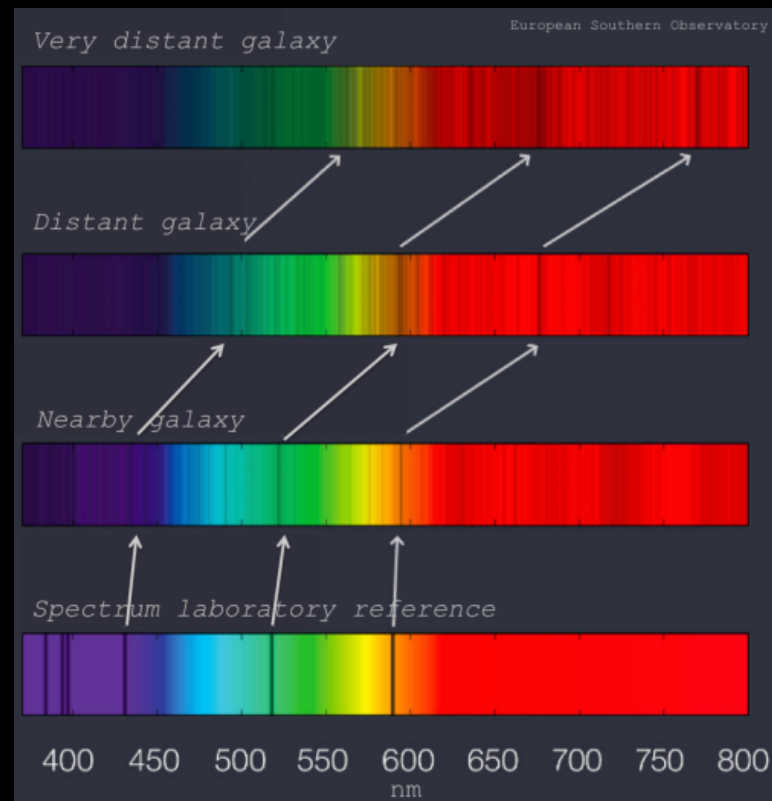
What is the advantage of the power spectrum over the correlation function?

The correlation function or power spectrum is the (co)variance of density, which is the lowest order irreducible moment after the mean. The central limit theorem implies that a density distribution is asymptotically Gaussian in the limit where the density results from the average of many independent random processes. A Gaussian is completely characterized by its mean and variance.



Heavy Elements 0.03%
Neutrinos 0.3%
Stars 0.5%
H He 4%

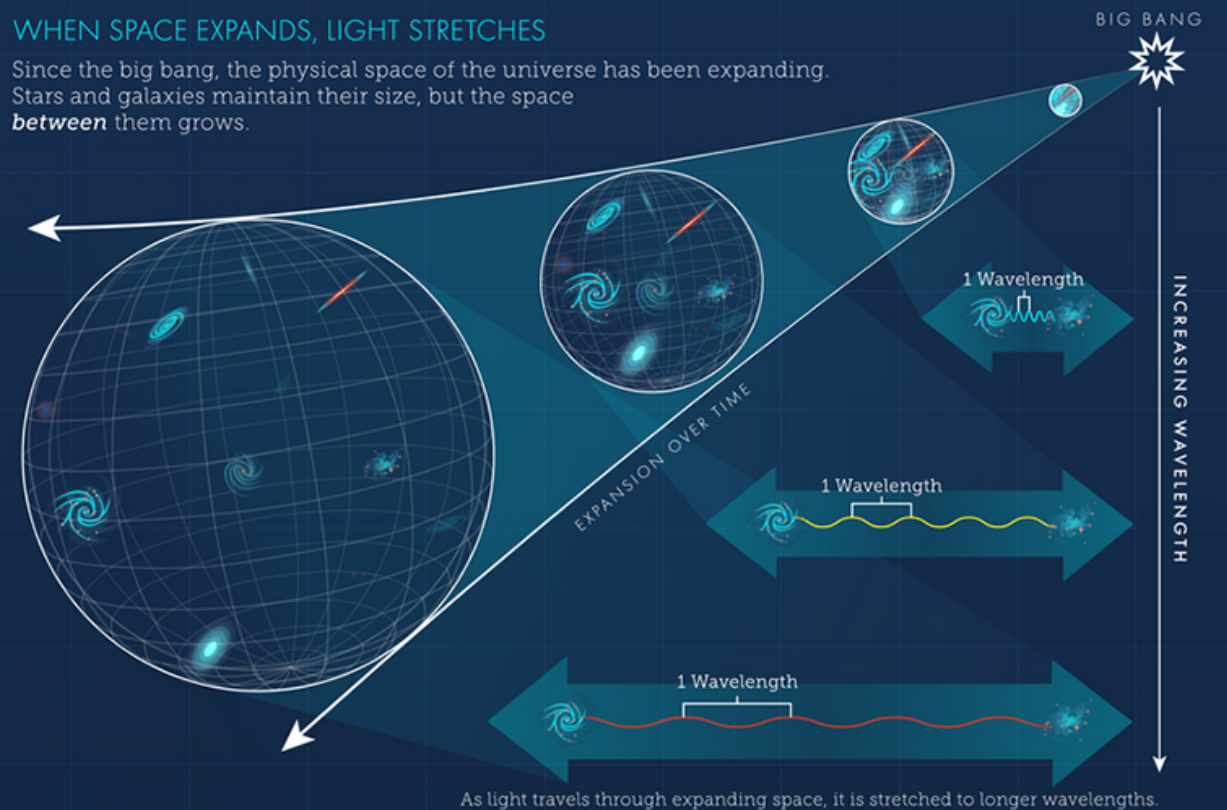
LSS Galaxy Redshift Survey



WHAT IS COSMOLOGICAL REDSHIFT?

WHEN SPACE EXPANDS, LIGHT STRETCHES

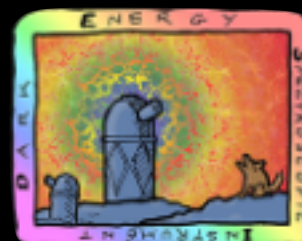
Since the big bang, the physical space of the universe has been expanding. Stars and galaxies maintain their size, but the space *between* them grows.



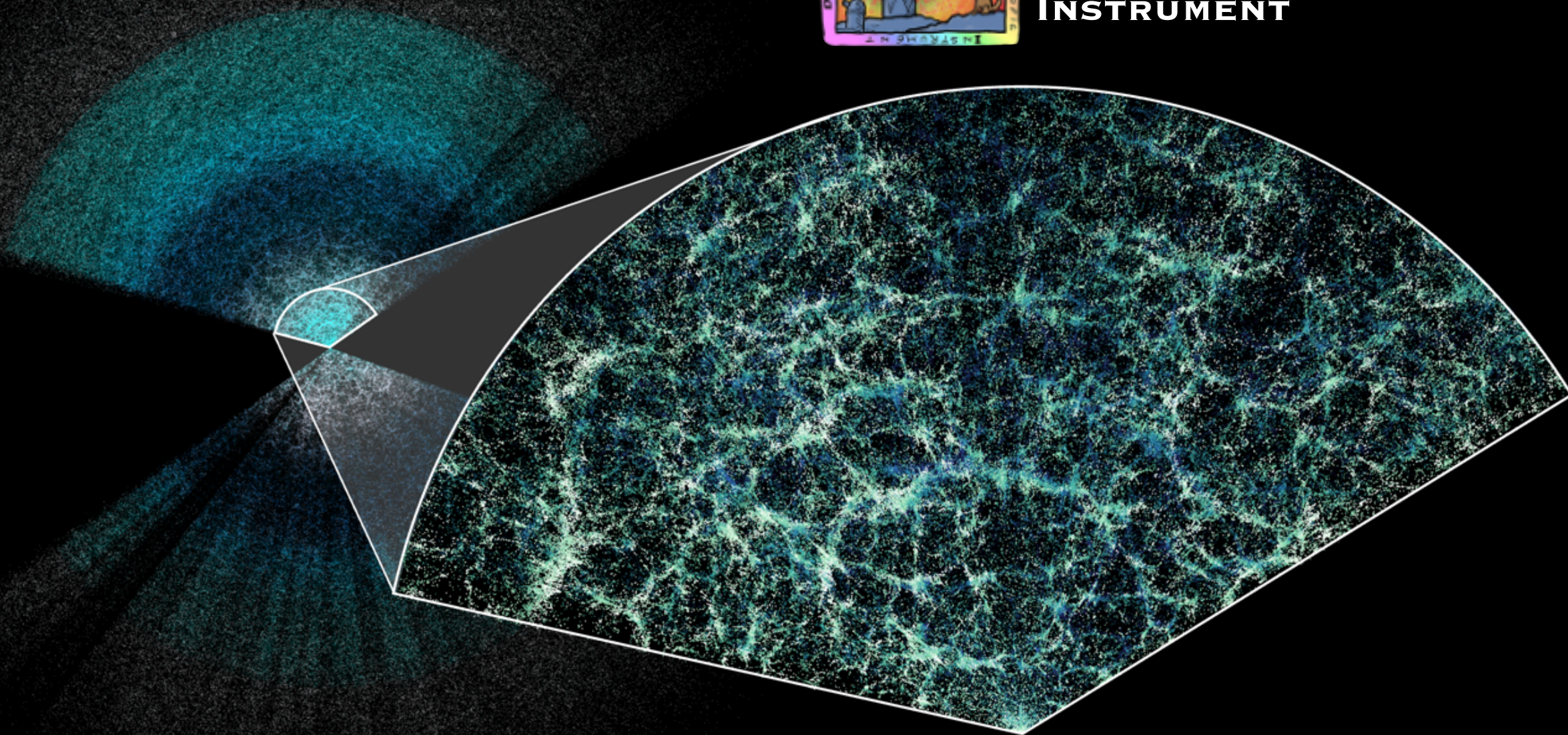
REDDER THAN RED

The longest visible wavelength is red. Beyond red are longer wavelengths that we can't see, starting with infrared. When light is stretched by the expansion of space, we say that it is **redshifted**—



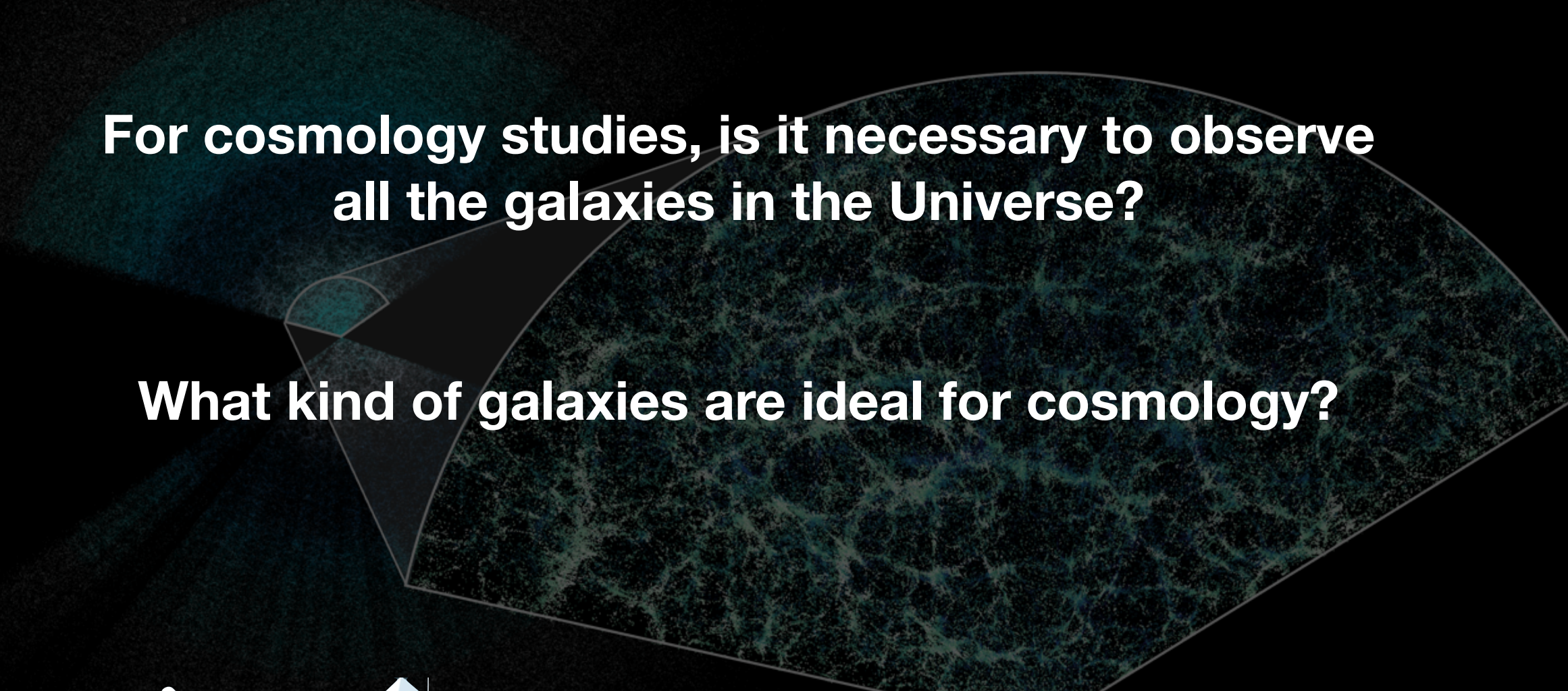


DARK ENERGY SPECTROSCOPIC INSTRUMENT



**For cosmology studies, is it necessary to observe
all the galaxies in the Universe?**



A large, semi-circular map of the Cosmic Microwave Background (CMB) fluctuations, showing a complex pattern of dark and light regions representing temperature variations across the sky. The map is tilted and partially obscured by the text.

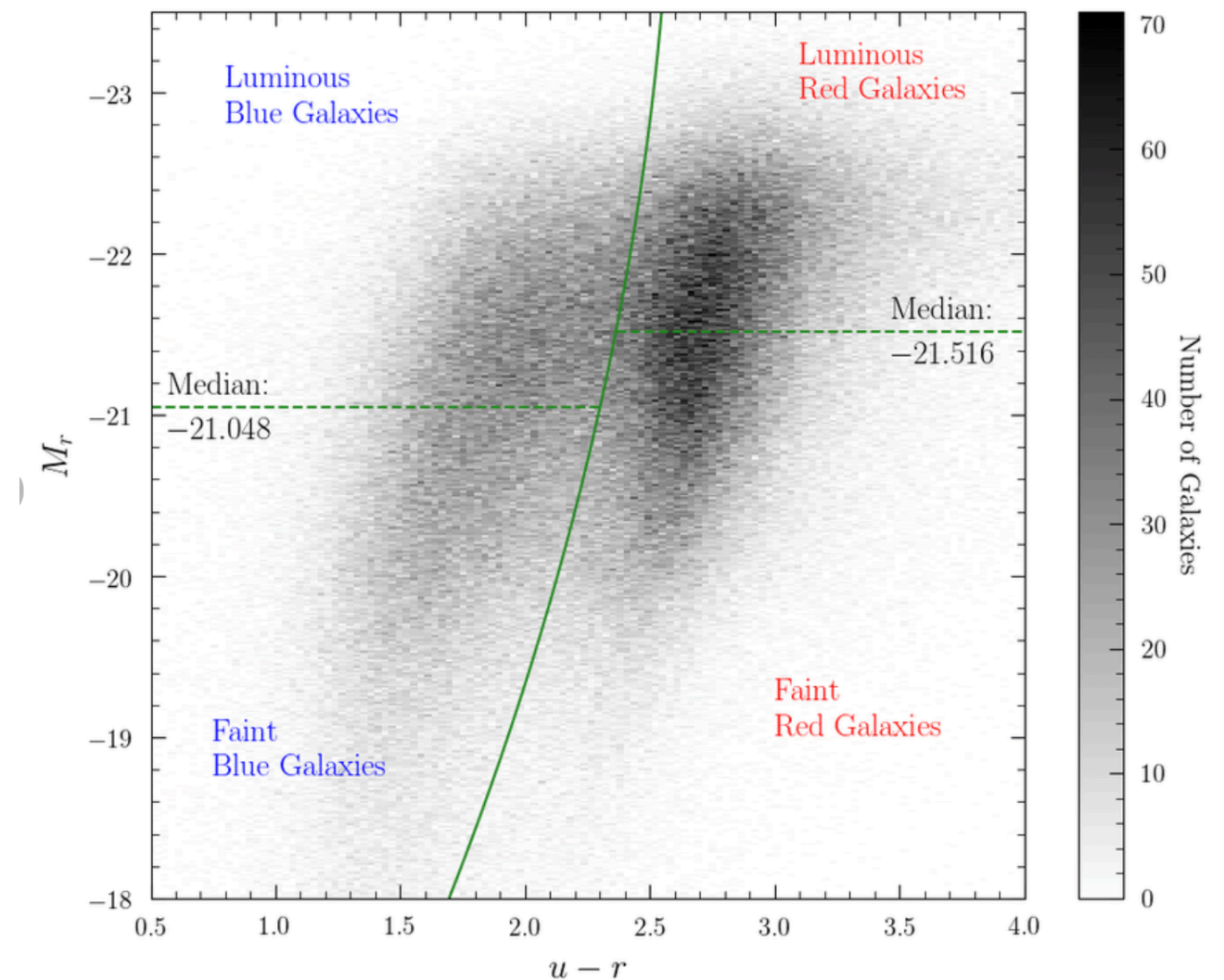
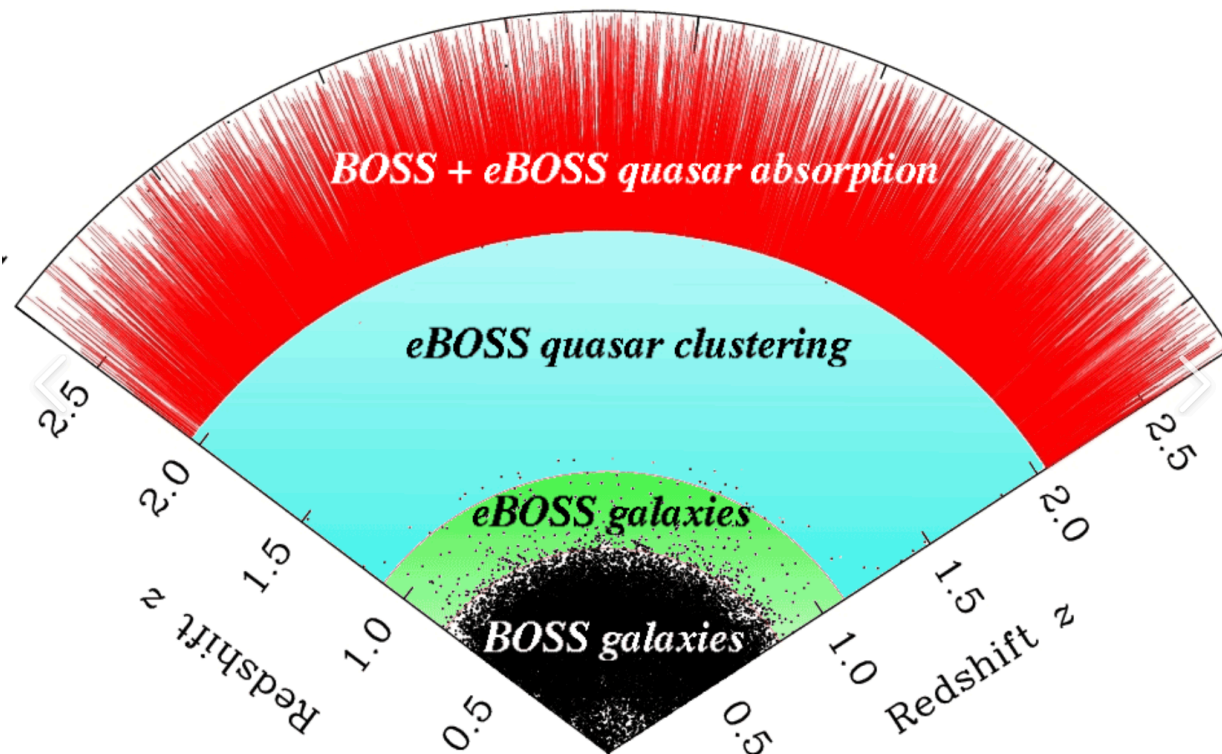
**For cosmology studies, is it necessary to observe
all the galaxies in the Universe?**

What kind of galaxies are ideal for cosmology?



The Galaxy Sample

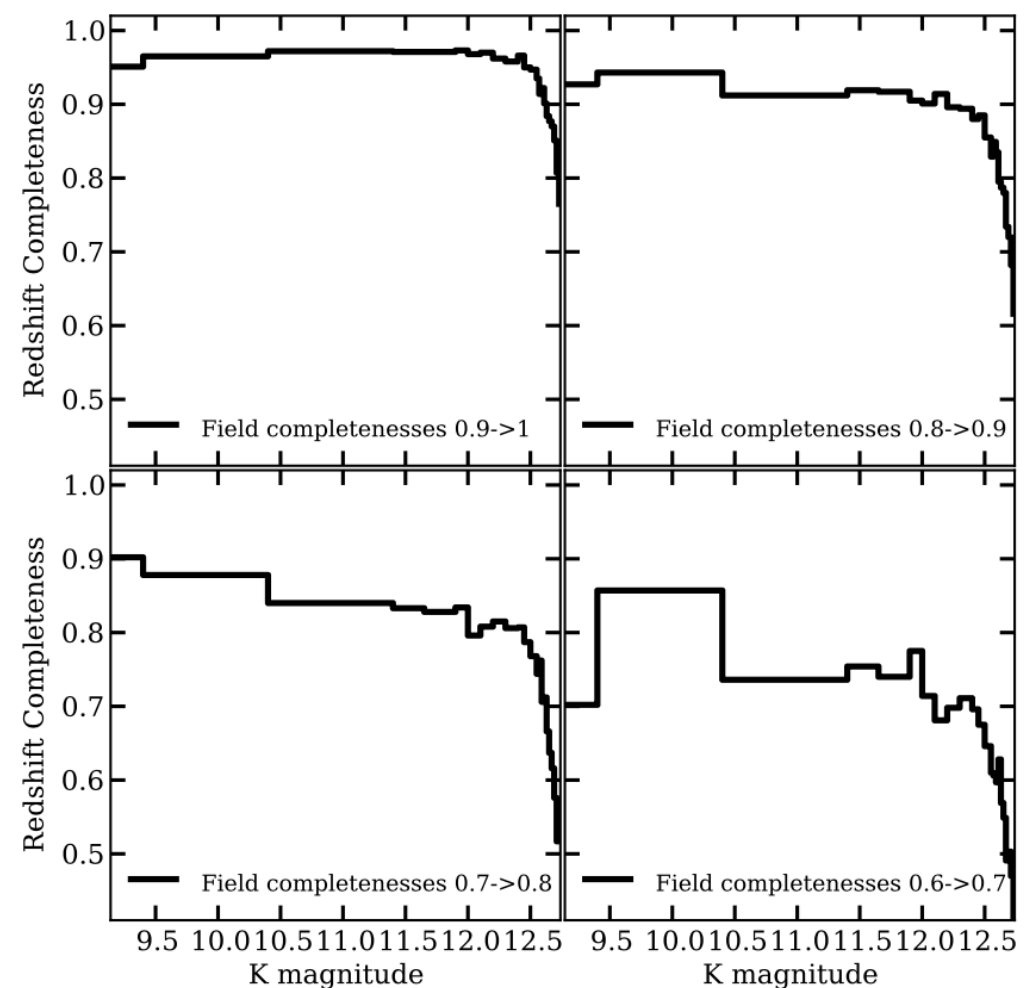
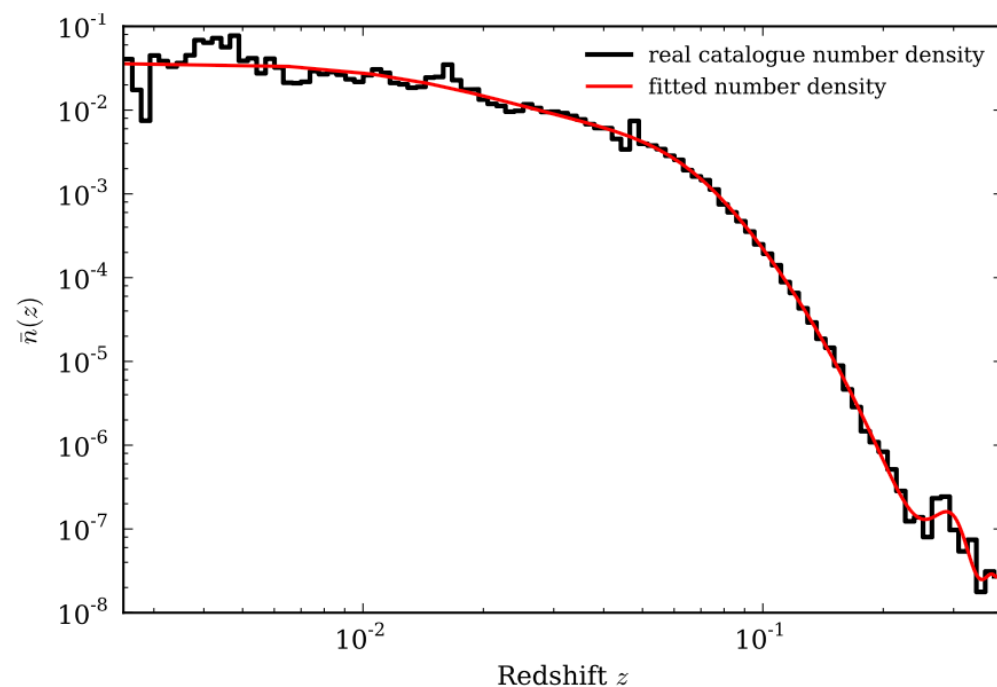
- Need to be a good sample of the underlying DM field
- Need to have enough number density
- Need to be distributed in large volumes
- Need to be at a high redshift
 - Luminous Red Galaxies
 - Emission Line Galaxies
 - Quasars



The Feldman-Kaiser-Peacock (1994) Method

- A galaxy survey samples only some fraction of the galaxies.
- Galaxies are selected randomly from some continuous underlying field.
- The Selection Function $\bar{n}(r)$

$$\bar{n}(r) = \bar{n}(z)\Phi(\theta, m)$$



The Feldman-Kaiser-Peacock (1994) Method

- A galaxy survey samples only some fraction of the galaxies.
- Galaxies are selected randomly from some continuous underlying field.
- The Shot Noise

$$\xi(r) = \langle \delta(x)\delta(x+r) \rangle$$

$$\langle \delta(r_i)\delta(r_j) \rangle_{\text{shot}} = \frac{\delta_D(r_{ij})}{\bar{n}(r_i)}$$

$$\langle \delta(k_i)\delta(k_j) \rangle_{\text{shot}} = \frac{1}{\bar{n}}(k_i + k_j)$$

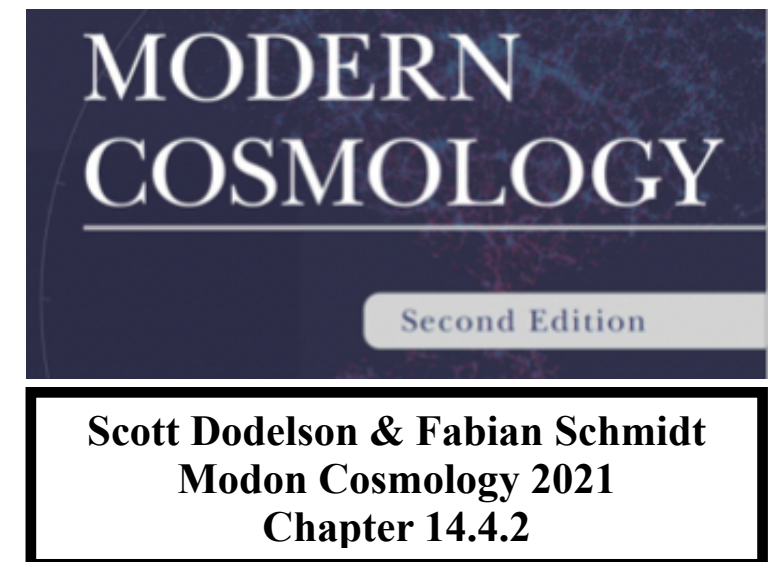
$$\frac{1}{\bar{n}}(k) = \int \frac{1}{\bar{n}}(r)e^{ikr}d^3r$$

$$\langle \delta(k)\delta(k') \rangle = 2\pi^2 P(k)\delta_D(k+k') + \frac{1}{\bar{n}}(k+k')$$

Shot Noise

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Cosmic Variance



$$\text{Cov}_{\alpha\beta} \equiv \langle \hat{P}_g(k_\alpha) \hat{P}_g(k_\beta) \rangle - \langle \hat{P}_g(k_\alpha) \rangle \langle \hat{P}_g(k_\beta) \rangle$$

$$= \frac{1}{m_{k,\alpha}} \sum_{\mathbf{k}} \frac{1}{m_{k,\beta}} \sum_{\mathbf{k}'} \left[\langle |\delta_g(\mathbf{k})|^2 |\delta_g(\mathbf{k}')|^2 \rangle - \langle |\delta_g(\mathbf{k})|^2 \rangle \langle |\delta_g(\mathbf{k}')|^2 \rangle \right], \quad ($$

$$\begin{aligned} \langle |\delta_g(\mathbf{k})|^2 |\delta_g(\mathbf{k}')|^2 \rangle &= \langle \delta_g(\mathbf{k}) \delta_g(-\mathbf{k}) \delta_g(\mathbf{k}') \delta_g(-\mathbf{k}') \rangle \\ &= \langle \delta_g(\mathbf{k}) \delta_g(-\mathbf{k}) \rangle \langle \delta_g(\mathbf{k}') \delta_g(-\mathbf{k}') \rangle \\ &\quad + \langle \delta_g(\mathbf{k}) \delta_g(\mathbf{k}') \rangle \langle \delta_g(-\mathbf{k}) \delta_g(-\mathbf{k}') \rangle \\ &\quad + \langle \delta_g(\mathbf{k}) \delta_g(-\mathbf{k}') \rangle \langle \delta_g(-\mathbf{k}) \delta_g(\mathbf{k}') \rangle \\ &\quad + \langle \delta_g(\mathbf{k}) \delta_g(-\mathbf{k}) \delta_g(\mathbf{k}') \delta_g(-\mathbf{k}') \rangle_{\text{conn}}. \end{aligned}$$

Wick's theorem

$$\langle \delta(k) \delta(k') \rangle = 2\pi^2 P(k) \delta_D(k + k') + \frac{1}{\bar{n}}(k + k')$$

Shot Noise

$$\sqrt{\text{Var}[\hat{P}_g(k_\alpha)]} = \sqrt{\frac{2}{m_{k,\alpha}}} [P_g(k_\alpha) + P_N].$$

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\tilde{\delta}(r) = w(r)(n(r) - \bar{n}(r)) = w(r)\bar{n}(r)\delta(r) = W(r)\delta(r)$$

$$\tilde{\delta}(k) = \int W(r)\delta(r)e^{ikr}d^3r$$

$$\langle \tilde{\delta}(k)\tilde{\delta}(k') \rangle = \int W(k_i - k)W(k_j + k)P(k)d^3k + N(k_i + k_j)$$

$$N(k) = \int w^2(r)\bar{n}^2\frac{1}{\bar{n}(r)}e^{ikr}d^3r$$

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\tilde{\delta}(r) = w(r)(n(r) - \bar{n}(r)) = w(r)\bar{n}(r)\delta(r) = W(r)\delta(r)$$

$$\tilde{\delta}(k) = \int W(r)\delta(r)e^{ikr}d^3r$$

$$\langle \tilde{\delta}(k)\tilde{\delta}(k') \rangle = \int W(k_i - k)W(k_j + k)P(k)d^3k + N(k_i + k_j)$$

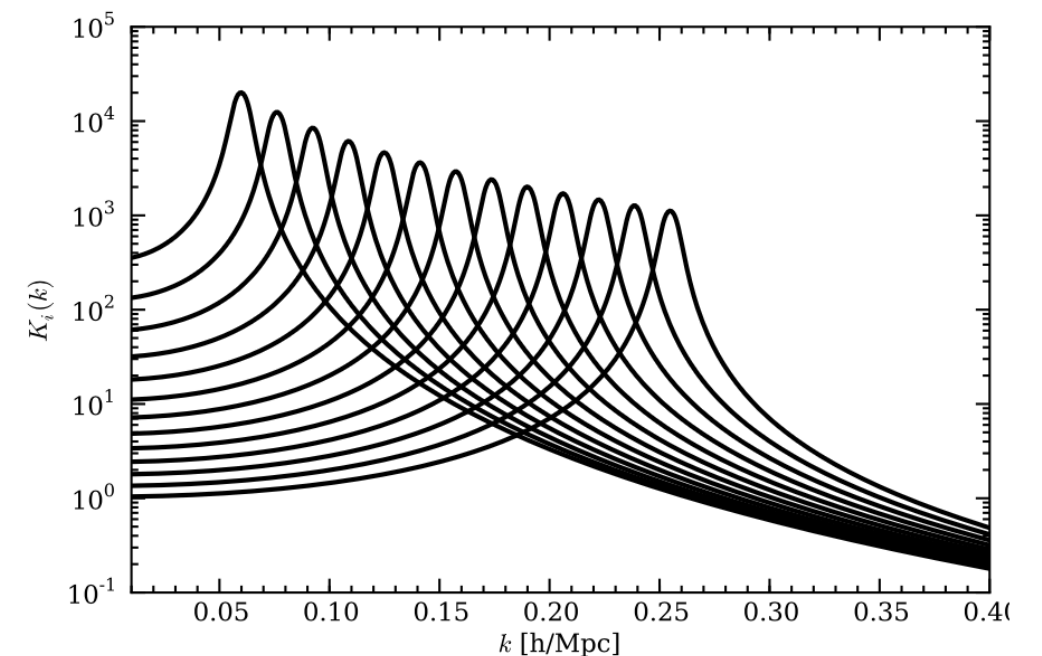
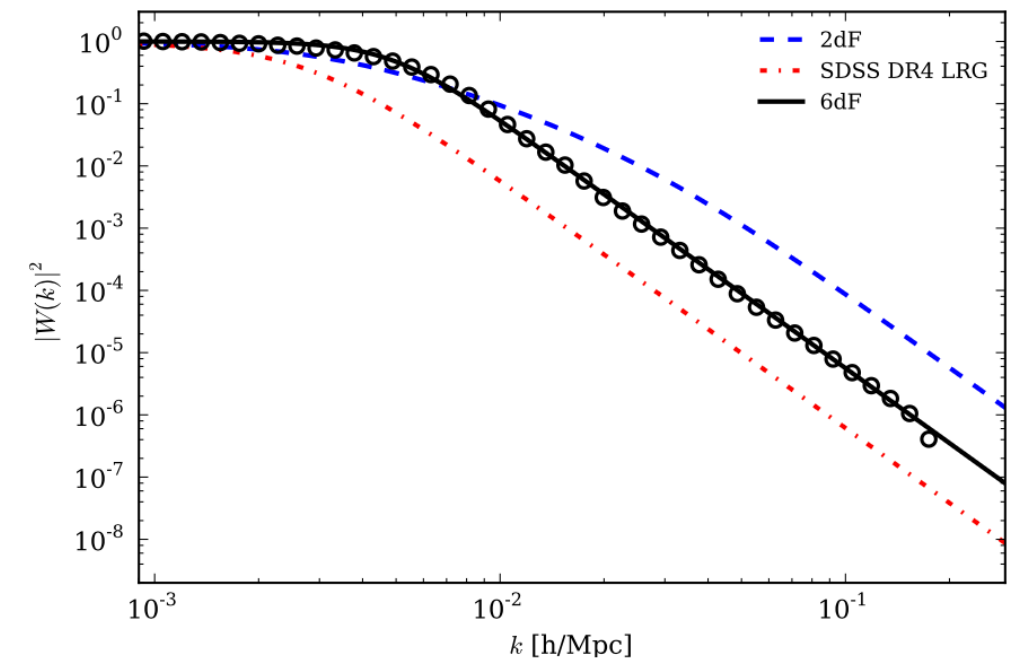
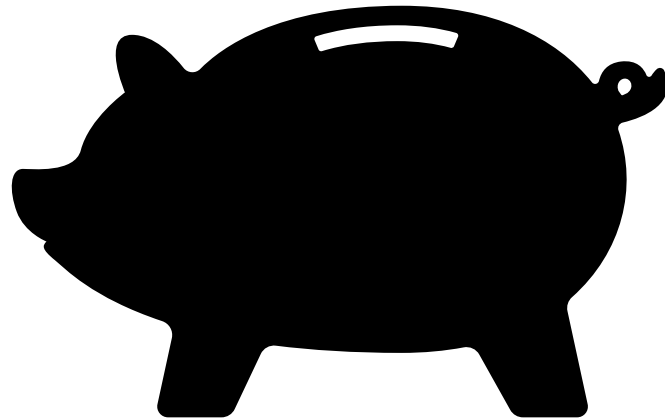
$$N(k) = \int w^2(r)\bar{n}^2\frac{1}{\bar{n}(r)}e^{ikr}d^3r$$

$$\langle \tilde{\delta}(k)\tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k')d^3k' + N(0)$$

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$



The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

- Inverse noise weight (FKP weight)

$$W(r) = \frac{1}{P(k) + 1/n(r)}$$

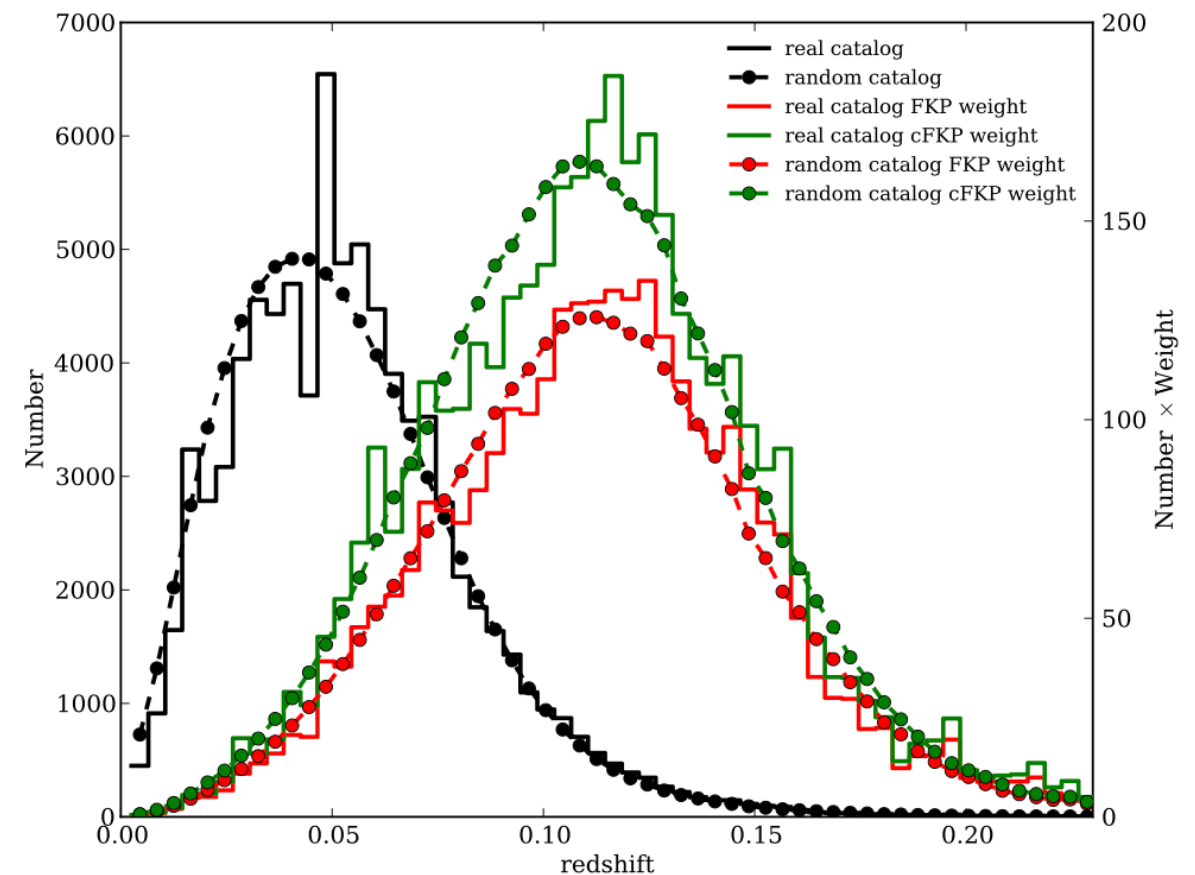
The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

- Inverse noise weight (FKP weight)

$$W(r) = \frac{1}{P(k) + 1/n(r)}$$



The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

- Inverse noise weight (FKP weight)

$$W(r) = \frac{1}{P(k) + 1/n(r)}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

- Inverse noise weight (FKP weight)

$$W(r) = \frac{1}{P(k) + 1/n(r)}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

- Estimation of Shot Noise

The Feldman-Kaiser-Peacock (1994) Method

- With finite survey volume
- Weighted overdensity

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

- Inverse noise weight (FKP weight)

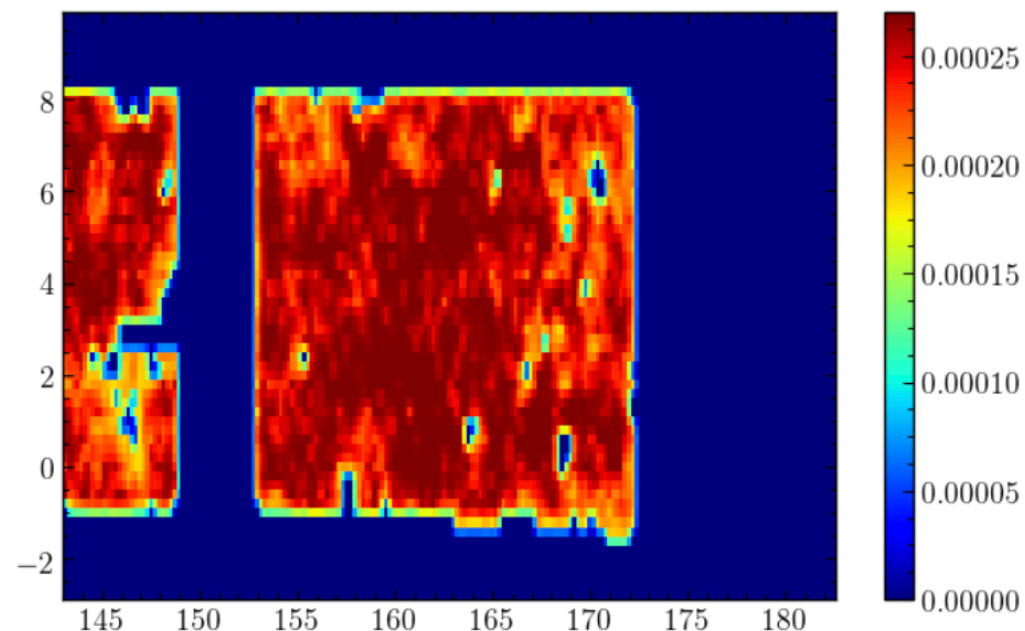
$$W(r) = \frac{1}{P(k) + 1/n(r)}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

- Estimation of Shot Noise
 - Via Simulated Mock Samples

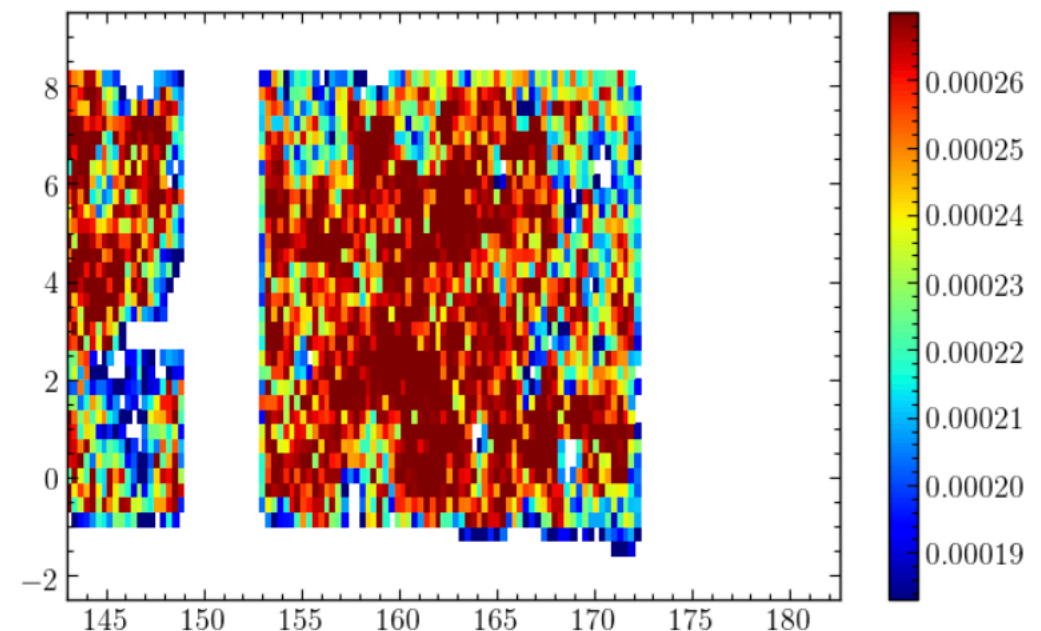
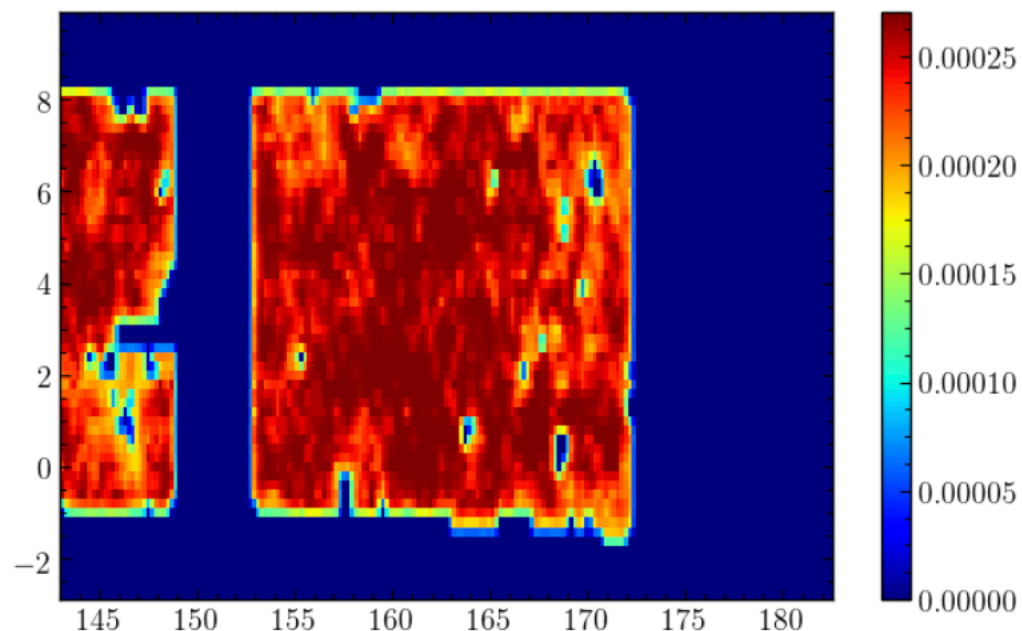
Shot Noise Estimation and Random Samples

- Estimation of Shot Noise
 - Via Simulated Random Samples
 - The random samples are supposed to have no LSS information
 - The random samples are supposed to have the same selection function $\bar{n}(r)$
- Generation of Random Samples
 - Using the known selection function $\bar{n}(r)$



The Feldman-Kaiser-Peacock (1994) Method

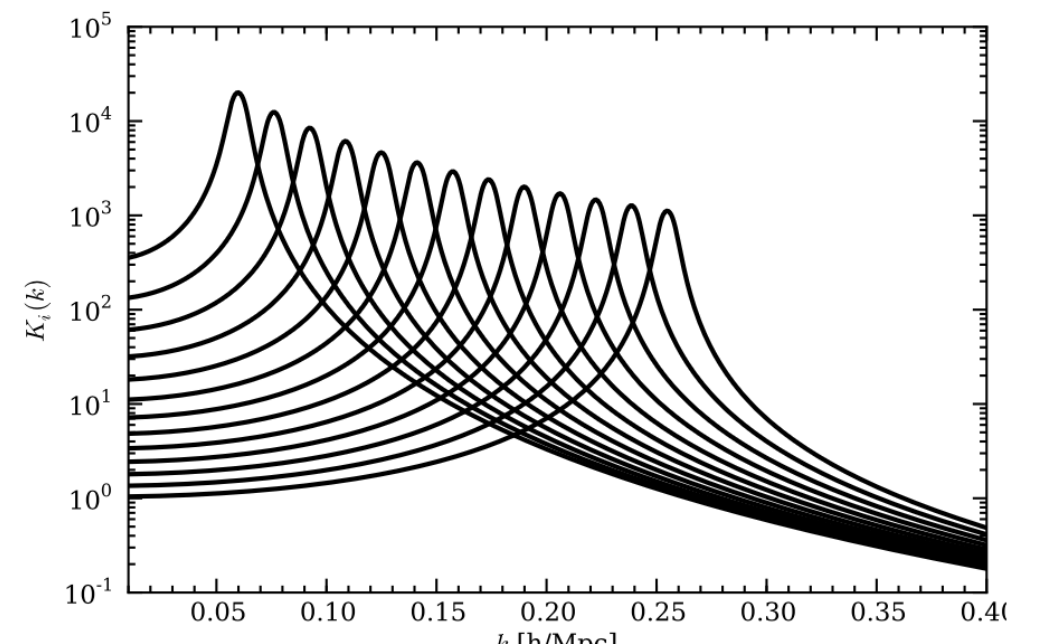
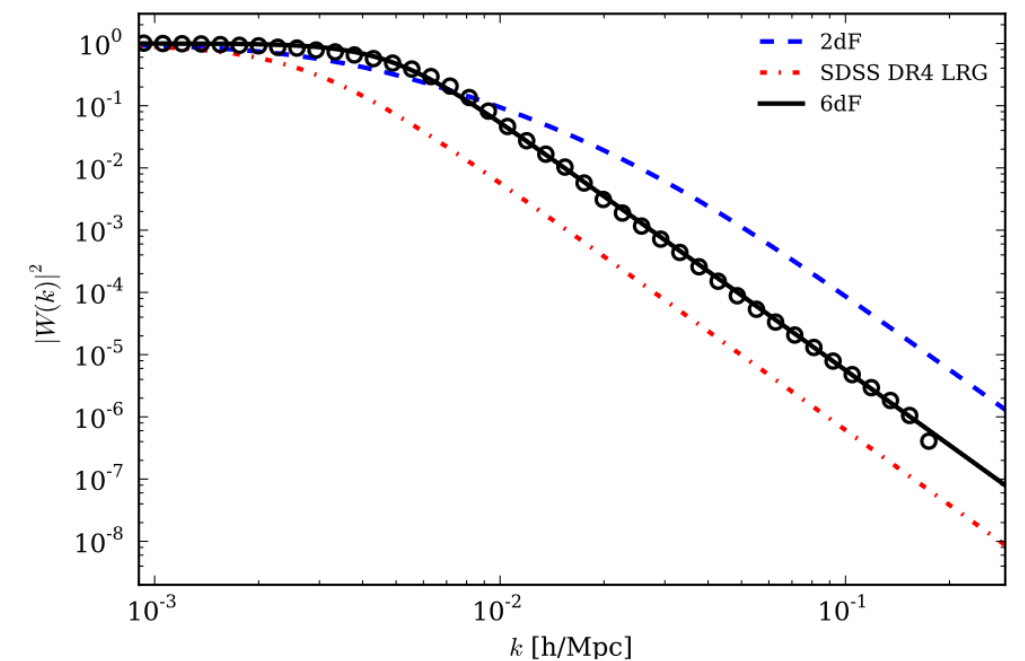
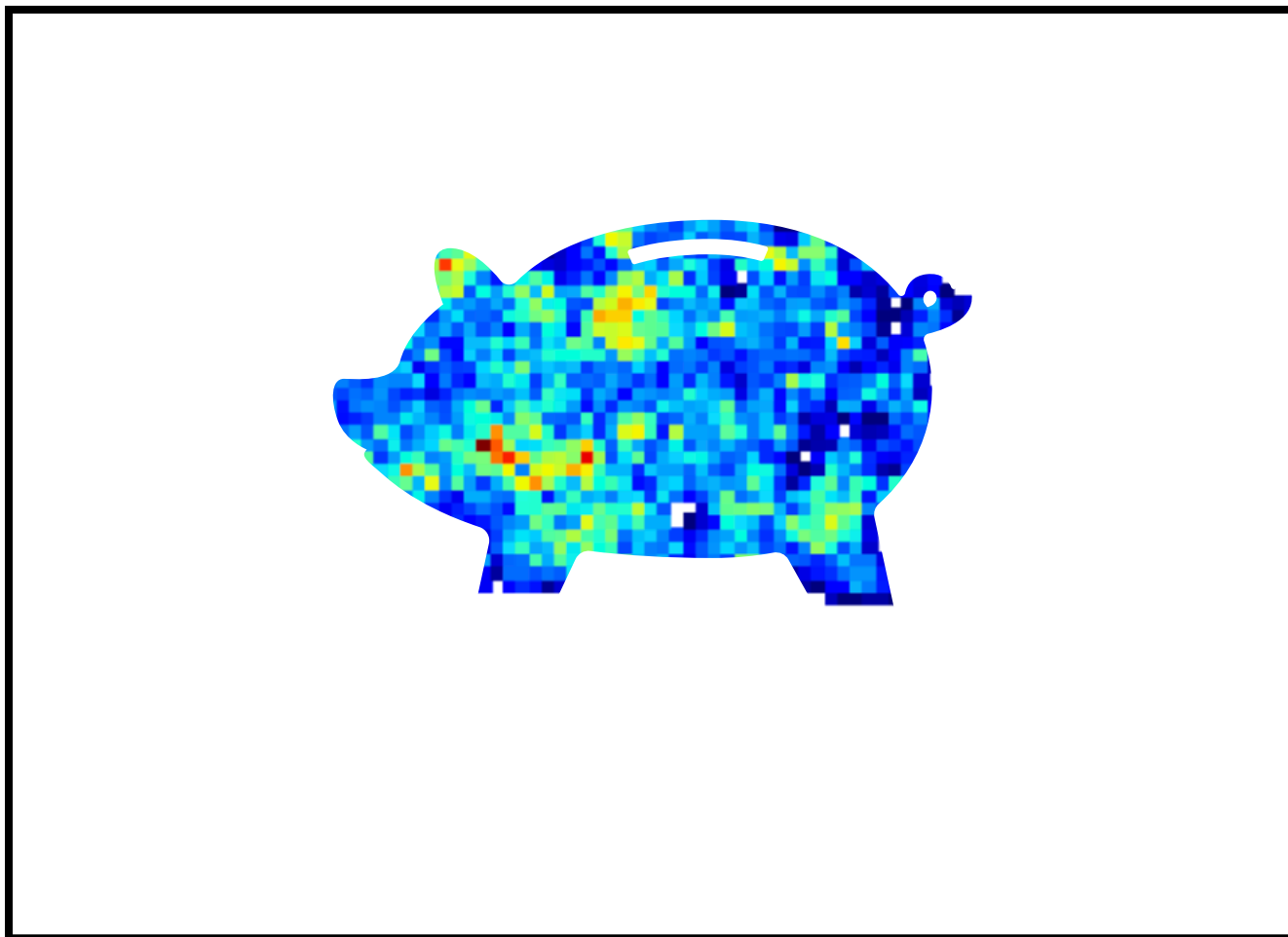
- Estimation of Shot Noise
 - Via Simulated Random Samples
 - The random samples are supposed to have no LSS information
 - The random samples are supposed to have the same selection function $\bar{n}(r)$
- Generation of Random Samples
 - Using the known selection function $\bar{n}(r)$
 - Using the real catalog and resampling
 - such as shuffle the position of the galaxies in the real catalog



The Estimation of the Window Function

- Estimate the Window Function with Random Samples
 - Random Samples need to have a larger volume than real data

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$



The Estimation of the Errors

- Estimate the Error
 - With Random Samples
 - With Simulation (N-body or Semi-analytical)
 - With Mock Samples.
 - Different from the Random Sample, the Mock Sample takes the LSS information.
 - Gaussian field

$$\tilde{\delta}(\mathbf{k}) = A(\mathbf{k}) + iB(\mathbf{k}),$$

$$\langle A(\mathbf{k})^2 \rangle = \langle B(\mathbf{k})^2 \rangle = \frac{1}{2}P(k),$$

$$\delta_G(\mathbf{x}) = \mathcal{F}^{-1}[\delta(\mathbf{k})],$$

Need to take care of the symmetry in the Fourier space to make sure a real number in the real (configuration) space

The Estimation of the Errors

- Estimate the Error
 - With Random Samples
 - With Simulation (N-body or Semi-analytical)
 - With Mock Samples.
 - Different from the Random Sample, the Mock Sample takes the LSS information.
 - Gaussian field
 - Log-normal field

$$\tilde{\delta}(\mathbf{k}) = A(\mathbf{k}) + iB(\mathbf{k}),$$

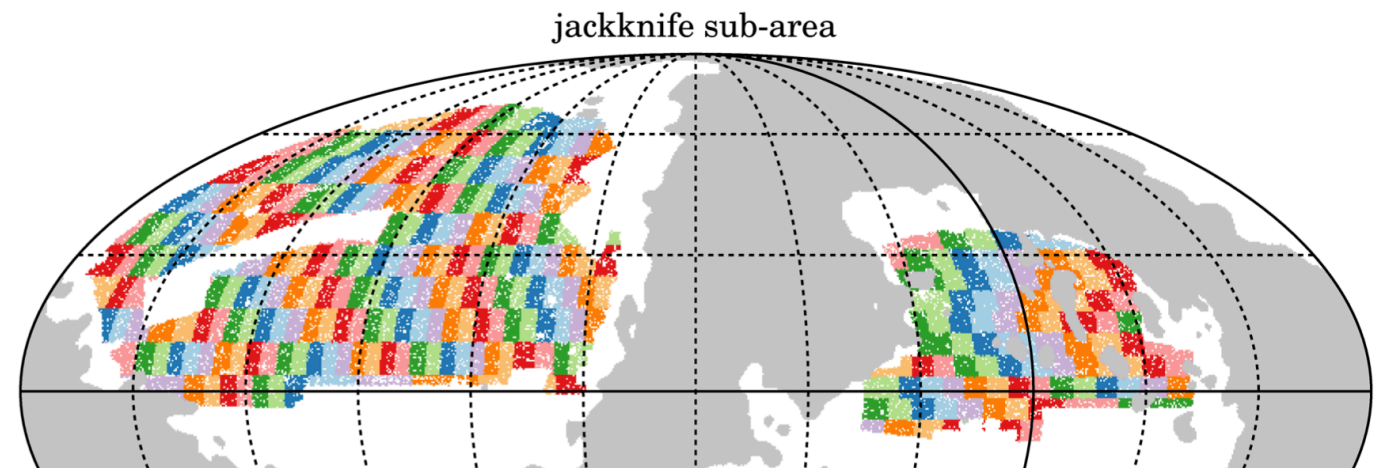
$$\langle A(\mathbf{k})^2 \rangle = \langle B(\mathbf{k})^2 \rangle = \frac{1}{2}P(k),$$

$$\delta_G(\mathbf{x}) = \mathcal{F}^{-1}[\delta(\mathbf{k})],$$

$$\rho(\mathbf{x}) = \exp \left[\delta(\mathbf{x}) - \frac{1}{2}\sigma^2 \right].$$

The Estimation of the Errors

- Estimate the Error
 - With Random Samples
 - With Simulation (N-body or Semi-analytical)
 - With Mock Samples.
 - Different from the Random Sample, the Mock Sample takes the LSS information.
 - Gaussian field
 - Log-normal field
 - Jackknife sample



$$\text{CovJK}(\theta_a, \theta_b) = \frac{N-1}{N} \sum_{i=1}^N \left(\theta_i^{(a)} - \bar{\theta}^{(a)} \right) \left(\theta_i^{(b)} - \bar{\theta}^{(b)} \right),$$

Correct the Alias Effect

- The Alias Effect with Gridding the Galaxy Catalog

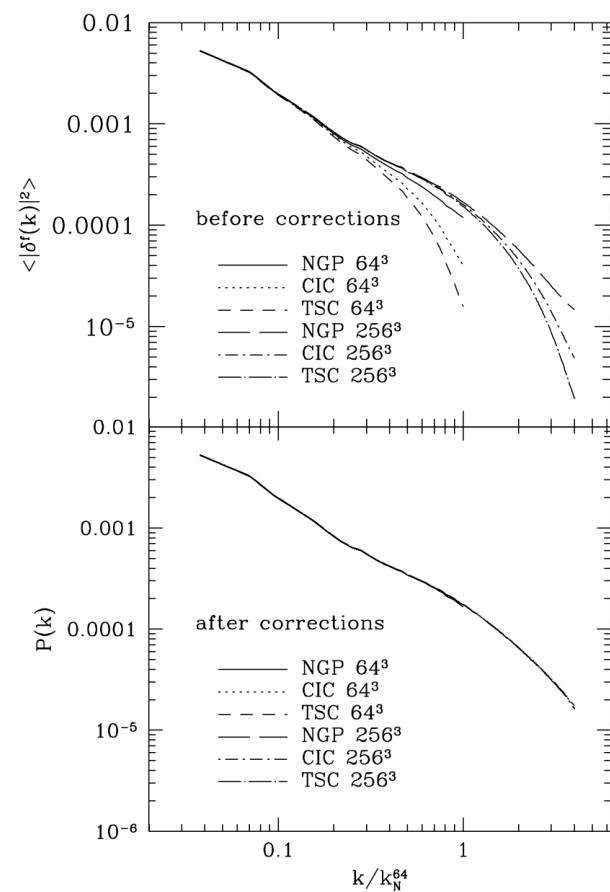


FIG. 2.—Six power spectra that we measured for an N -body simulation sample using three mass assignments (NGP, CIC, and TSC) and two grids (64^3 and 256^3 grid points), where k_N^{64} is the Nyquist wavenumber for 64^3 grid points. *Top*: Raw power spectra $\langle |\delta^f(k)|^2 \rangle$ estimated directly from the FFT (eq. [8]). *Bottom*: True power spectra $P(k)$ reconstructed from the $\langle |\delta^f(k)|^2 \rangle$ following the procedure described in the text. The six reconstructed $P(k)$ agree so well that their curves overlay each other.

THE ASTROPHYSICAL JOURNAL, 620:559–563, 2005 February 20

© 2005. The American Astronomical Society. All rights reserved. Printed in U.S.A.

CORRECTING FOR THE ALIAS EFFECT WHEN MEASURING THE POWER SPECTRUM USING A FAST FOURIER TRANSFORM

Y. P. JING

Shanghai Astronomical Observatory, The Partner Group of the Max-Planck-Institut für Astrophysik, Nandan Road 80, Shanghai 200030, China

Received 2004 September 10; accepted 2004 October 28

ABSTRACT

Because of mass assignment onto grid points in the measurement of the power spectrum using a fast Fourier transform (FFT), the raw power spectrum $\langle |\delta^f(k)|^2 \rangle$ estimated with the FFT is not the same as the true power spectrum $P(k)$. In this paper we derive a formula that relates $\langle |\delta^f(k)|^2 \rangle$ to $P(k)$. For a sample of N discrete objects, the formula reads $\langle |\delta^f(k)|^2 \rangle = \sum_n |W(\mathbf{k} + 2\mathbf{k}_N \mathbf{n})|^2 P(\mathbf{k} + 2\mathbf{k}_N \mathbf{n}) + 1/N |W(\mathbf{k} + 2\mathbf{k}_N \mathbf{n})|^2$, where $W(\mathbf{k})$ is the Fourier transform of the mass assignment function $W(\mathbf{r})$, k_N is the Nyquist wavenumber, and $\mathbf{n}$ is an integer vector. The formula is different from that in some previous works in which the summation over $\mathbf{n}$ is neglected. For the nearest grid point, cloud-in-cell, and triangular-shaped cloud assignment functions, we show that the shot-noise term $\sum_n (1/N) |W(\mathbf{k} + 2\mathbf{k}_N \mathbf{n})|^2$ can be expressed by simple analytical functions. To reconstruct $P(k)$ from the alias sum $\sum_n |W(\mathbf{k} + 2\mathbf{k}_N \mathbf{n})|^2 P(\mathbf{k} + 2\mathbf{k}_N \mathbf{n})$, we propose an iterative method. We test the method by applying it to an N -body simulation sample and show that the method can successfully recover $P(k)$. The discussion is further generalized to samples with observational selection effects.

Subject headings: galaxies: clusters: general — large-scale structure of universe — methods: data analysis — methods: statistical

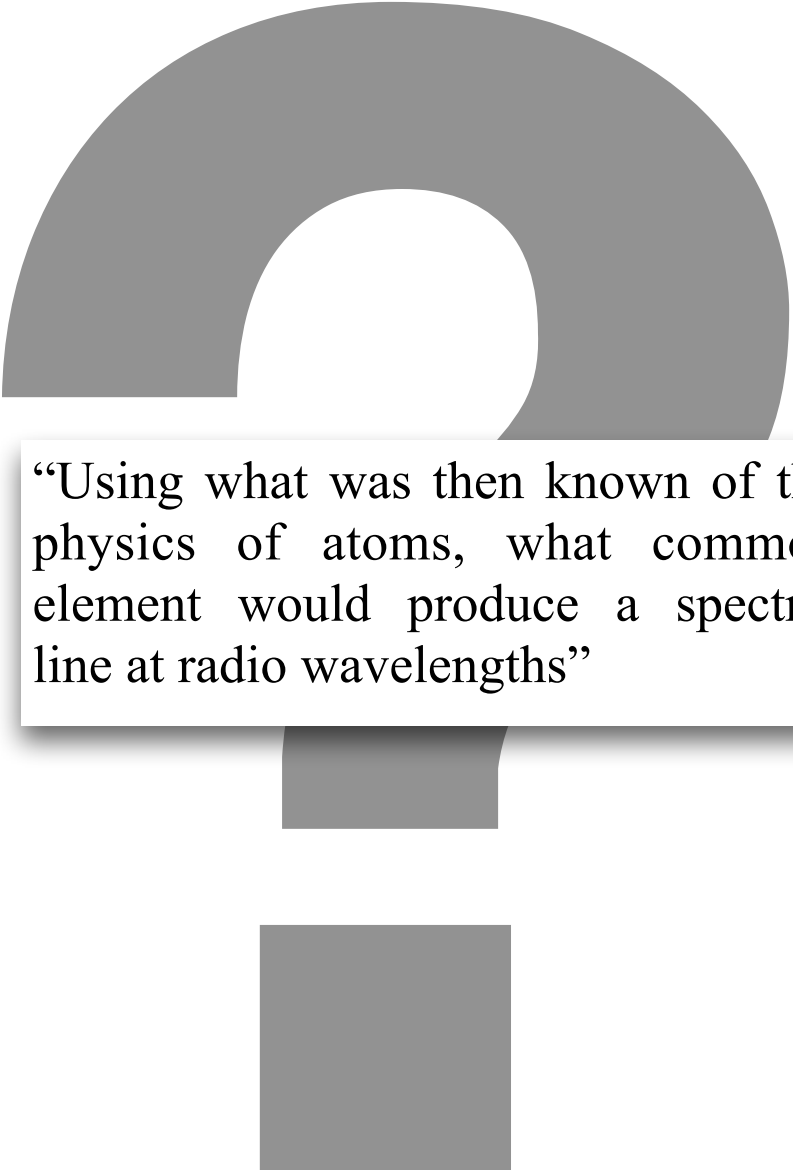
Summary for Galaxy Survey PS Estimation

- Real catalog for PS estimation
- Random catalog for shot noise and window function estimation
- Mock catalog for error estimation
 - Gaussian field
 - Log-normal field
 - Jackknife sample
- Correct the Alian Effects

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$
$$W(r) = \frac{1}{P(k) + 1/n(r)}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

Optical v.s. Radio



“Using what was then known of the physics of atoms, what common element would produce a spectral line at radio wavelengths”

The HI 21 cm Line

- The HI in the Milky Way
 - Jan Oort 1932 measured the rotation of the Milky Way via stars in the optical band

Jan Oort



Jan Oort in May 1961

Born	28 April 1900 Franeker, Friesland
Died	5 November 1992 (aged 92) Leiden, South Holland
Nationality	Dutch
Known for	Oort cloud Dark matter
Awards	Vetlesen Prize (1966) Kyoto Prize (1987)
Scientific career	
Fields	Astronomy
Doctoral advisor	Jacobus Cornelius Kapteyn

The HI 21 cm Line

- The HI in the Milky Way
 - Jan Oort 1932 measured the rotation of the Milky Way via stars in the optical band

Karl Guthe Jansky



Born October 22, 1905
[Norman, Oklahoma, U.S.](#)

Died February 14, 1950 (aged 44)
[Red Bank, New Jersey, U.S.](#)

Nationality American

Known for [Radio astronomy](#)

Fields [Physics](#)
[Radio astronomy](#)

Grote Reber



Born December 22, 1911
[Wheaton, Illinois, United States](#)

Died December 20, 2002 (aged 90)
[Ouse, Tasmania](#)

Nationality [American](#)

Alma mater [Illinois Institute of Technology](#)

Known for Pioneering work in radio astronomy

Awards [Elliott Cresson Medal](#) (1963)

Fields [Radio astronomy](#)

Jan Oort



Jan Oort in May 1961

Born 28 April 1900
[Franeker, Friesland](#)

Died 5 November 1992 (aged 92)
[Leiden, South Holland](#)

Nationality [Dutch](#)

Known for [Oort cloud](#)
[Dark matter](#)

Awards [Vetlesen Prize](#) (1966)
[Kyoto Prize](#) (1987)

Scientific career

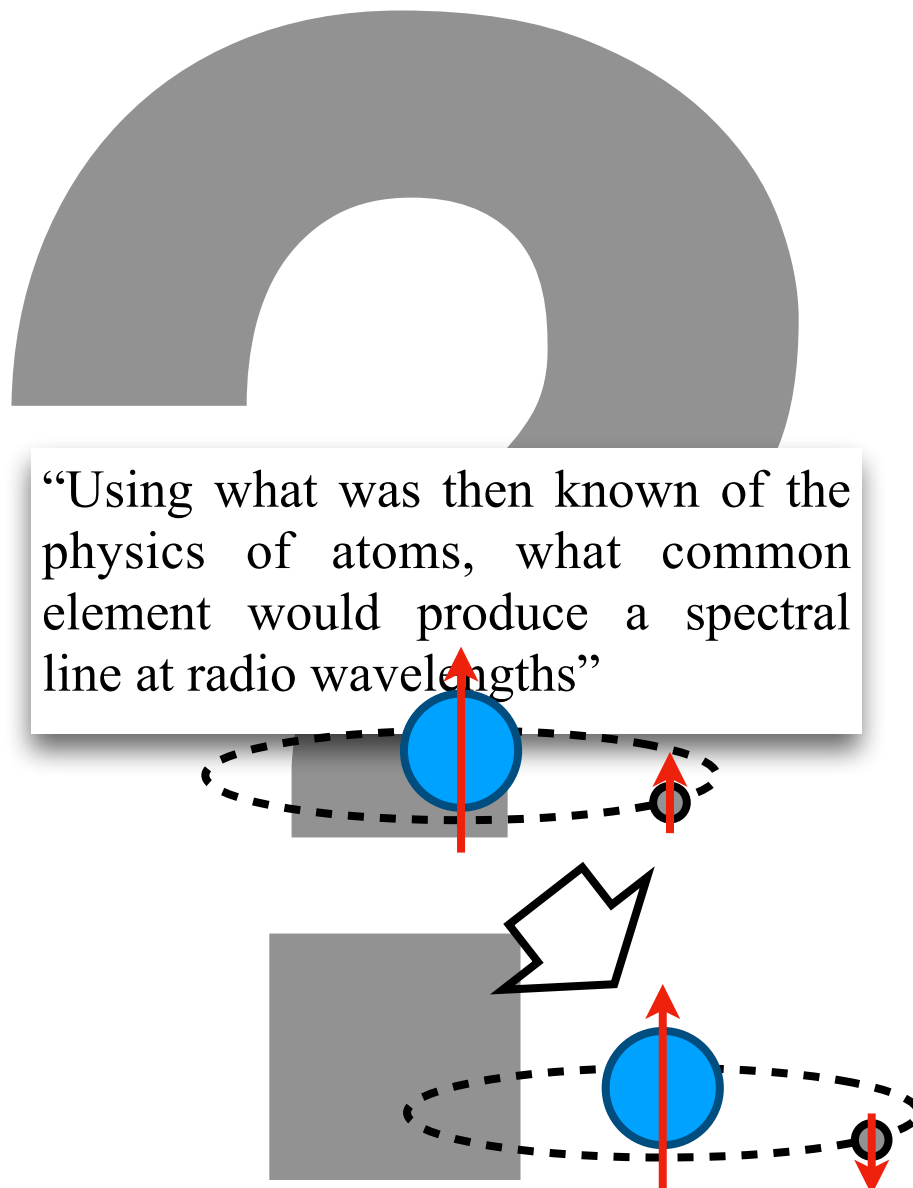
Fields [Astronomy](#)

Doctoral advisor [Jacobus Cornelius Kapteyn](#)

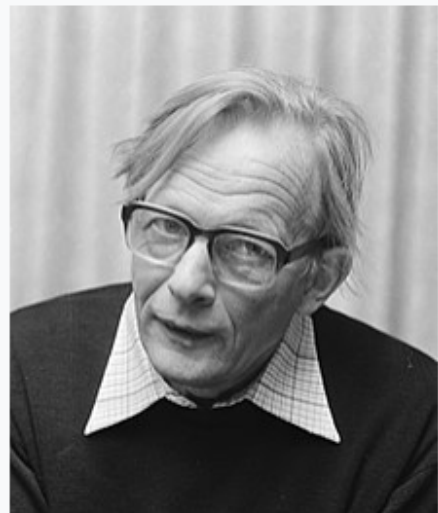
The HI 21 cm Line

- The HI in the Milky Way
 - Hendrik van de Hulst 1944

- Hendrik van de Hulst 1944
 - HI 21cm line
 - 1420.406 MHz
 - forbidden line
 - $A = 2.87 \times 10^{-15} \text{ s}^{-1}$
 - life time 11 million years



Hendrik C. van de Hulst



Hendrik C. van de Hulst in 1977

Born	19 November 1918 Utrecht, the Netherlands
Died	31 July 2000 (aged 81) Leiden, the Netherlands
Nationality	Dutch
Known for	21 cm hyperfine line Anomalous diffraction theory
Awards	Henry Draper Medal (1955) Eddington Medal (1955) Rumford Medal (1964) Bruce Medal (1978) Karl Schwarzschild Medal (1995)
Scientific career	

Jan Oort



Jan Oort in May 1961

Born	28 April 1900 Franeker, Friesland
Died	5 November 1992 (aged 92) Leiden, South Holland
Nationality	Dutch
Known for	Oort cloud Dark matter
Awards	Vetlesen Prize (1966) Kyoto Prize (1987)
Scientific career	
Fields	Astronomy
Doctoral advisor	Jacobus Cornelius Kapteyn

The HI 21 cm Line

- The HI in the Milky Way
 - Even and Purcell at Harvard made the discovery of Milky Way HI in 1951



Harold I. Ewen



Harold I. Ewen

Birthdate

1922/03/05

Birthplace

Chicopee Falls, MA, USA

Associated organizations

Harvard, Ewen Knight Corporation

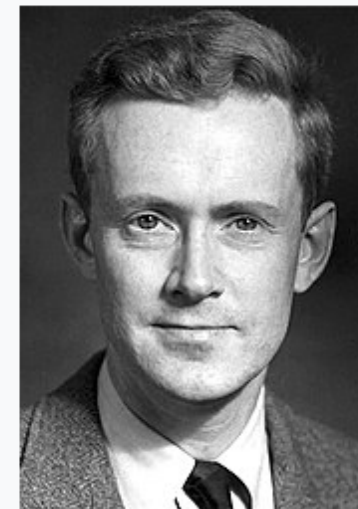
Fields of study

Measurement

Awards

IEEE Morris E. Leeds Award

Edward Mills Purcell



Purcell in 1952

Born

August 30, 1912

[Taylorville, Illinois](#), United States

Died

March 7, 1997 (aged 84)

[Cambridge, Massachusetts](#), United States

Nationality

American

Alma mater

[Purdue University](#) (BSEE)

[Harvard University](#) (M.A.)

[Harvard University](#) (Ph.D)

Known for

[Nuclear magnetic resonance \(NMR\)](#)

[Smith-Purcell effect](#)

[21 cm line](#)

[Scallop theorem](#)

Jan Oort



Jan Oort in May 1961

Born

28 April 1900

[Franeker, Friesland](#)

Died

5 November 1992

(aged 92)

[Leiden, South Holland](#)

Nationality

[Dutch](#)

Known for

[Oort cloud](#)

[Dark matter](#)

Awards

[Vetlesen Prize](#) (1966)

[Kyoto Prize](#) (1987)

Scientific career

Fields

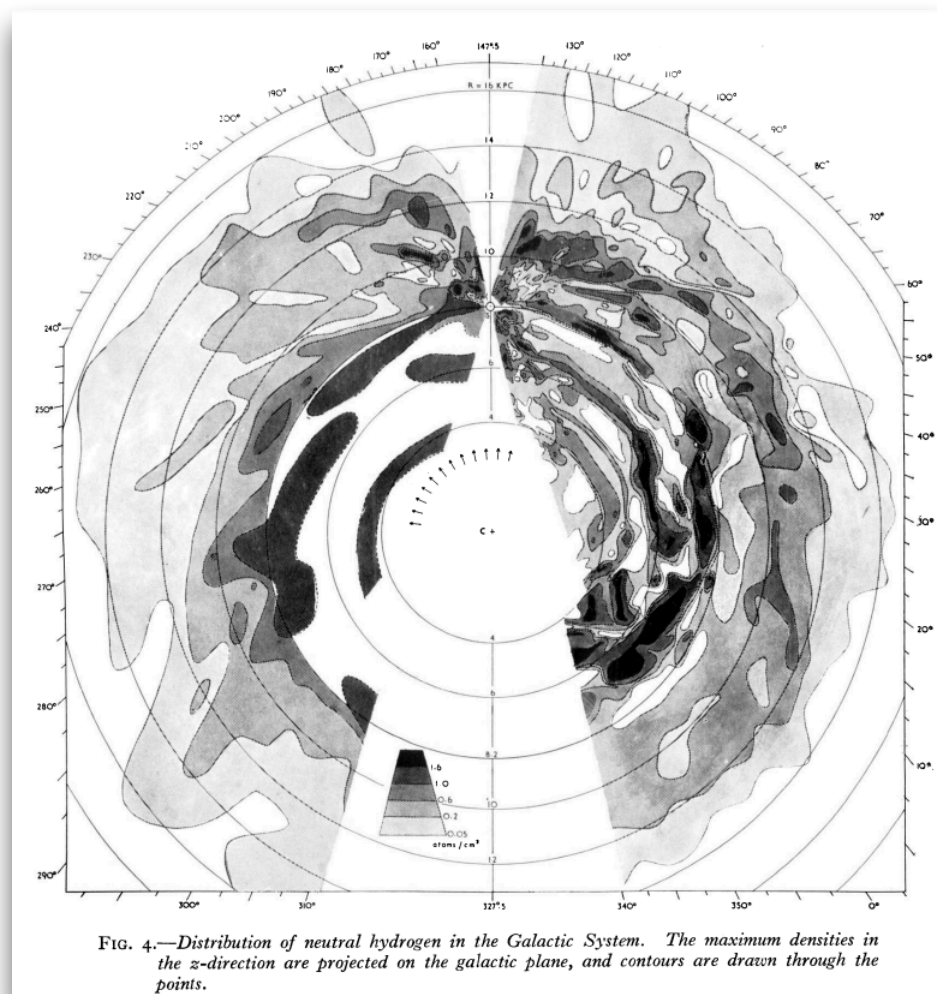
[Astronomy](#)

Doctoral advisor

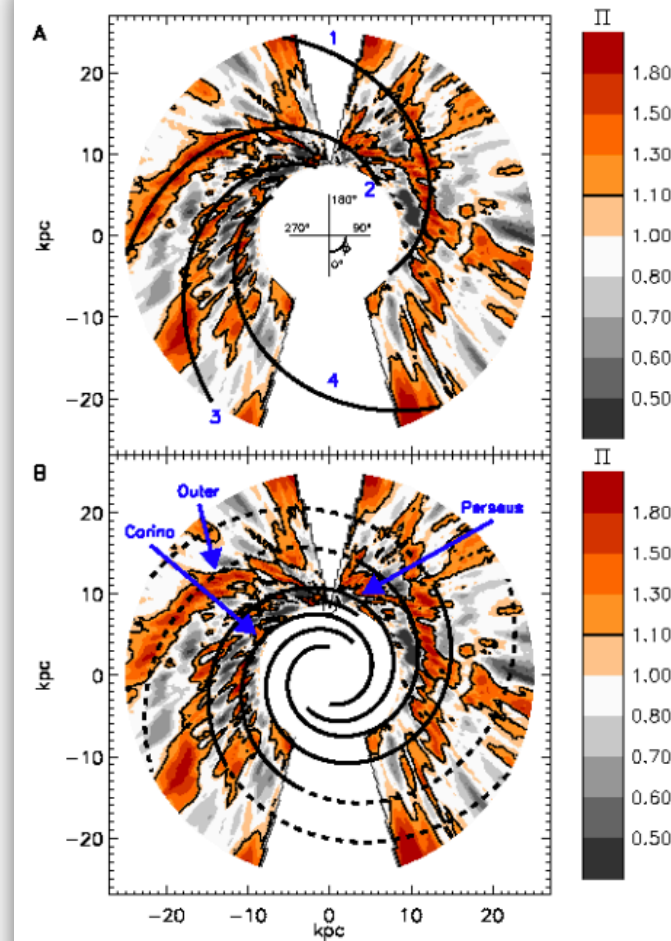
[Jacobus Cornelius Kapteyn](#)

The HI 21 cm Line

- The HI in the Milky Way
- Mapping HI in the Milky Way.



Oort, J. H., Kerr, F. J., and Westerhout, G., “The galactic system as a spiral nebula (Council Note)”, *Monthly Notices of the Royal Astronomical Society*, vol. 118, p. 379, 1958. doi:10.1093/mnras/118.4.379.



Levine, E. S., Blitz, L., and Heiles, C., “The Spiral Structure of the Outer Milky Way in Hydrogen”, *Science*, vol. 312, no. 5781, pp. 1773–1777, 2006. doi:10.1126/science.1128455.

Jan Oort



Jan Oort in May 1961

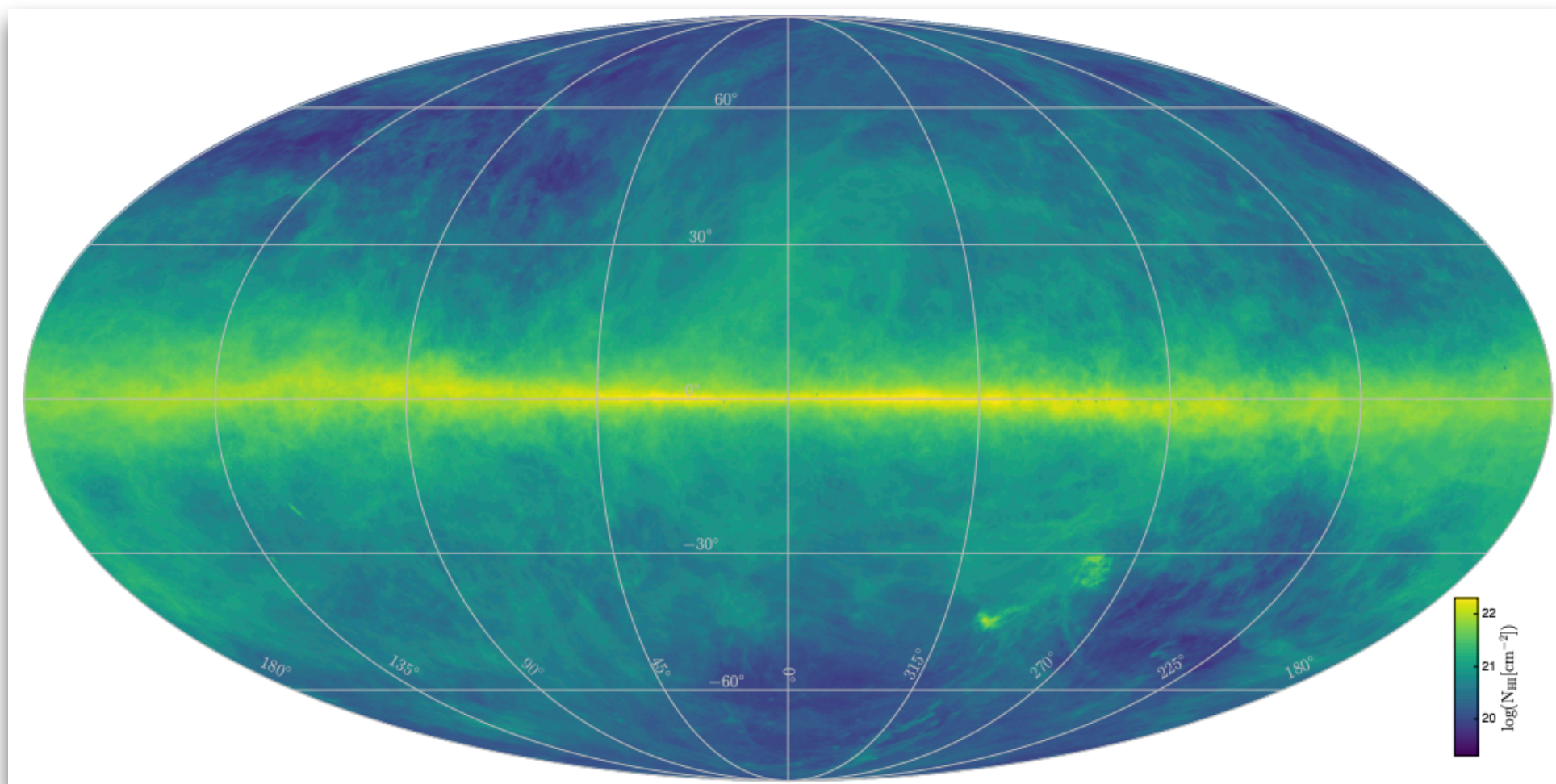
Born	28 April 1900 Franeker, Friesland
Died	5 November 1992 (aged 92) Leiden, South Holland
Nationality	Dutch
Known for	Oort cloud Dark matter
Awards	Vetlesen Prize (1966) Kyoto Prize (1987)

Scientific career

Fields	Astronomy
Doctoral advisor	Jacobus Cornelius Kapteyn

The HI 21 cm Line

- The HI in the Milky Way
 - Mapping HI in the Milky Way.



HI4PI collaboration: HI4PI: A full-sky H i survey based on EBHIS and GASS 2016 [arxiv:1610.06175](https://arxiv.org/abs/1610.06175)

The HI 21 cm Line

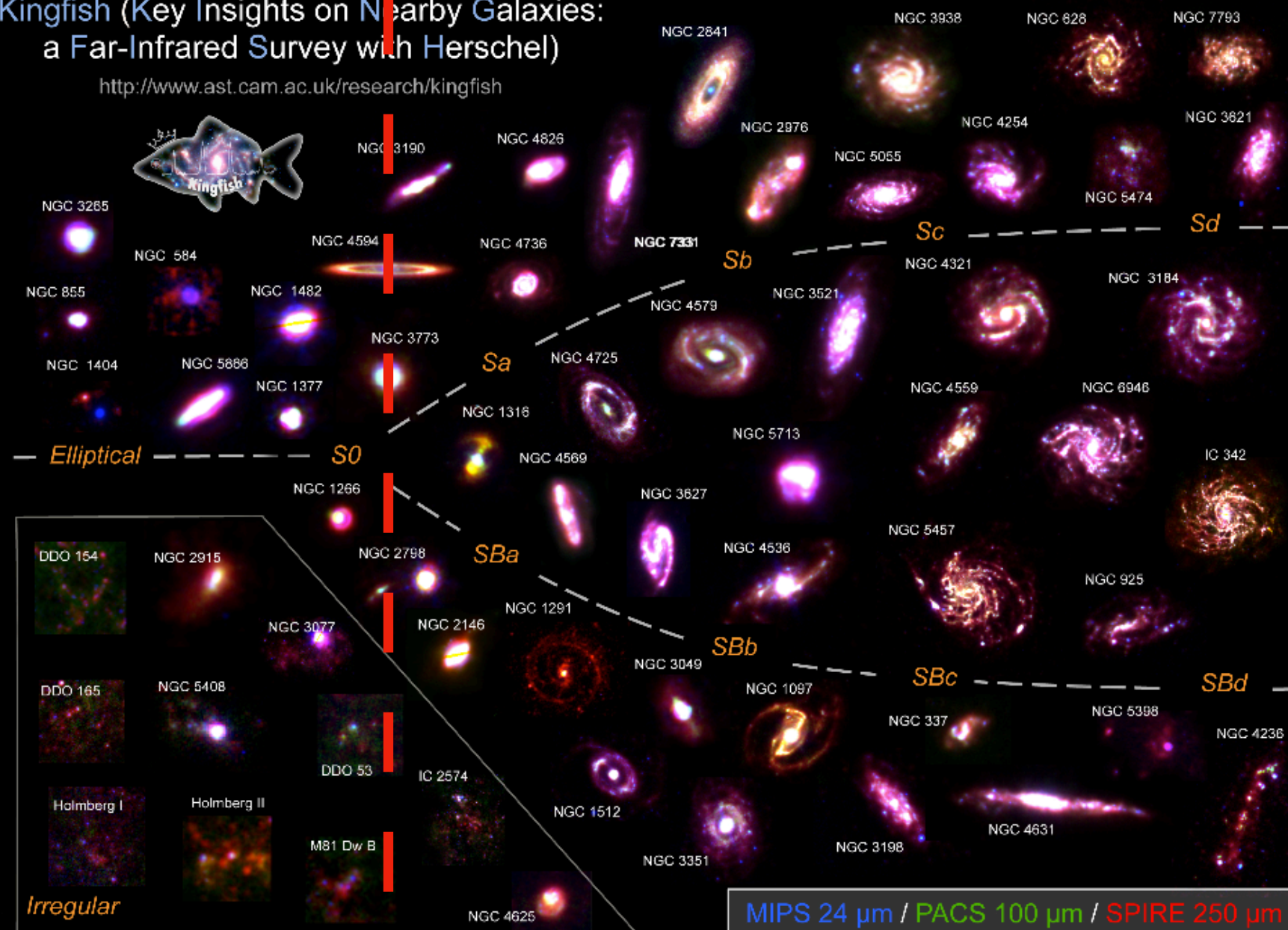
- HI in galaxies

Early Type Galaxy
Old Galaxies

Late Type Galaxy
Young Galaxies

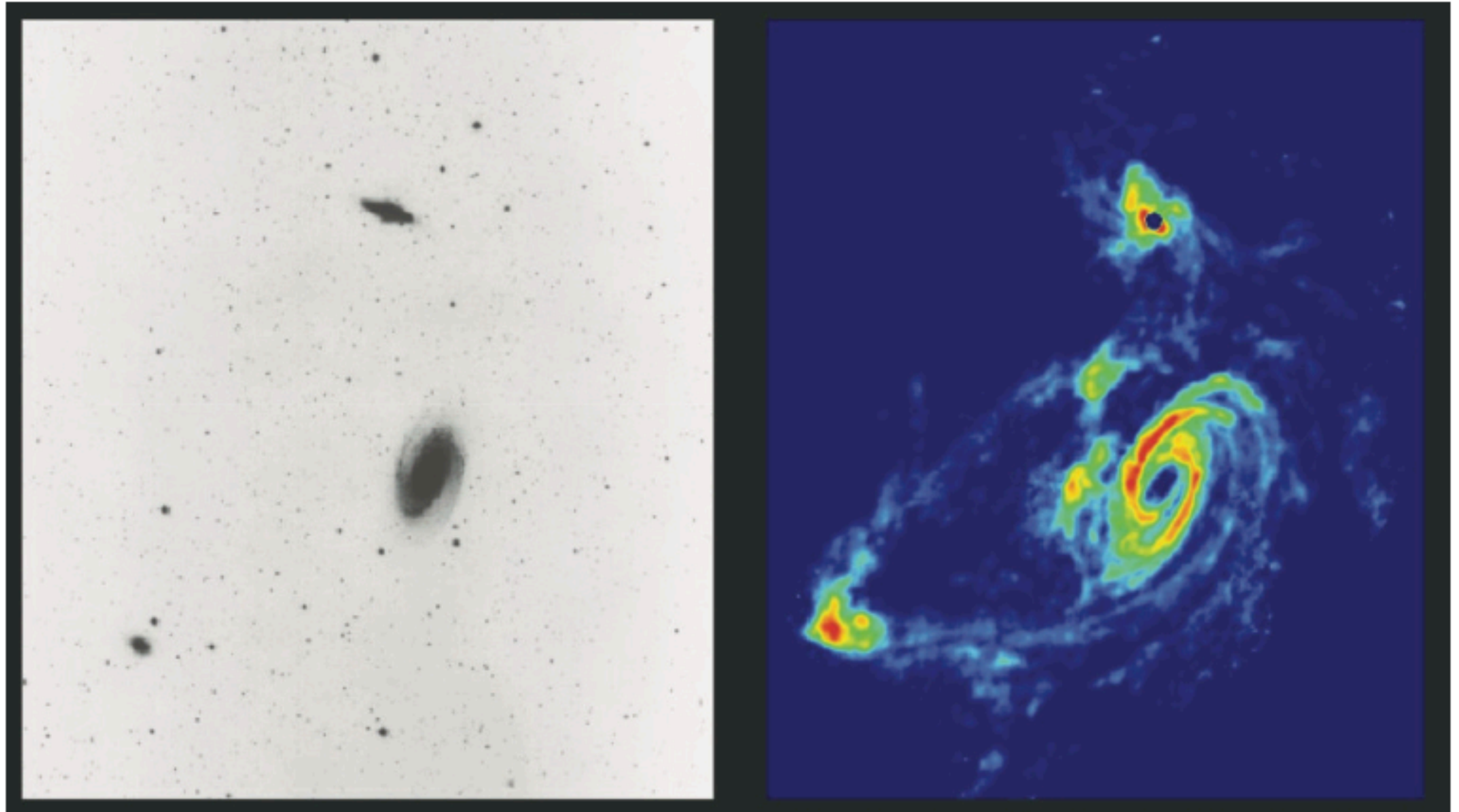
Kingfish (Key Insights on Nearby Galaxies:
a Far-Infrared Survey with Herschel)

<http://www.ast.cam.ac.uk/research/kingfish>



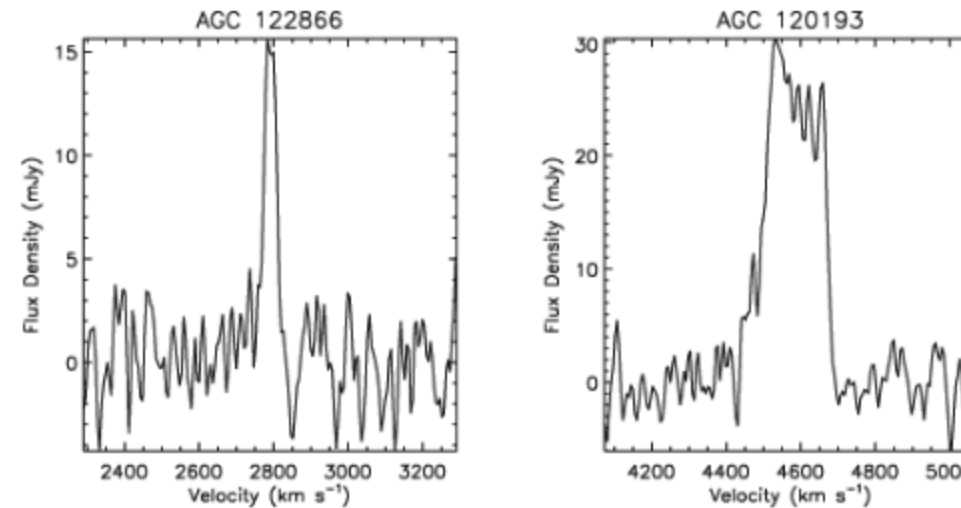
The HI 21 cm Line

- HI in IGM

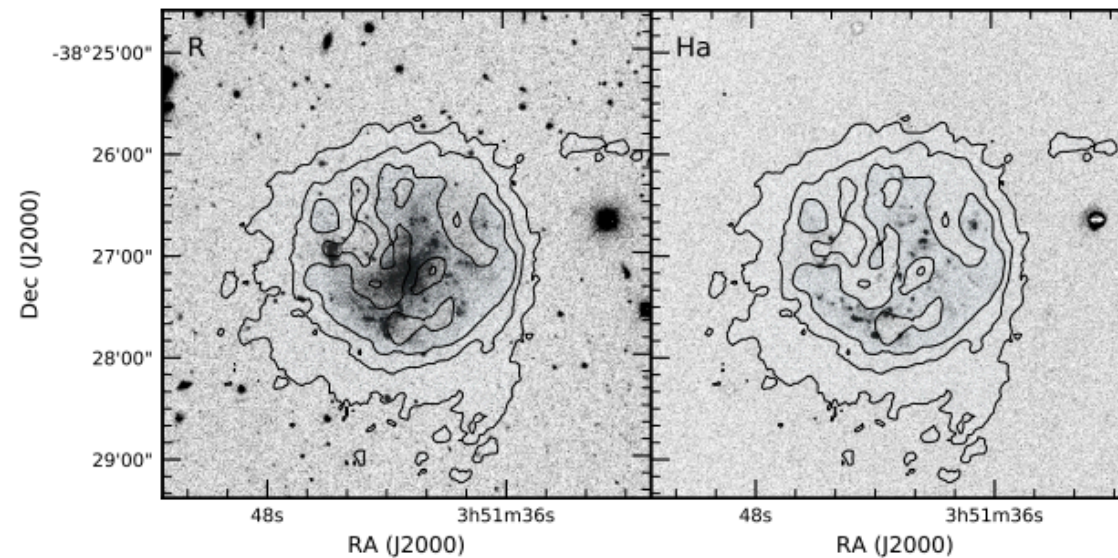


Neutral Hydrogen (HI)

- The HI astronomy



ALFALFA Survey

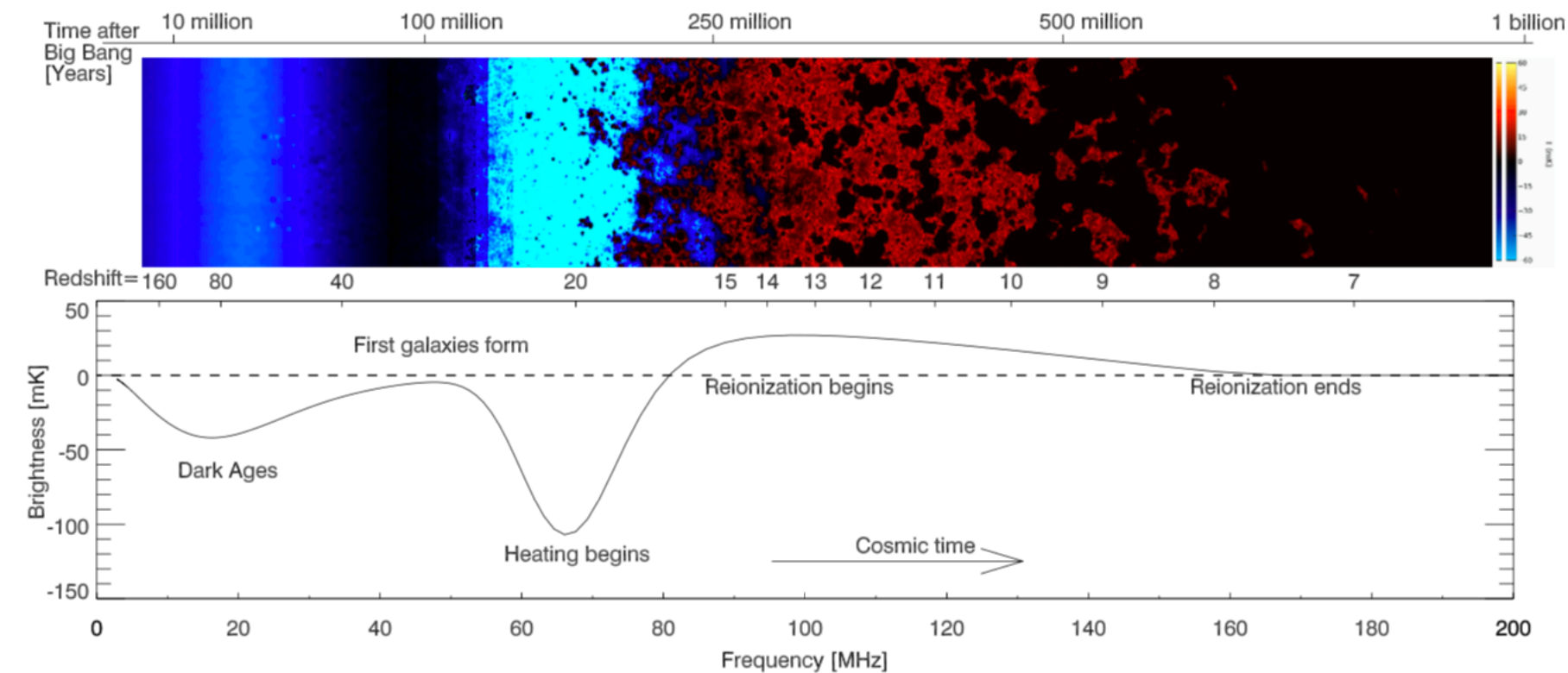


MeerKAT HI commissioning
observations of MHONGOOSE
galaxy ESO 302-G014
arXiv:2009.09766

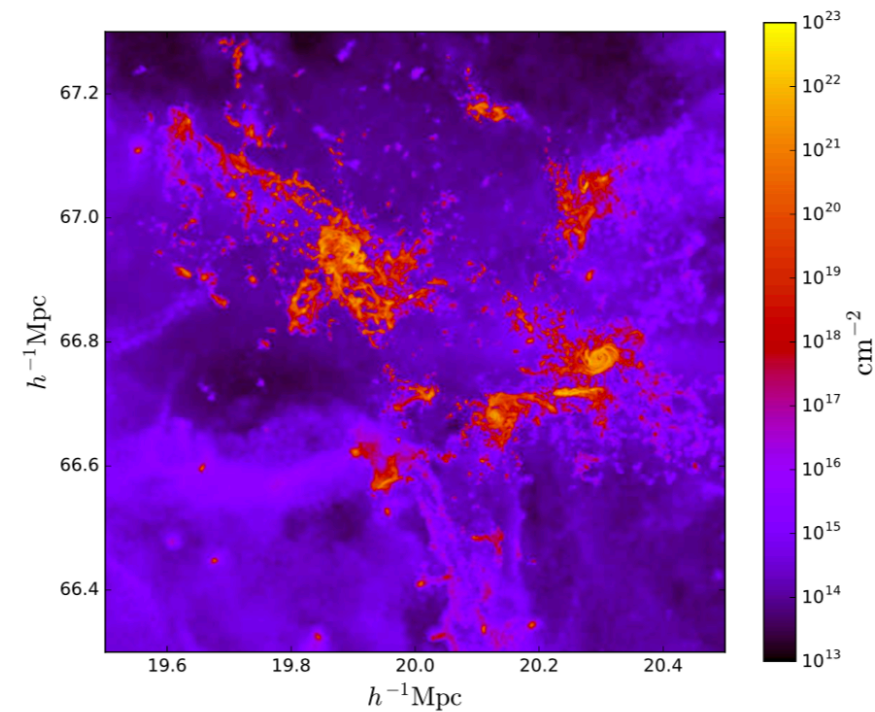
Fig. 6. Overlays of the MeerKAT $r = 0.5$ integrated HI map on SINGG R-band (left panel) and H α (right panel) images. For clarity, only a limited number of column density contours are shown. These are $(1.0, 5.0, 10.0) \cdot 10^{20} \text{ cm}^{-2}$. Darker shading indicates the highest column densities.

Neutral Hydrogen (HI)

- The HI astronomy
- The HI cosmology



Pritchard & Loeb 2012



Villaescusa-Navarro 2018

Neutral Hydrogen (HI)

- The HI astronomy
- The HI cosmology
- HI Galaxy survey
 - ALFALFA
 - HIJASS
 - HIPASS
 - VLA
 - ASKAP/MeerKAT

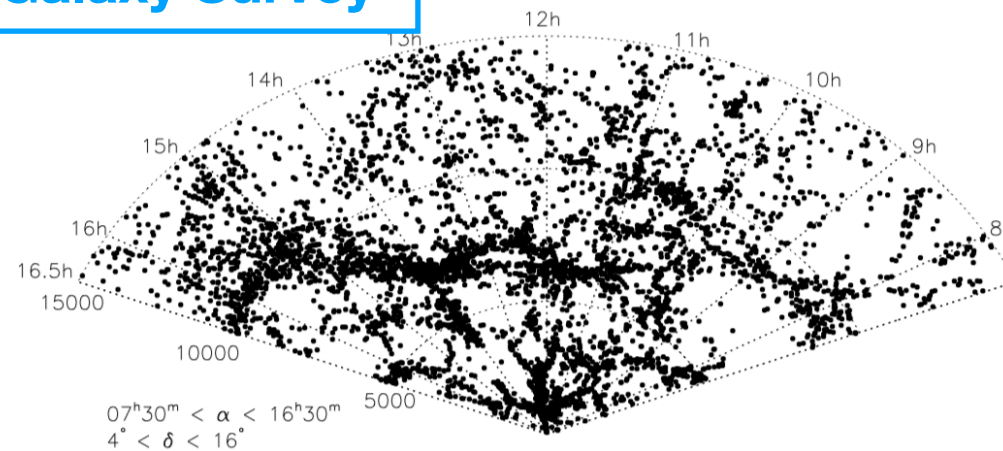
Radio Telescope : Not able to resolve individual galaxy

Neutral Hydrogen (HI)

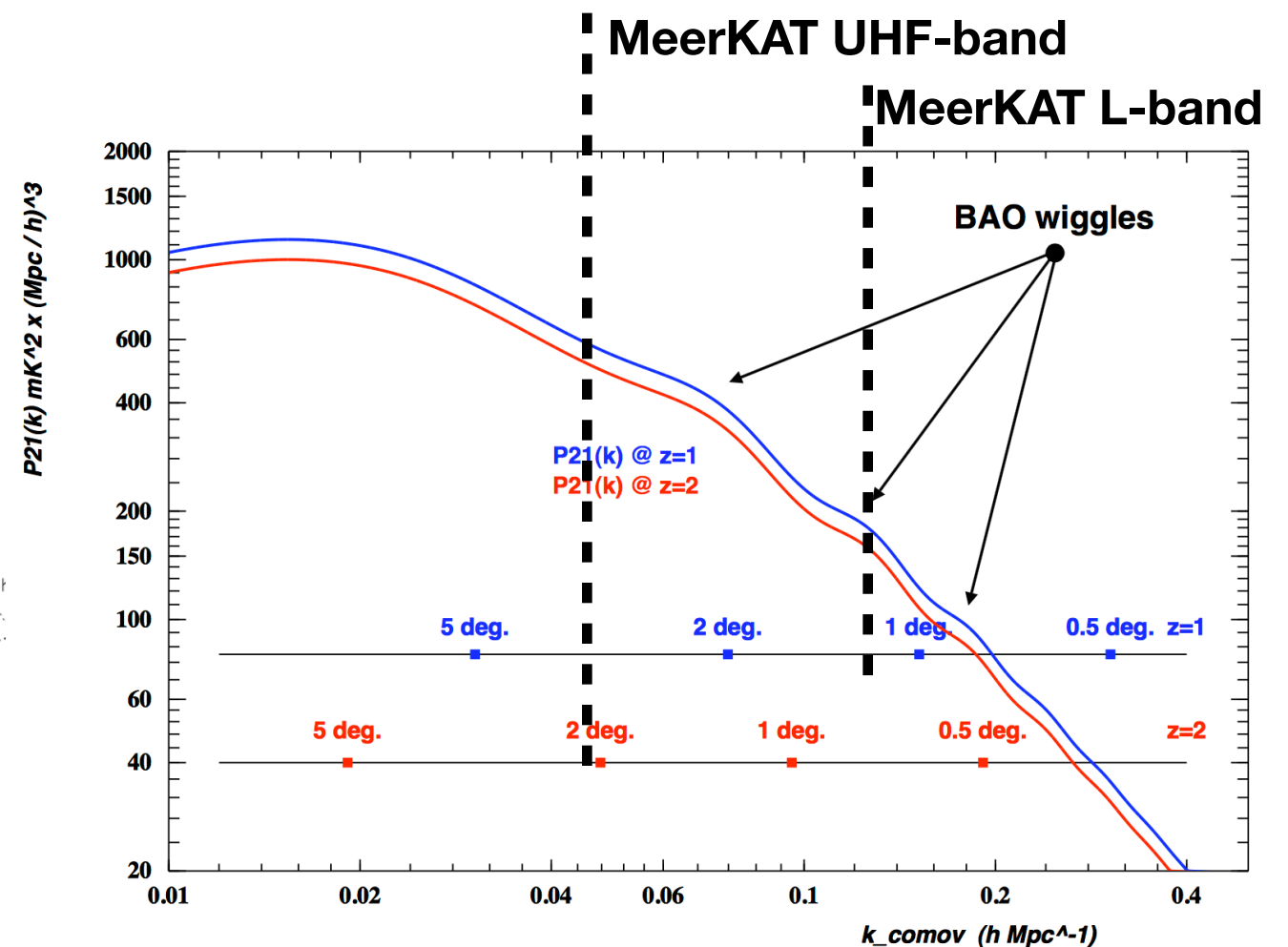
- The HI astronomy
- The HI cosmology
- HI Galaxy survey
 - ALFALFA
 - HIJASS
 - HIPASS
 - VLA
 - ASKAP/MeerKAT
- HI Intensity Mapping

HI Intensity Mapping (IM)

Galaxy Survey



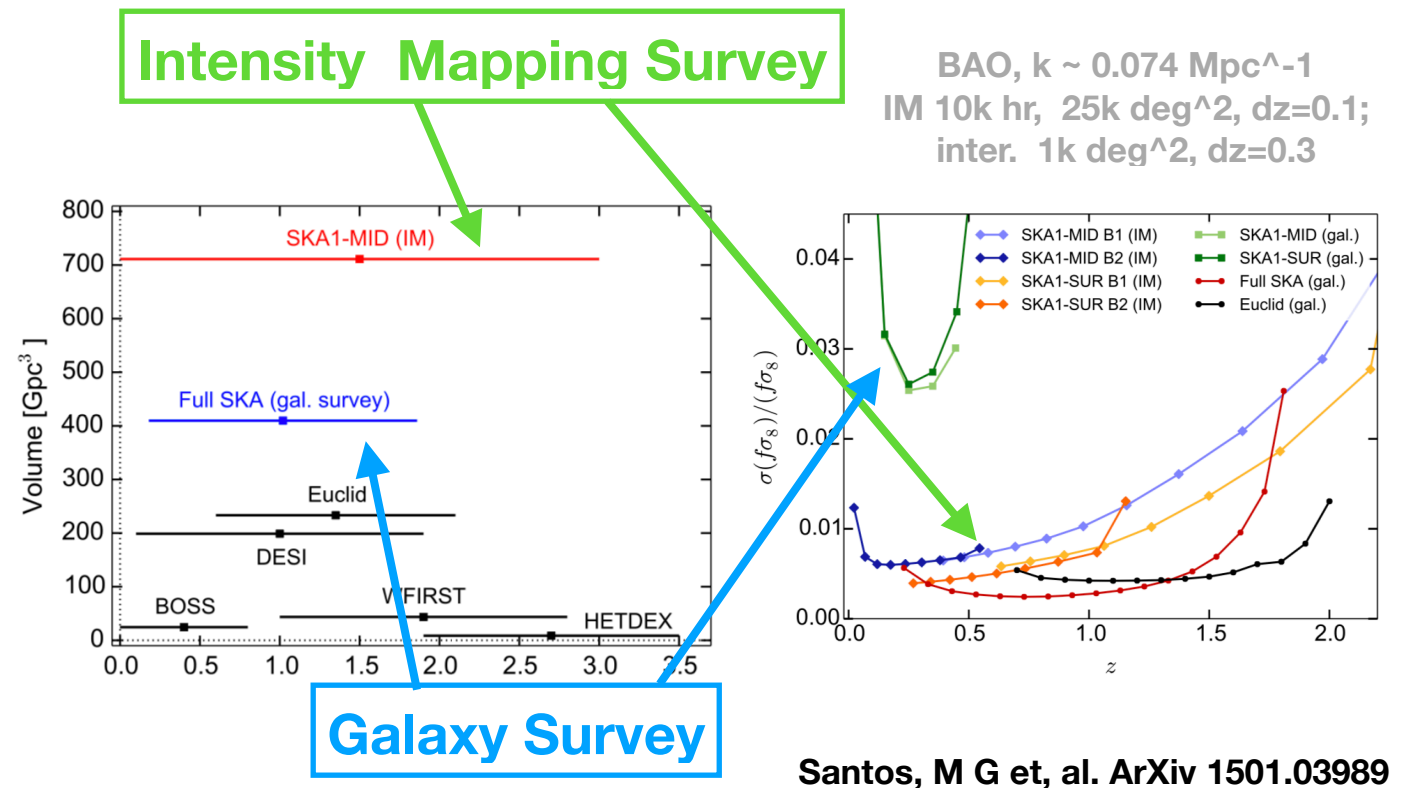
Intensity Mapping Survey



Ansari et. al. 1108.1474

HI IM for cosmology

- **Provide huge observation volume for cosmology studies**
- Multi redshifts
- Multi tracer
- Cosmological Large-scale structure (LSS)
- Baryon Acoustic Oscillation (BAO)
- Redshift space distortion (RSD)
- Dark Energy
- Omega H I
- primordial non-Gaussianity
- EoR



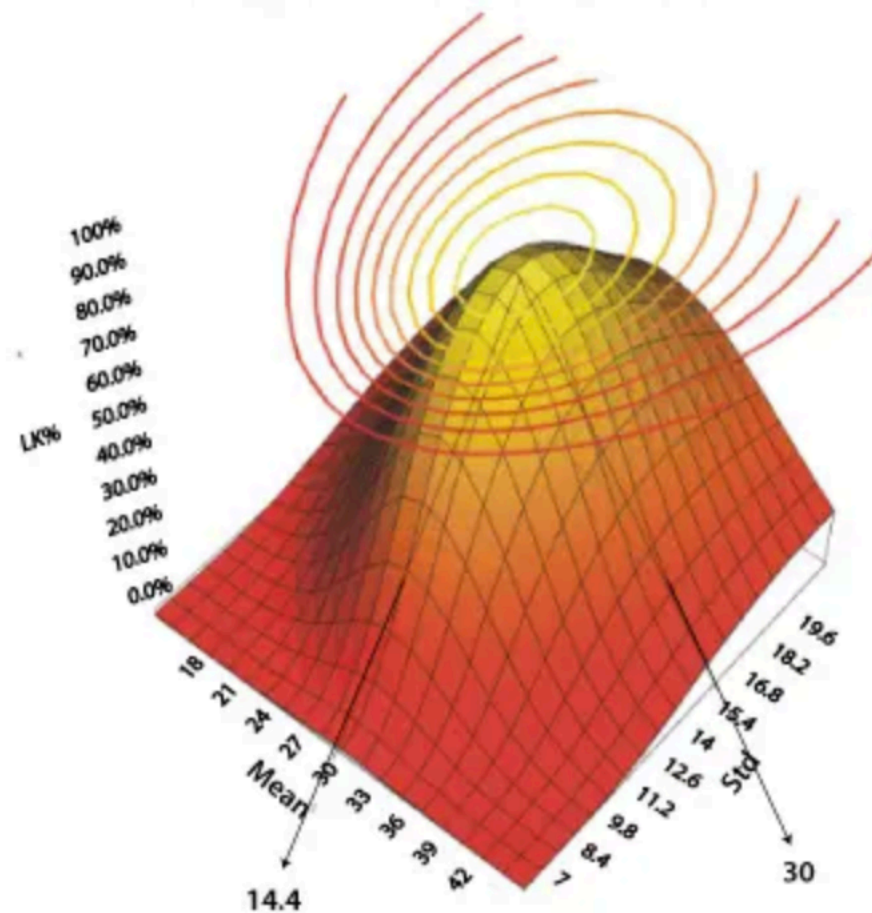
Forecast

The Fisher Information Matrix is given by

$$F_{ij} = \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \right\rangle$$

Where $L = \exp(-0.5\chi^2)$ is the likelihood function.

Likelihood Function Surface



Forecast

$$F_{ij} = \left\langle \frac{\partial^2 \ln \mathcal{L}}{\partial \theta_i \partial \theta_j} \right\rangle$$

$$\chi^2 = \sum_k \frac{(\hat{P}(k) - P^{\text{th}}(k, \{\lambda_\alpha\}))^2}{\text{Var} [\hat{P}(k)]} = \chi^2(\bar{\lambda}) + \mathcal{F}(\lambda - \bar{\lambda})^2$$

$$\mathcal{F} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \lambda^2} \Big|_{\lambda=\bar{\lambda}} = \sum_k \frac{1}{\text{Var} [\hat{P}(k)]} \left(\left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda} \right)^2 + \left(P^{\text{th}}(k) - \hat{P}(k) \right) \frac{\partial^2 P^{\text{th}}(k)}{\partial \lambda^2} \right)$$

$$F_{\alpha\beta} = \sum_k \frac{1}{\text{Var} [\hat{P}(k)]} \left(\left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\alpha} \right) \left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\beta} \right) \right)$$

Forecast

$$P(\mathbf{k}) = \langle \delta(\mathbf{k}) \delta^*(\mathbf{k}) \rangle = |\delta(\mathbf{k})|^2$$

$$\hat{P}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{\mathbf{k}}^{||\mathbf{k}| - k_\alpha| < \Delta k/2} |\delta(\mathbf{k})|^2, \quad m_{k,\alpha} = \frac{1}{2} \frac{4\pi k_\alpha^2 \Delta k}{k_F^3} = \frac{1}{4\pi^2} V k_\alpha^2 \Delta k$$

$$\delta(\mathbf{k}) = \delta_{\text{HI}}(\mathbf{k}) + \delta_{\text{Noise}}(\mathbf{k})$$

$$\hat{P}_{\text{HI}}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{\mathbf{k}}^{||\mathbf{k}| - k_\alpha| < \Delta k/2} |\delta(\mathbf{k})|^2 - P_N$$

$$\text{Cov}_{\alpha\beta} = \langle \hat{P}_{\text{HI}}(k_\alpha) \hat{P}_{\text{HI}}(k_\beta) \rangle - \langle \hat{P}_{\text{HI}}(k_\alpha) \rangle \langle \hat{P}_{\text{HI}}(k_\beta) \rangle = \dots = \frac{2}{m_{k,\alpha}} (P_{\text{HI}}(k_\alpha) + P_N)^2 \delta_{\alpha\beta}$$

Forecast

$$P(\mathbf{k}) = \langle \delta(\mathbf{k}) \delta^*(\mathbf{k}) \rangle = |\delta(\mathbf{k})|^2$$

$$\hat{P}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{|\mathbf{k}| - k_\alpha < \Delta k/2} |\delta(\mathbf{k})|^2, \quad m_{k,\alpha} = \frac{1}{2} \frac{4\pi k_\alpha^2 \Delta k}{k_F^3} = \frac{1}{4\pi^2} V k_\alpha^2 \Delta k$$

$$\delta(\mathbf{k}) = \delta_{\text{HI}}(\mathbf{k}) + \delta_{\text{Noise}}(\mathbf{k})$$

Scott Dodelson & Fabian Schmidt
Modern Cosmology 2021
Chapter 14.4.2

$$\hat{P}_{\text{HI}}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{|\mathbf{k}| - k_\alpha < \Delta k/2} |\delta(\mathbf{k})|^2 - P_N$$

$$\text{Cov}_{\alpha\beta} = \langle \hat{P}_{\text{HI}}(k_\alpha) \hat{P}_{\text{HI}}(k_\beta) \rangle - \langle \hat{P}_{\text{HI}}(k_\alpha) \rangle \langle \hat{P}_{\text{HI}}(k_\beta) \rangle = \dots = \frac{2}{m_{k,\alpha}} (P_{\text{HI}}(k_\alpha) + P_N)^2 \delta_{\alpha\beta}$$

$$\sqrt{\text{Var} [\hat{P}_{\text{HI}}(k_\alpha)]} = \frac{2}{m_{k,\alpha}} (P_{\text{HI}}(k) + P_N)$$

Forecast

$$\hat{P}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{|\mathbf{k}| - k_\alpha < \Delta k/2} |\delta(\mathbf{k})|^2, \quad m_{k,\alpha} = \frac{1}{2} \frac{4\pi k_\alpha^2 \Delta k}{k_F^3} = \frac{1}{4\pi^2} V k_\alpha^2 \Delta k$$

$$\sqrt{\text{Var} [\hat{P}_{\text{HI}}(k_\alpha)]} = \frac{2}{m_{k,\alpha}} (P_{\text{HI}}(k) + P_N)$$

$$\chi^2 = \sum_k \frac{(\hat{P}(k) - P^{\text{th}}(k, \{\lambda_\alpha\}))^2}{\text{Var} [\hat{P}(k)]} = \chi^2(\bar{\lambda}) + \mathcal{F}(\lambda - \bar{\lambda})^2$$

$$\mathcal{F} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial \lambda^2} \Big|_{\lambda=\bar{\lambda}} = \sum_k \frac{1}{\text{Var} [\hat{P}(k)]} \left(\left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda} \right)^2 + \left(P^{\text{th}}(k) - \hat{P}(k) \right) \frac{\partial^2 P^{\text{th}}(k)}{\partial \lambda^2} \right)$$

$$F_{\alpha\beta} = \sum_k \frac{1}{\text{Var} [\hat{P}(k)]} \left(\left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\alpha} \right) \left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\beta} \right) \right)$$

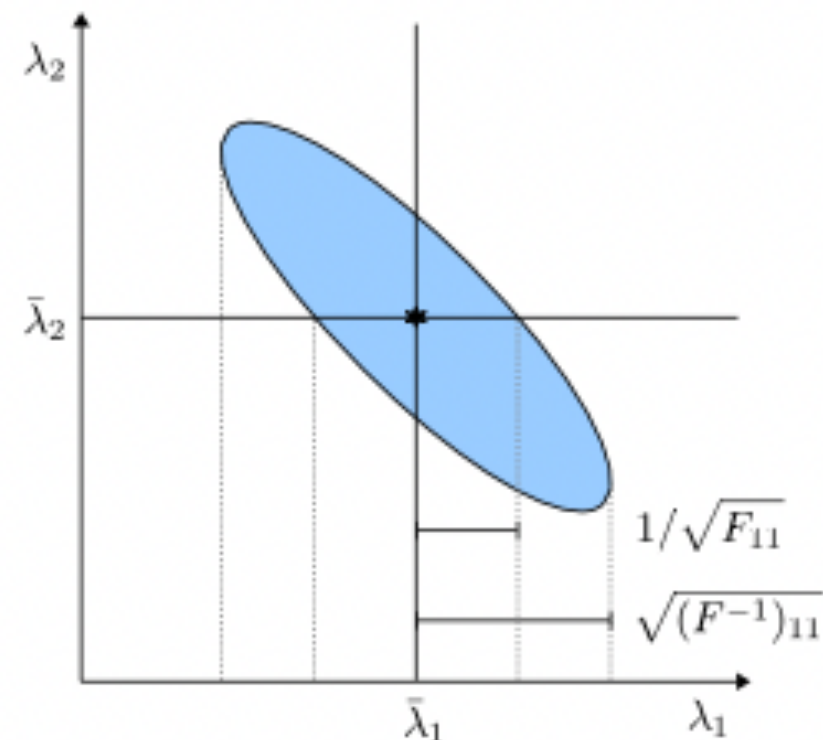
Forecast

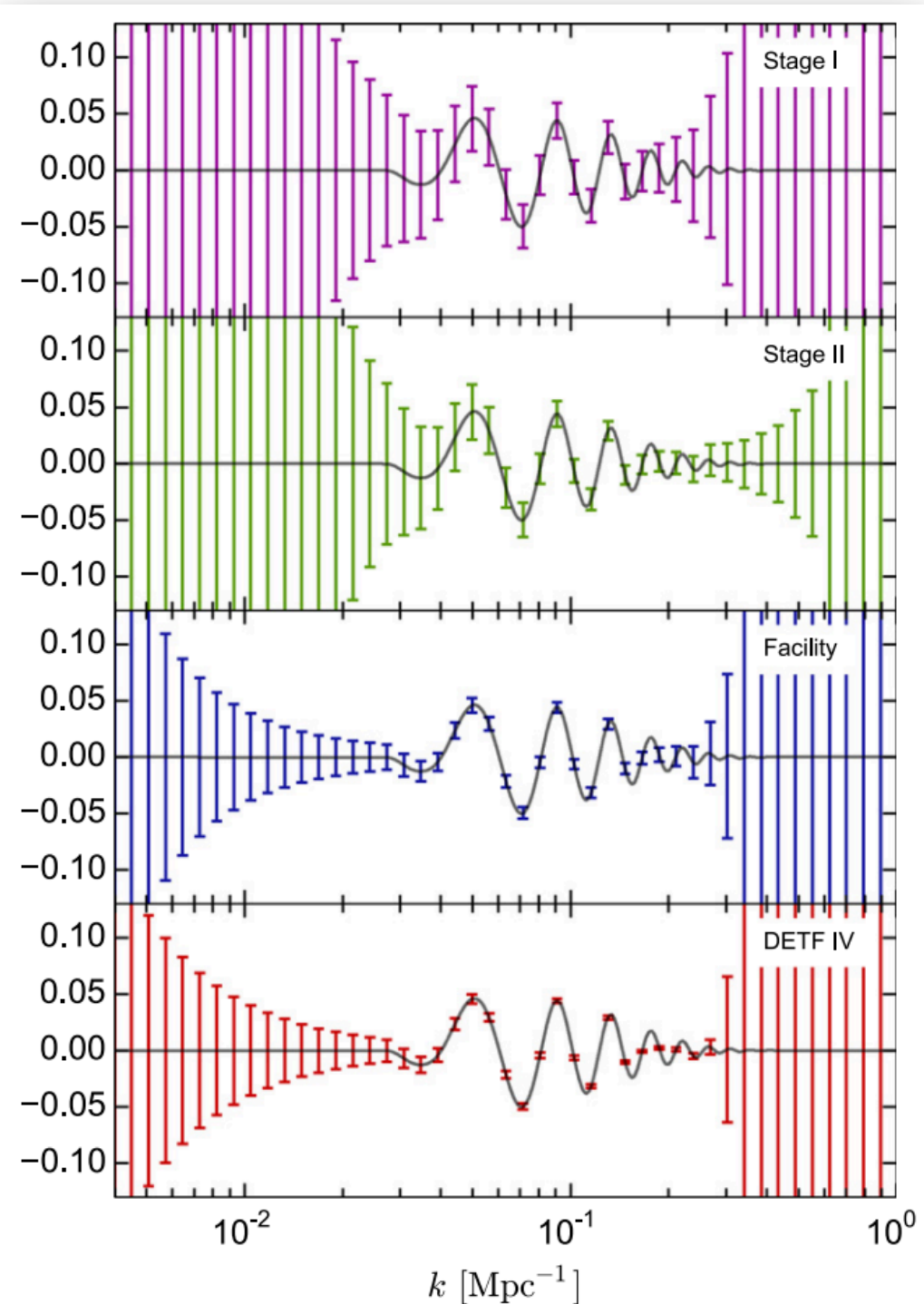
$$\hat{P}(k_\alpha) = \frac{1}{m_{k,\alpha}} \sum_{\mathbf{k} \mid ||\mathbf{k}| - k_\alpha| < \Delta k/2} |\delta(\mathbf{k})|^2, \quad m_{k,\alpha} = \frac{1}{2} \frac{4\pi k_\alpha^2 \Delta k}{k_F^3} = \frac{1}{4\pi^2} V k_\alpha^2 \Delta k$$

$$\boxed{\sqrt{\text{Var} [\hat{P}_{\text{HI}}(k_\alpha)]} = \frac{2}{m_{k,\alpha}} (P_{\text{HI}}(k) + P_N)}$$

$$F_{\alpha\beta} = \sum_k \frac{1}{\text{Var} [\hat{P}(k)]} \left(\left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\alpha} \right) \left(\frac{\partial P^{\text{th}}(k)}{\partial \lambda_\beta} \right) \right)$$

$$\left(\frac{\Delta P_a}{P_a} \right)^2 = \left(\frac{1}{2} V_{\text{bin}} \int \frac{k^2 d\mu dk}{(2\pi)^2} \left(\frac{P^{\text{HI}}(k, \mu)}{P^{\text{HI}}(k, \mu) + N(k, \mu)} \right)^2 \right)^{-1}$$





GBT
Parkes

MeerKAT
ASKAP

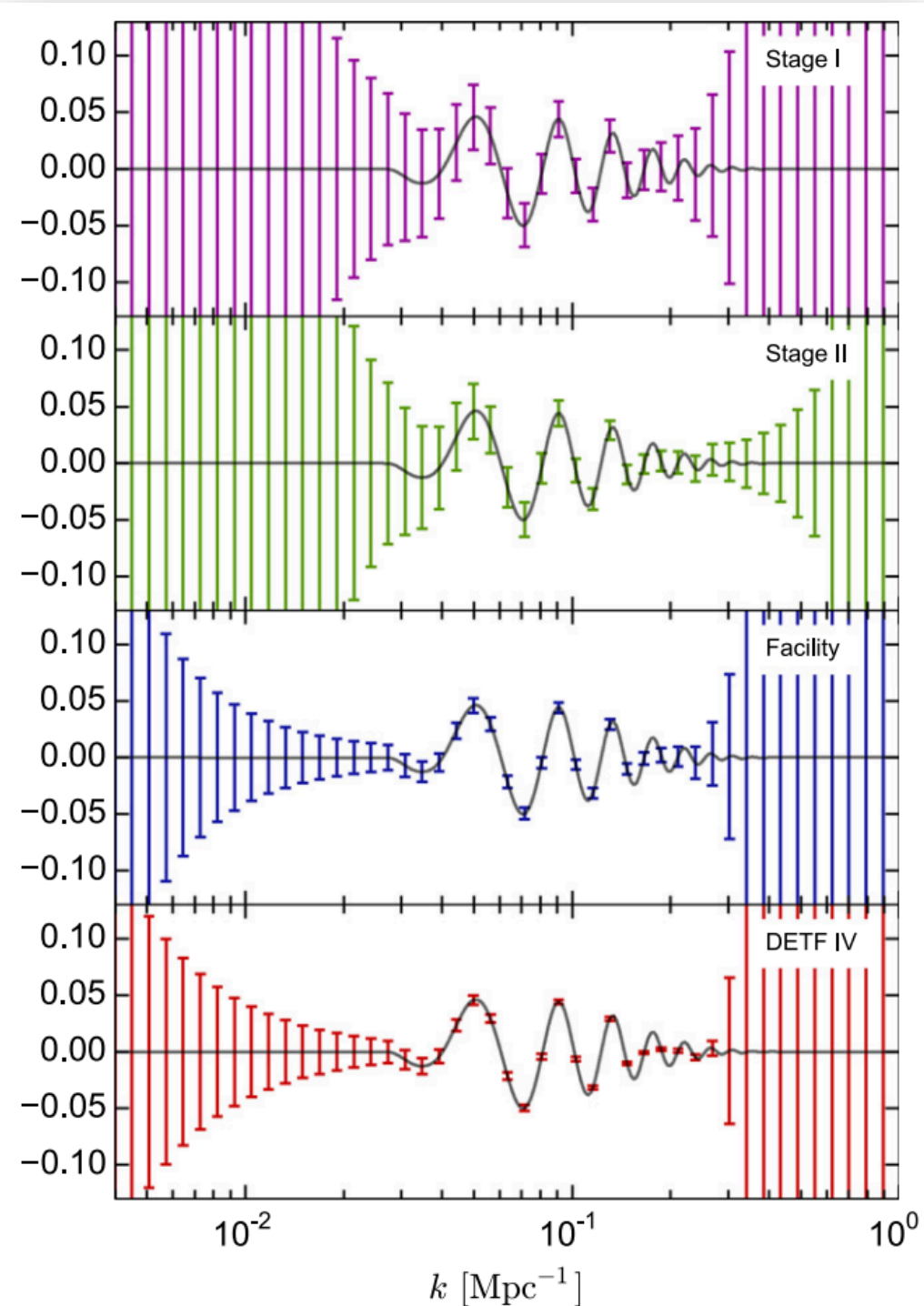
Tianlai
CHIME
BINGO

SKA
Full Tianlai
Full CHIME

$$\left(\frac{\Delta P_a}{P_a} \right)^2 = \left(\frac{1}{2} V_{\text{bin}} \int \frac{k^2 d\mu dk}{(2\pi)^2} \left(\frac{P^{\text{HI}}(k, \mu)}{P^{\text{HI}}(k, \mu) + N(k, \mu)} \right)^2 \right)^{-1}$$

- Late-time cosmology with 21 cm intensity mapping experiments

Bull P. et al 2015, ApJ, 803, 21.
arXiv: 1405.1452



**GBT
Parkes**

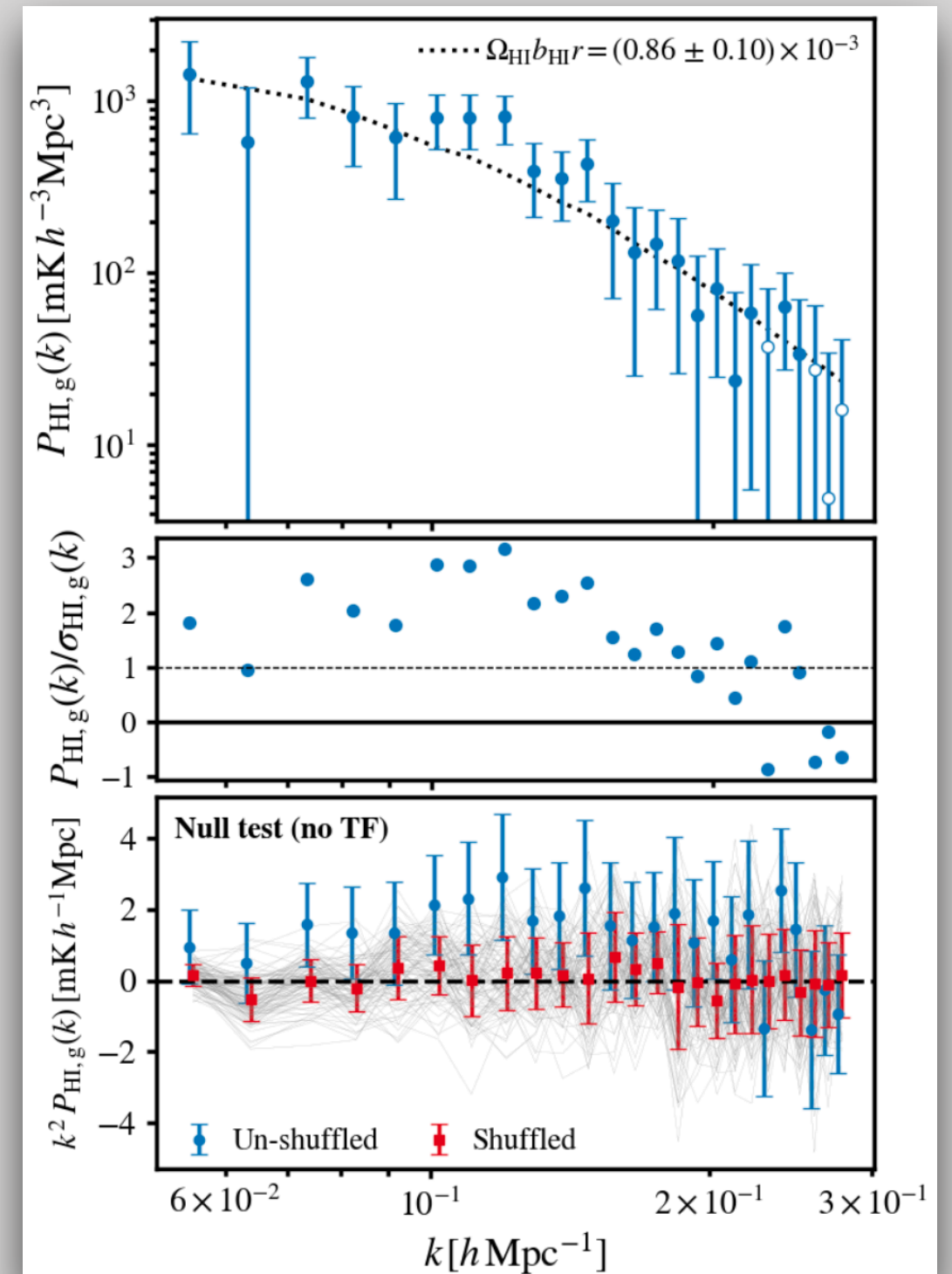
**MeerKAT
ASKAP**

**Tianlai
CHIME
BINGO**

**SKA
Full Tianlai
Full CHIME**

- Late-time cosmology with 21 cm intensity mapping experiments

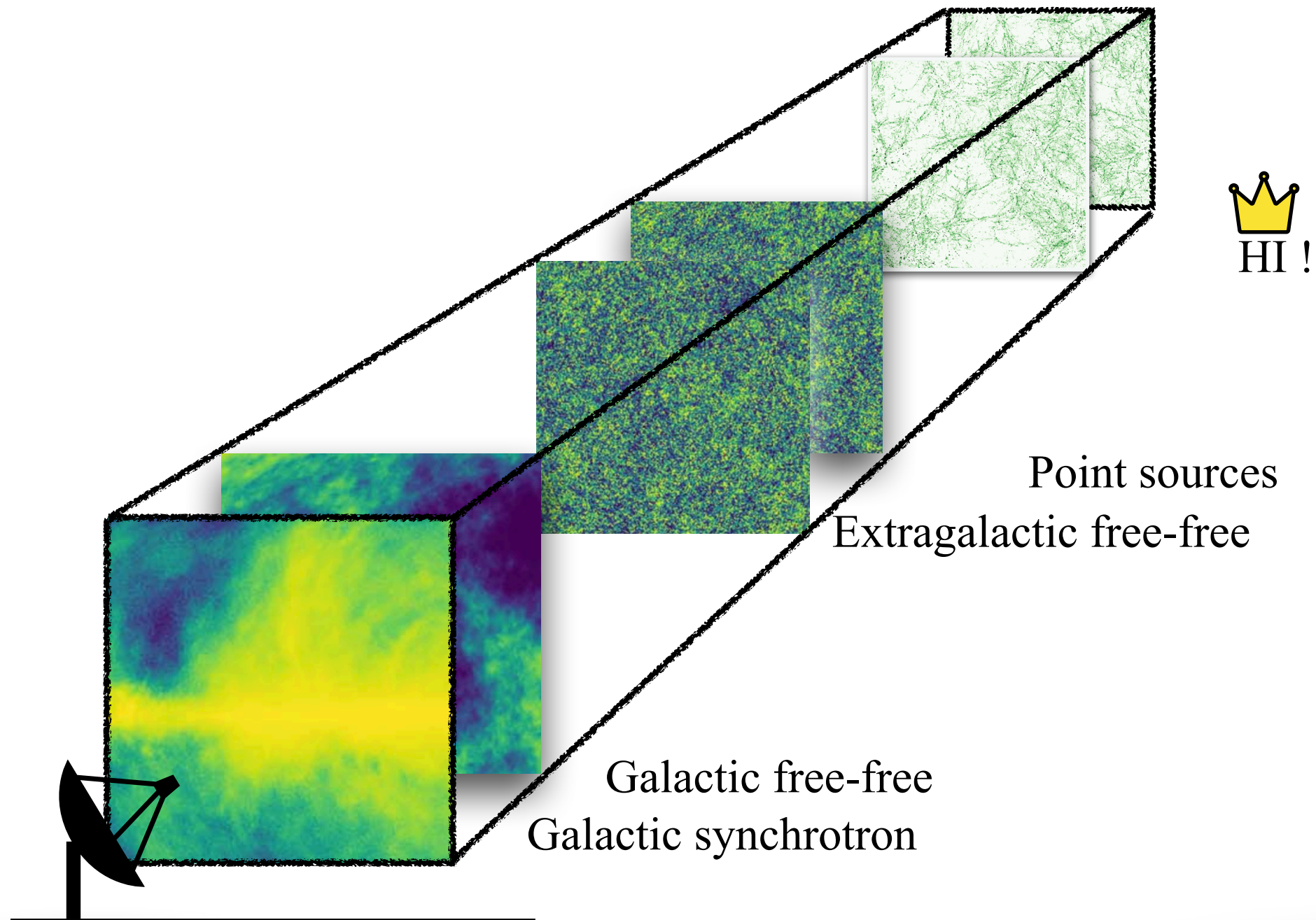
Bull P. et al 2015, ApJ, 803, 21.
arXiv: 1405.1452



- 10.5 hour HI IM observation using MeerKAT
64 dishes, we achieve 7.7σ detection of the cross-correlation power spectrum;

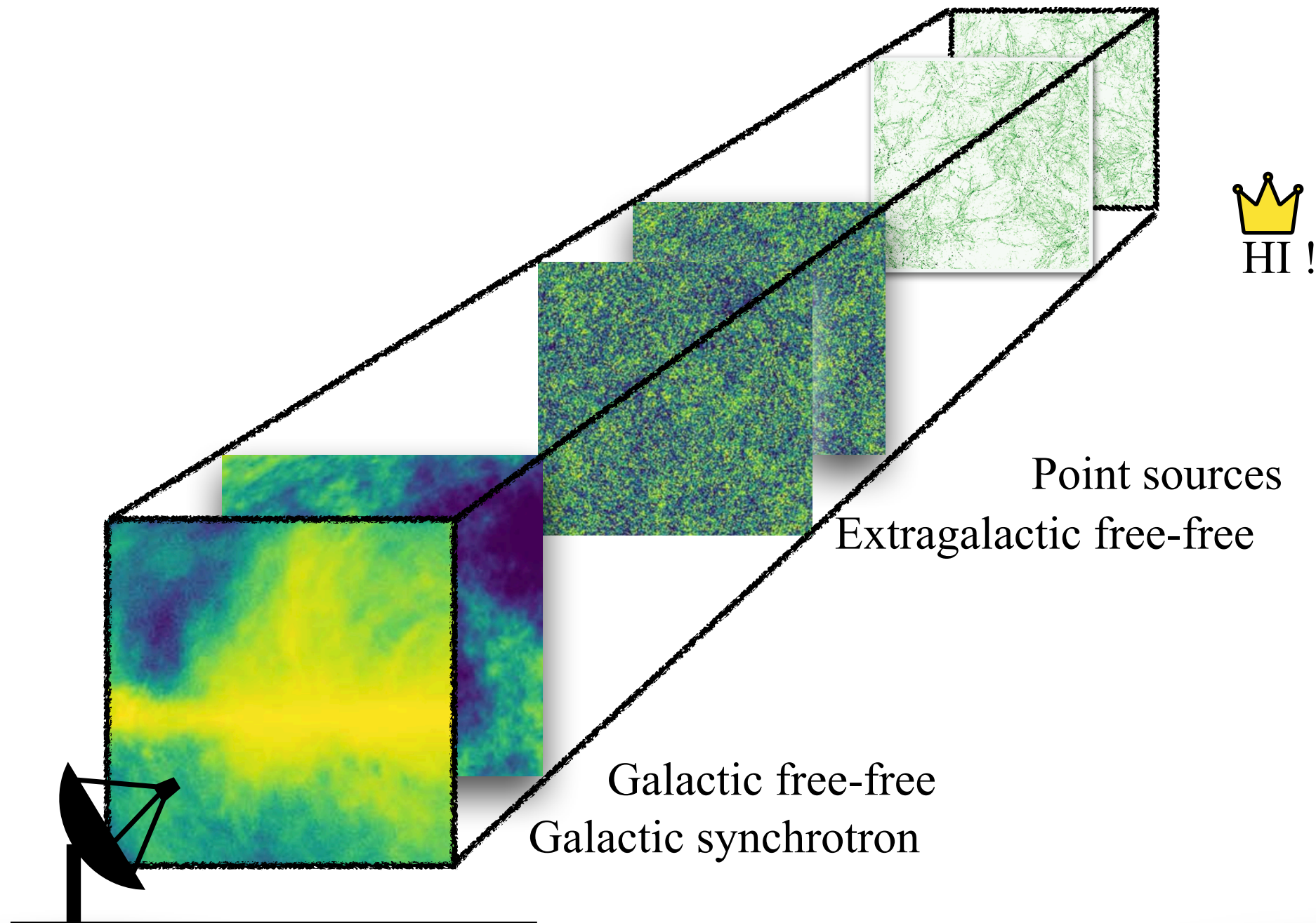
Cunnington S., Li Y., Santos M.~G., et al.
arXiv: 2206.01579

The Foreground



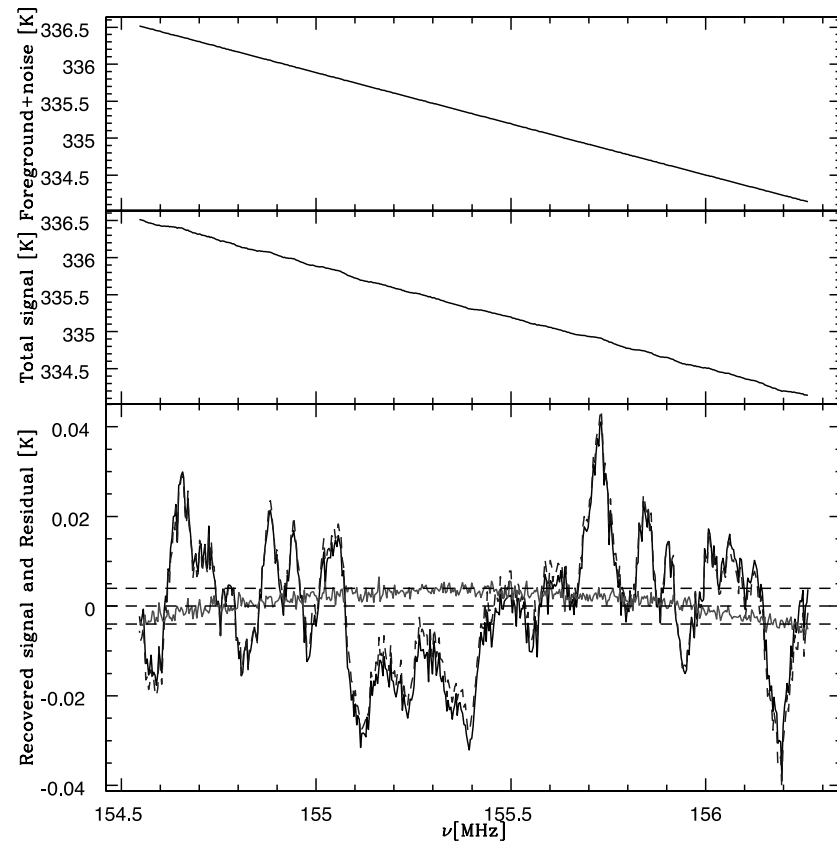
The Foreground

The secret to successfully recovering the underlying HI fluctuation is to correctly remove the bright foreground contamination.



FG Subtraction

- Model dependent foreground subtraction method
 - Logarithmic polynomial fitting



Wang, X., et. al (2006). *ApJ*, 650(2), 529–537.

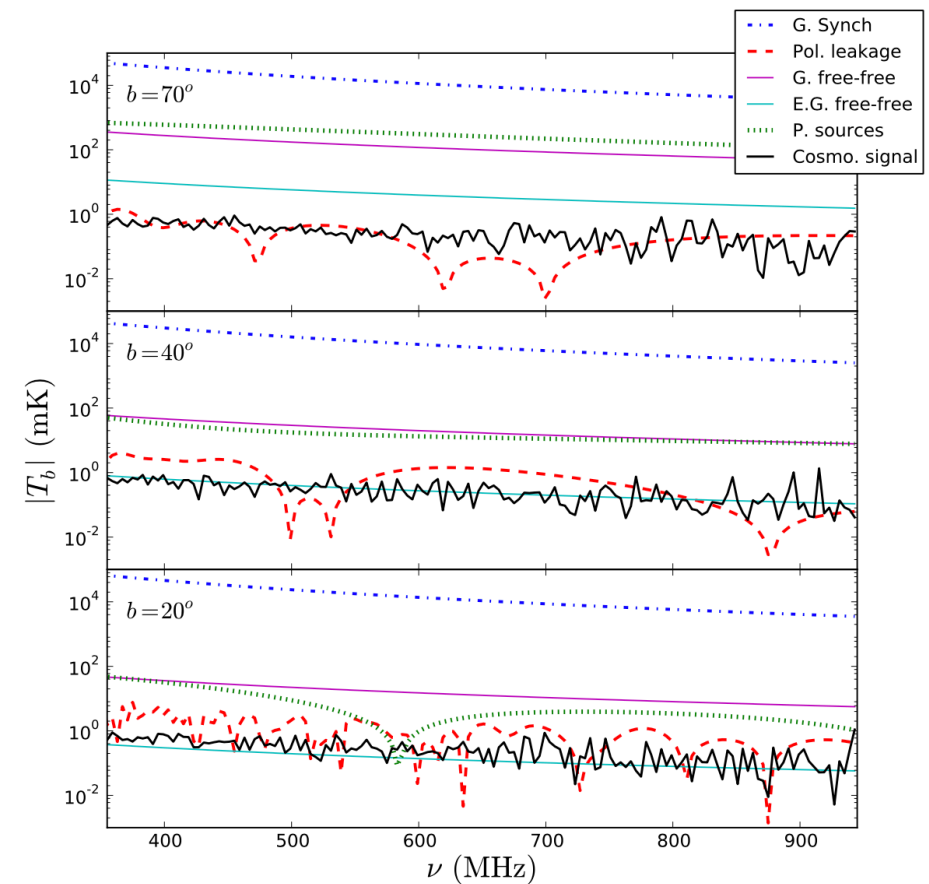
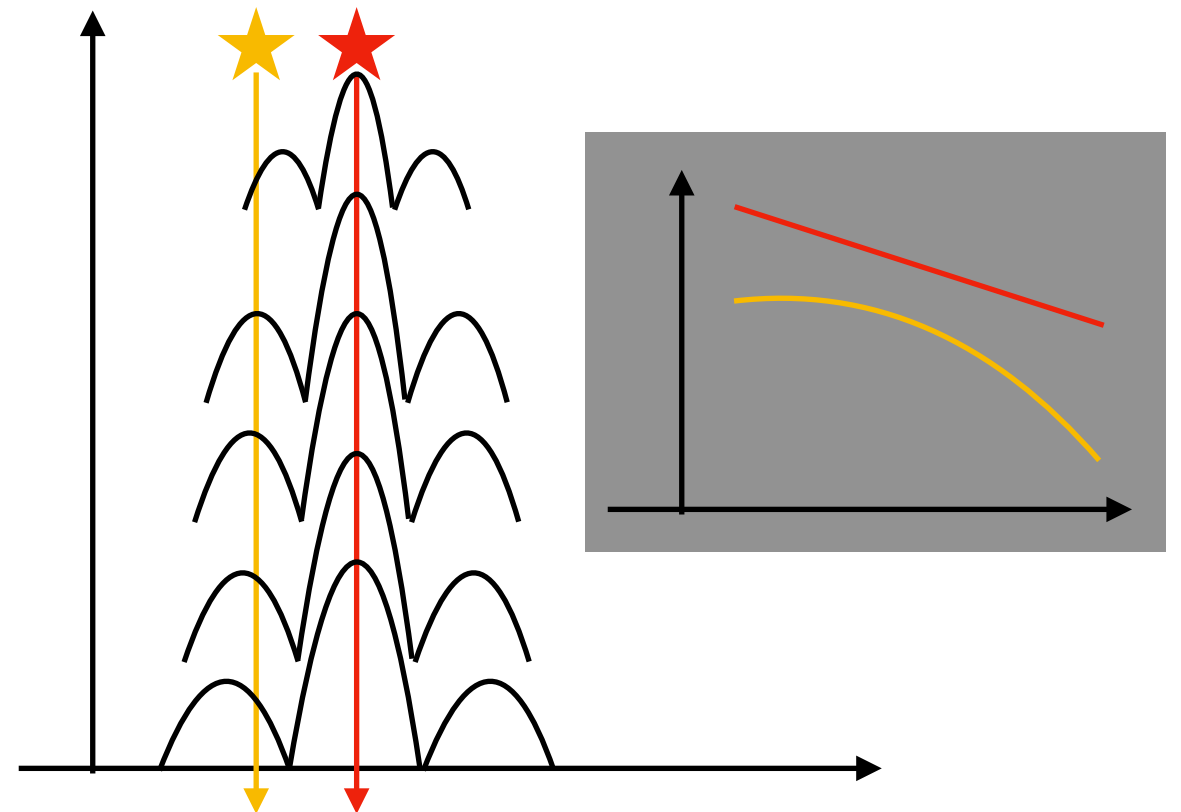
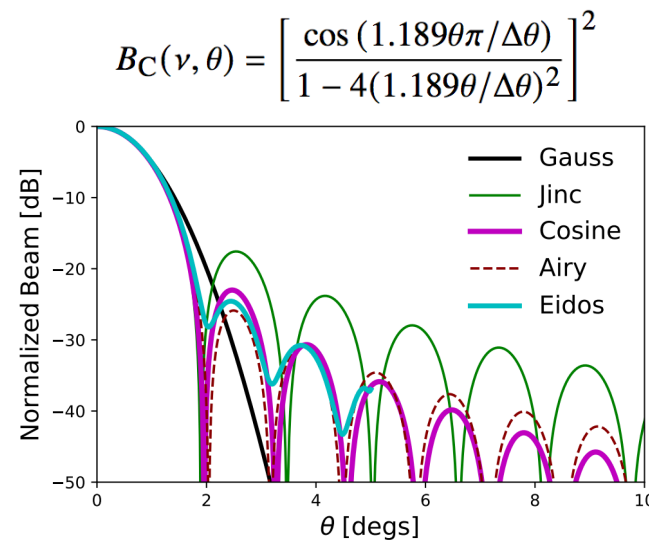
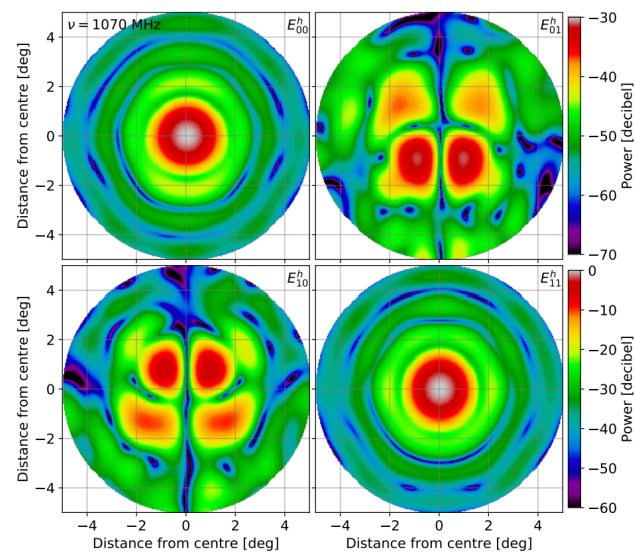


Figure 8. Frequency dependence of the different foregrounds and the cosmological signal along LOS with different galactic latitudes (given in the top-right corner of each panel). The effect of Faraday decorrelation increases as we approach the galactic plane, making the subtraction of the polarization leakage more challenging.

Alonso, D. et al. (2014) *MNRAS* 444, 3183

FG Subtraction

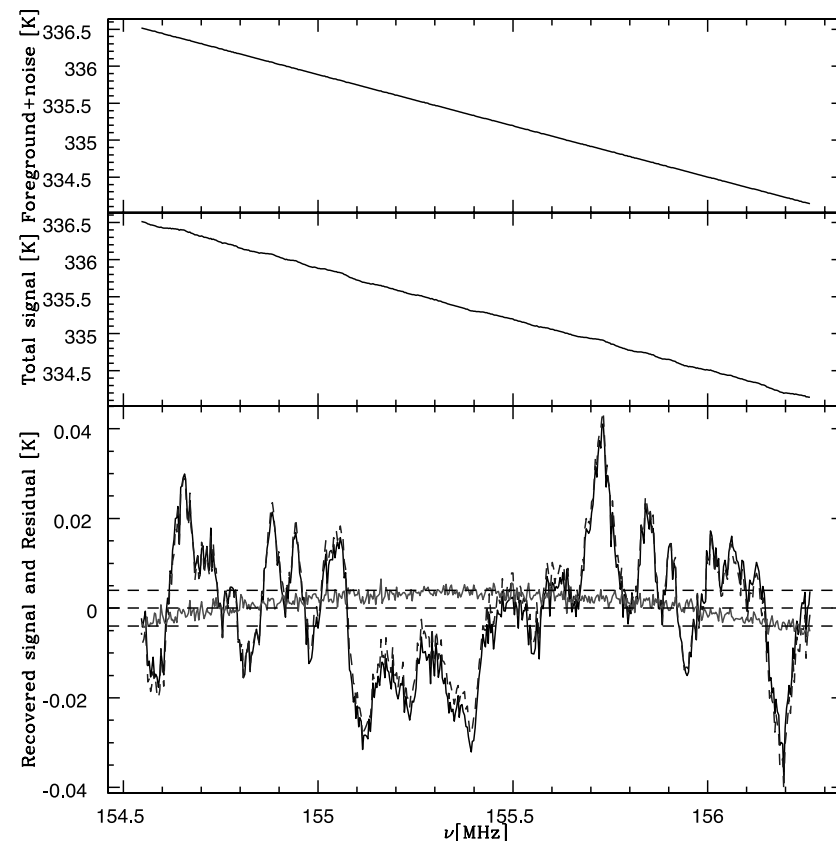
- Model dependent foreground subtraction method
 - Logarithmic polynomial fitting



FG Subtraction

Model dependent method failed when there is systematic effect.

- Model dependent foreground subtraction method
 - Logarithmic polynomial fitting



Wang, X., et. al (2006). *ApJ*, 650(2), 529–537.

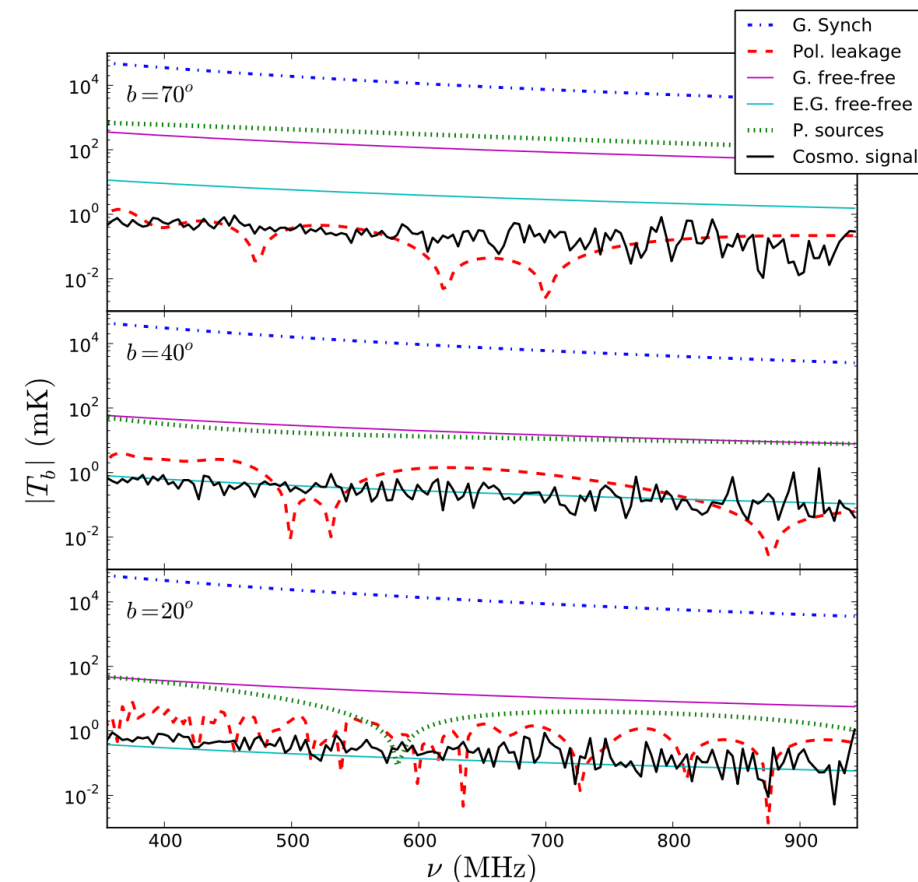


Figure 8. Frequency dependence of the different foregrounds and the cosmological signal along LOS with different galactic latitudes (given in the top-right corner of each panel). The effect of Faraday decorrelation increases as we approach the galactic plane, making the subtraction of the polarization leakage more challenging.

Alonso, D. et al. (2014) *MNRAS* 444, 3183

FG Subtraction

- Model independent foreground subtraction method
 - Principle Component Analysis (PCA)

Assume that FG is strongly correlated across frequency

$$C_{ij} = \frac{1}{N} \sum_p m(\nu_i, n_p) m(\nu_j, n_p)$$
$$\mathbf{C}\mathbf{V} = \mathbf{C}\mathbf{\Lambda}$$

Cross-correlation function with GBT HIIM x DEEP2
T.-C. Chang et al. 2010 Nature Vol 466

Cross PS GBT HIIM x WiggleZ
K. Masui et al 2013 ApJ 763L 20M

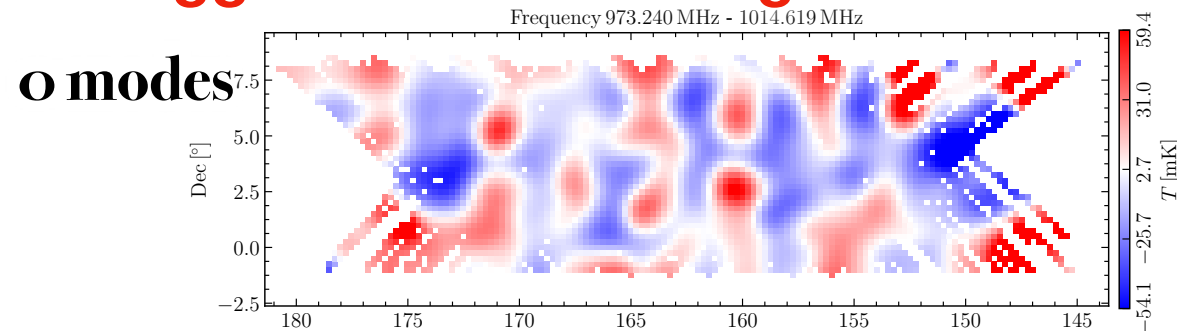
Cross PS PKS x 2dF
C. Anderson, N. J. Luciw, Y. Li et, al 2018, MNRAS, 476, 3382.

Cross PS MeerKAT x WiggleZ
Cunnington S., Li Y., Santos M.~G., arXiv: 2206.01579

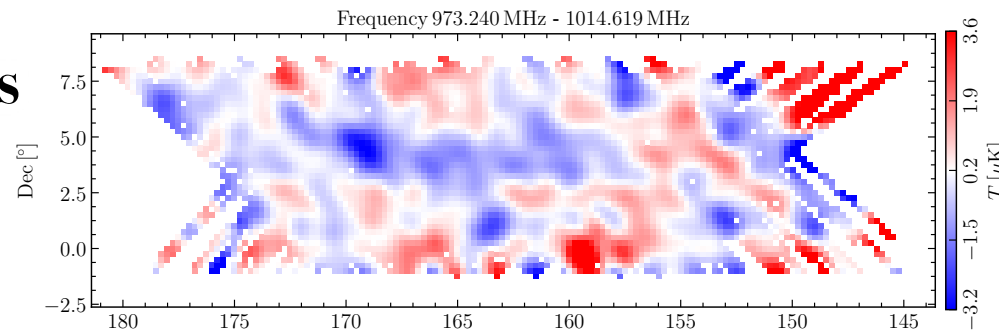
GBT Auto PS
Switzer E.~R., et al., 2013, MNRAS, 434, L46.

MeerKAT HI IM

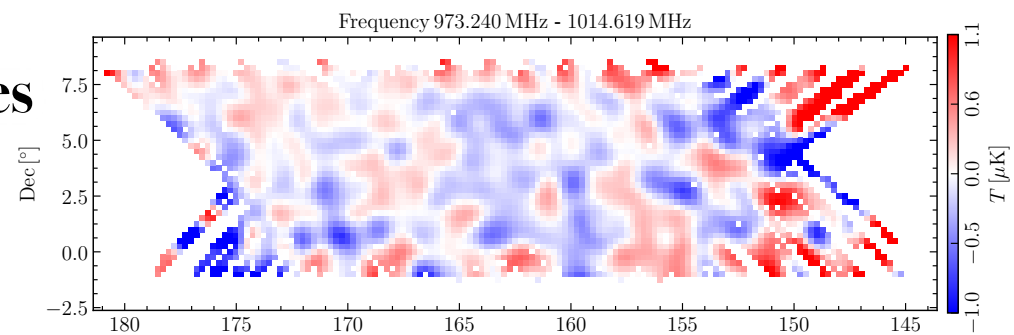
Aggressive PCA foreground subtraction kills HI signals



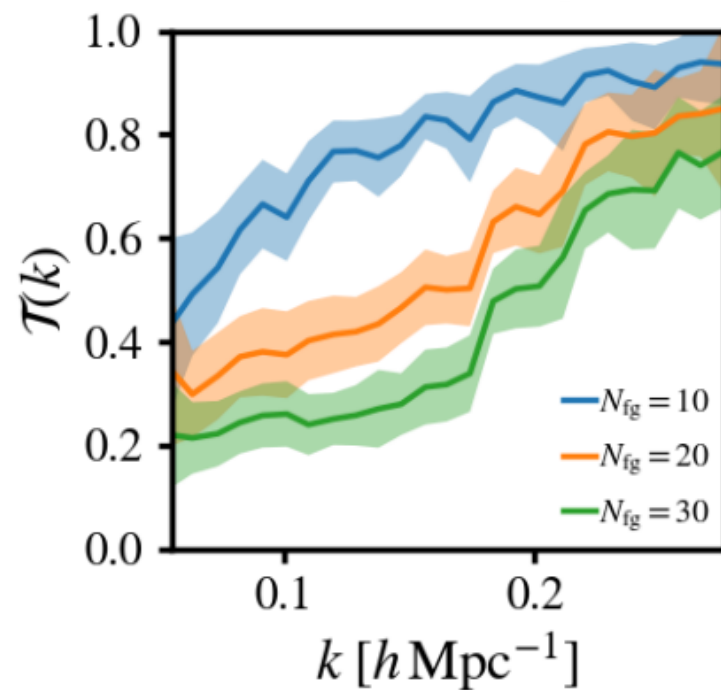
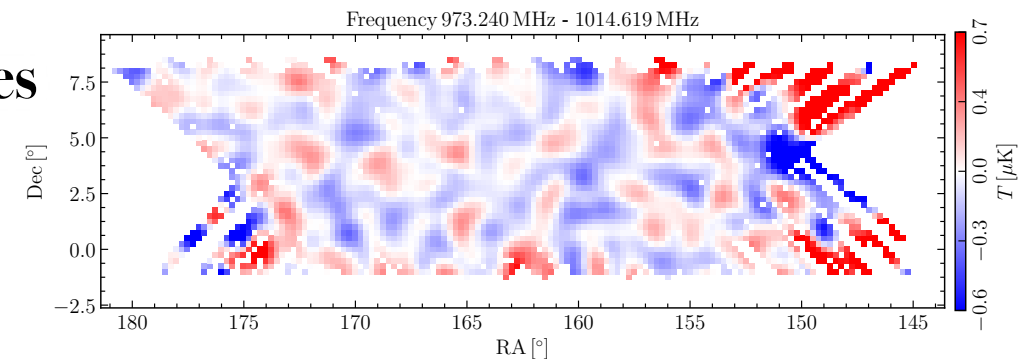
5 modes



20 modes



30 modes



Cross PS MeerKAT x WiggleZ
Cunnington S., Li Y., Santos M.~G., arXiv: 2206.01579

Power Spectrum Estimation with HI IM

- The noise inverse weight

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

$$W(r) = \frac{1}{P(k) + 1/n(r)} \Rightarrow W(r) = \frac{1}{P(k) + 1/n(r) + P_N}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

Power Spectrum Estimation with HI IM

- The noise inverse weight

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3 k' + N(0)$$

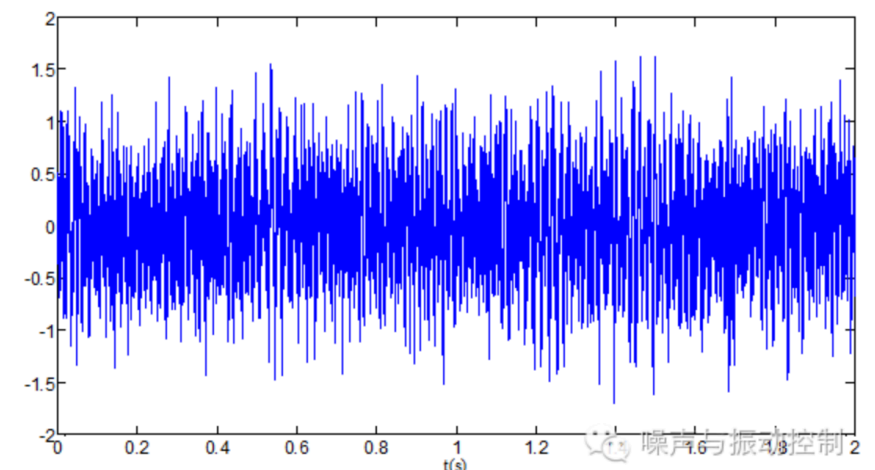
$$W(r) = \frac{1}{P(k) + 1/n(r)} \Rightarrow W(r) = \frac{1}{P(k) + 1/n(r) + P_N}$$

$$\int |W(k - k')|^2 P(k') d^3 k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

天线温度

噪声功率 vs 噪声方差

- $V = \alpha G k \Delta \nu (T_A + T_{\text{sys}})$
 - 噪声抬高系统的平均功率
 - 噪声本身产生电压的随机涨落
- $\sigma = \frac{T_{\text{sys}}}{\sqrt{\delta t \delta \nu}}$
- 表征系统灵敏度



Power Spectrum Estimation with HI IM

- The noise inverse weight

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

$$W(r) = \frac{1}{P(k) + 1/n(r)} \Rightarrow W(r) = \frac{1}{P(k) + 1/n(r) + P_N}$$

$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

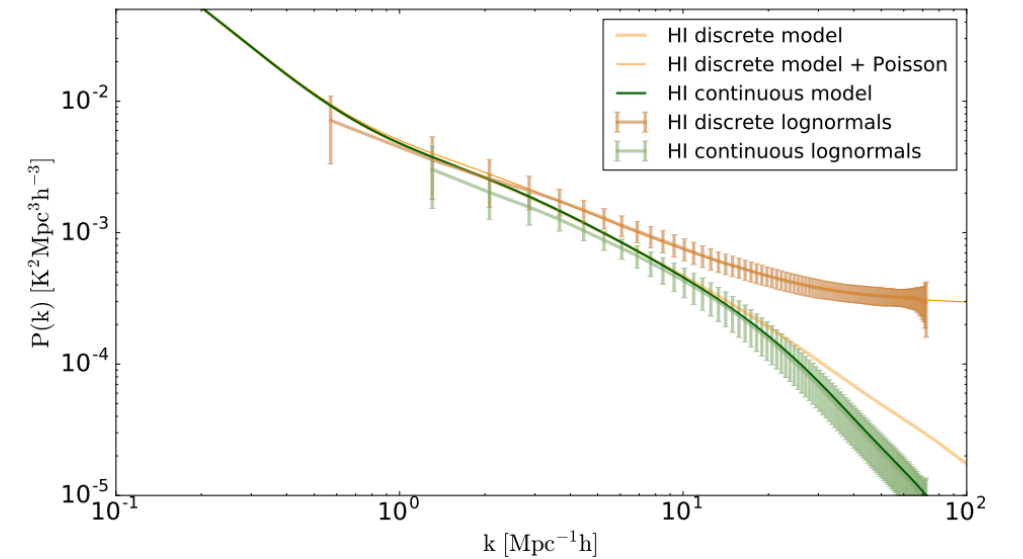


Figure 4. The HI power spectra predicted by HALOMOD for the HI continuous model and the HI discrete model in comparison with a average power spectrum of 100 lognormal realisations with a box of 15Mpc/h drawn from the respective galaxy HOD. Note that the HI continuous model does not include a scale-independent Poisson noise contribution since it is estimated from a continuum field. We show HALOMOD predictions including and excluding Poisson noise contribution.

Power Spectrum Estimation with HI IM

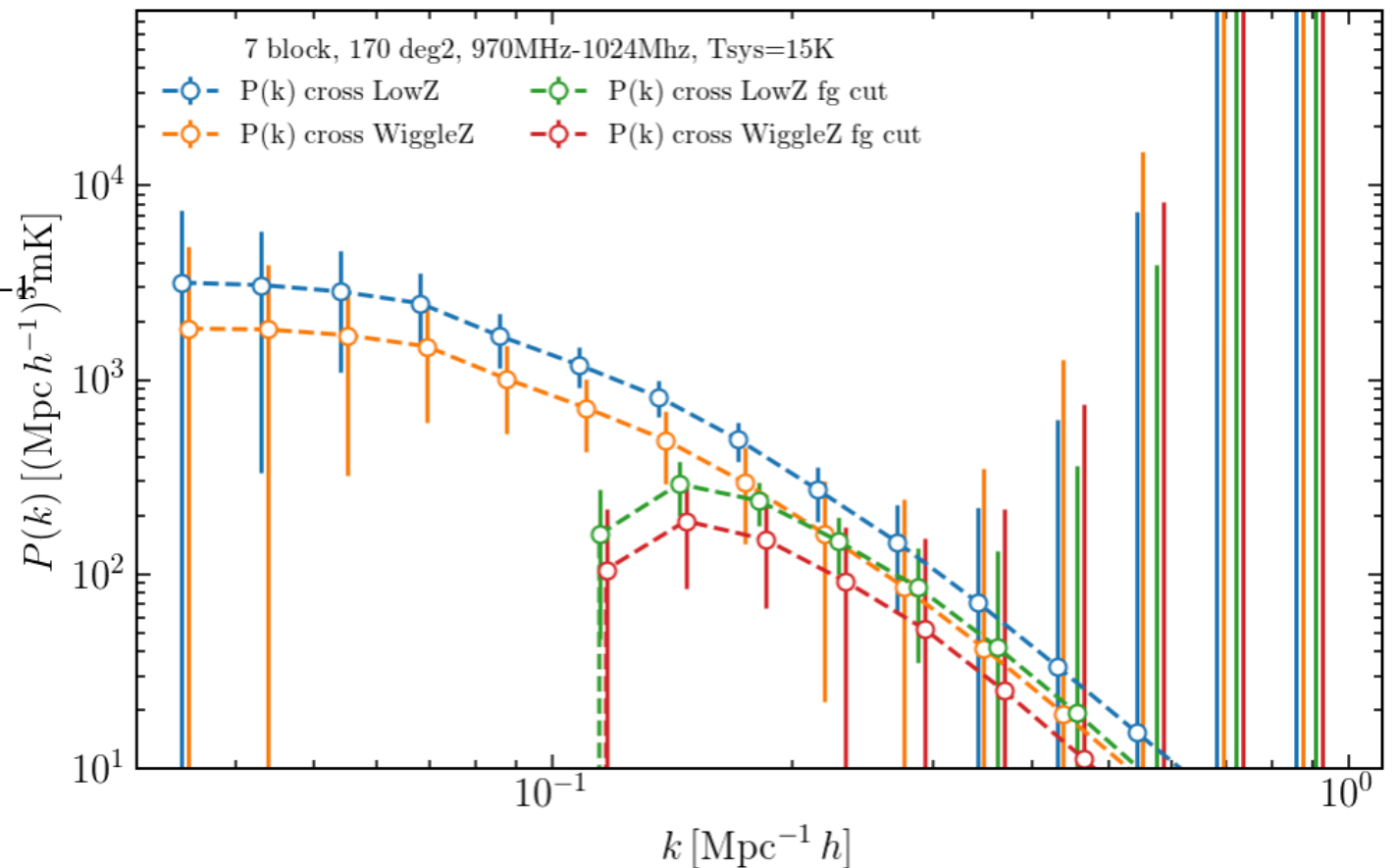
- The noise inverse weight

$$\langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle = \int |W(k - k')|^2 P(k') d^3k' + N(0)$$

$$W(r) = \frac{1}{P(k) + 1/n(r)} \Rightarrow W(r) = \frac{1}{P(k) + 1/n(r) + P_N}$$

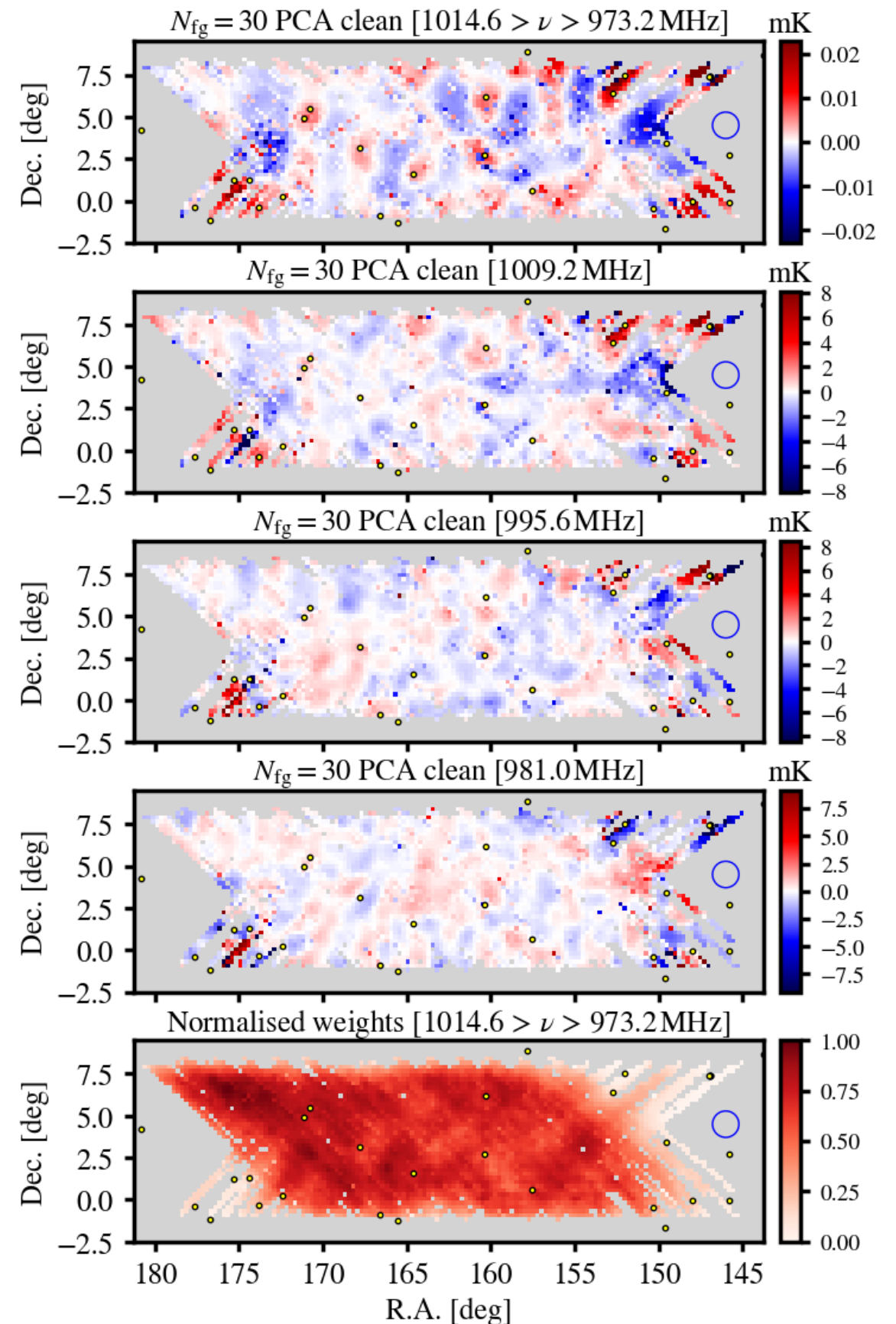
$$\int |W(k - k')|^2 P(k') d^3k' = \langle \tilde{\delta}(k) \tilde{\delta}^*(k) \rangle - N(0)$$

$$\left(\frac{\Delta P_a}{P_a} \right)^2 = \left(\frac{1}{2} V_{\text{bin}} \int \frac{k^2 d\mu dk}{(2\pi)^2} \left(\frac{P^{\text{HI}}(k, \mu)}{P^{\text{HI}}(k, \mu) + N(k, \mu)} \right)^2 \right)^{-1} \frac{1}{\sigma_{\text{mK}}^2}$$



Power Spectrum Estimation with HI IM

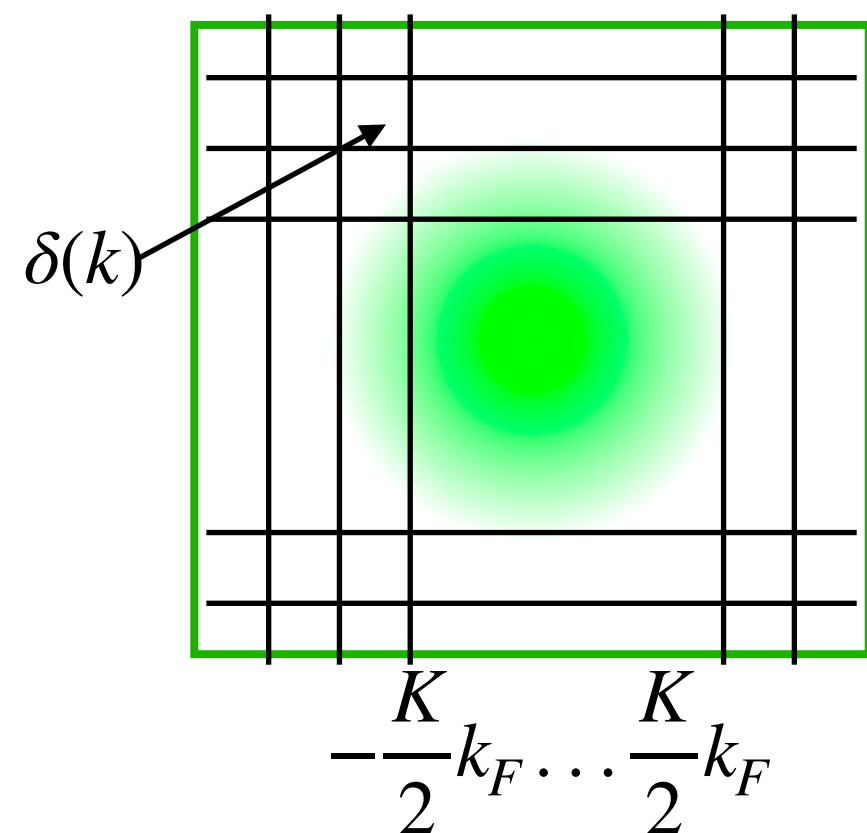
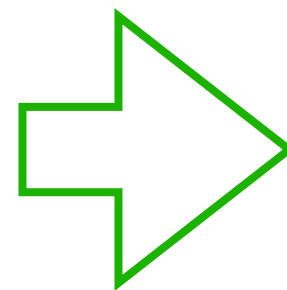
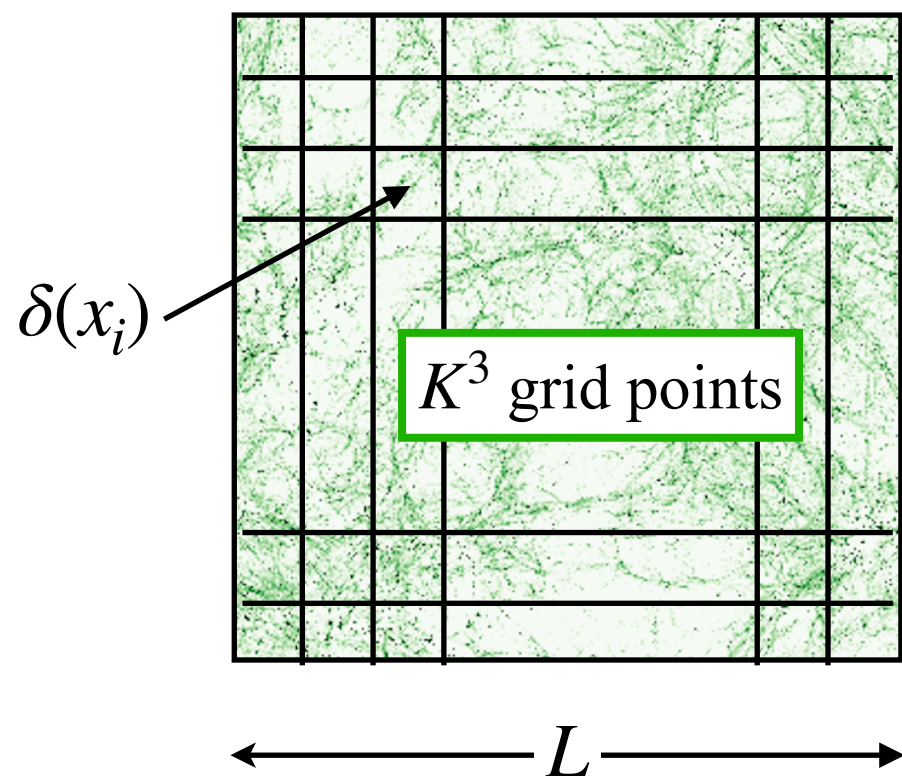
- The HI IM maps
 - Zero-mean (pre-whitened)



Power Spectrum Estimation with HI IM

- Fourier transform vs Fast Fourier Transform (FFT)

$$\delta(k) = \int \frac{d^3x}{(2\pi)^{3/2}} \delta(x) e^{-ik \cdot x}$$



$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)

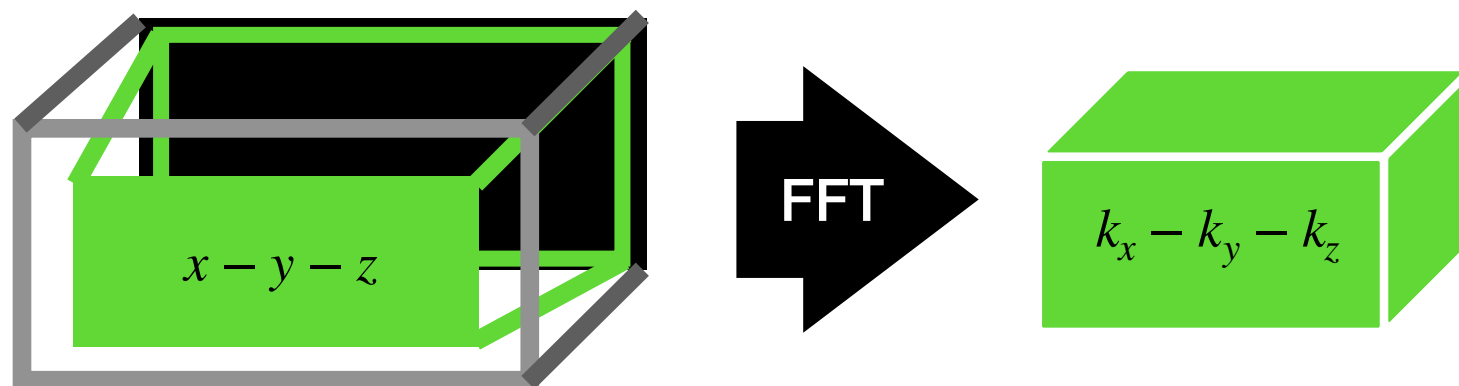


Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT

$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

$$P(\mathbf{k}) = \text{Real} [\delta(\mathbf{k})\delta^*(\mathbf{k})]$$



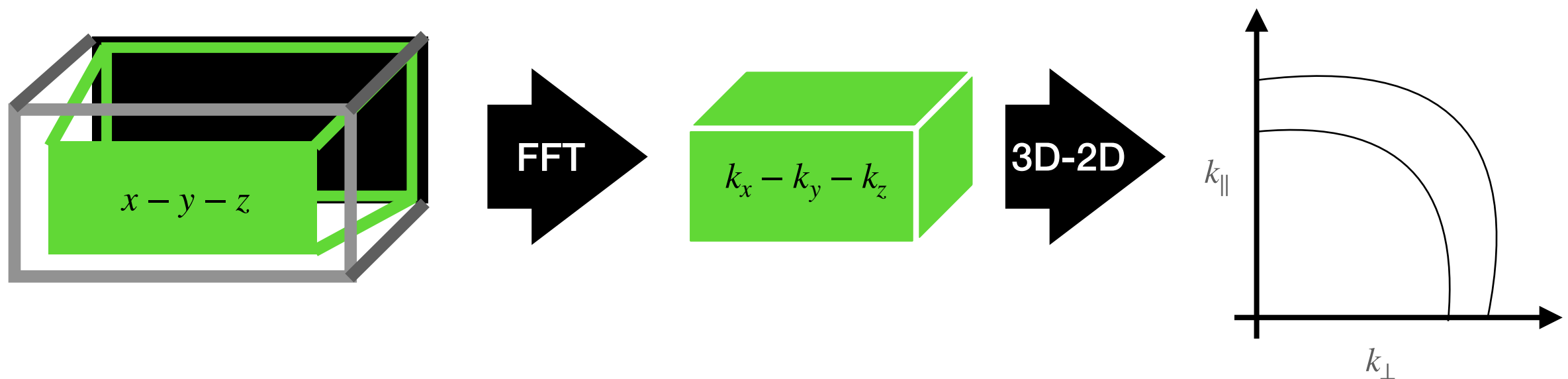
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT

$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

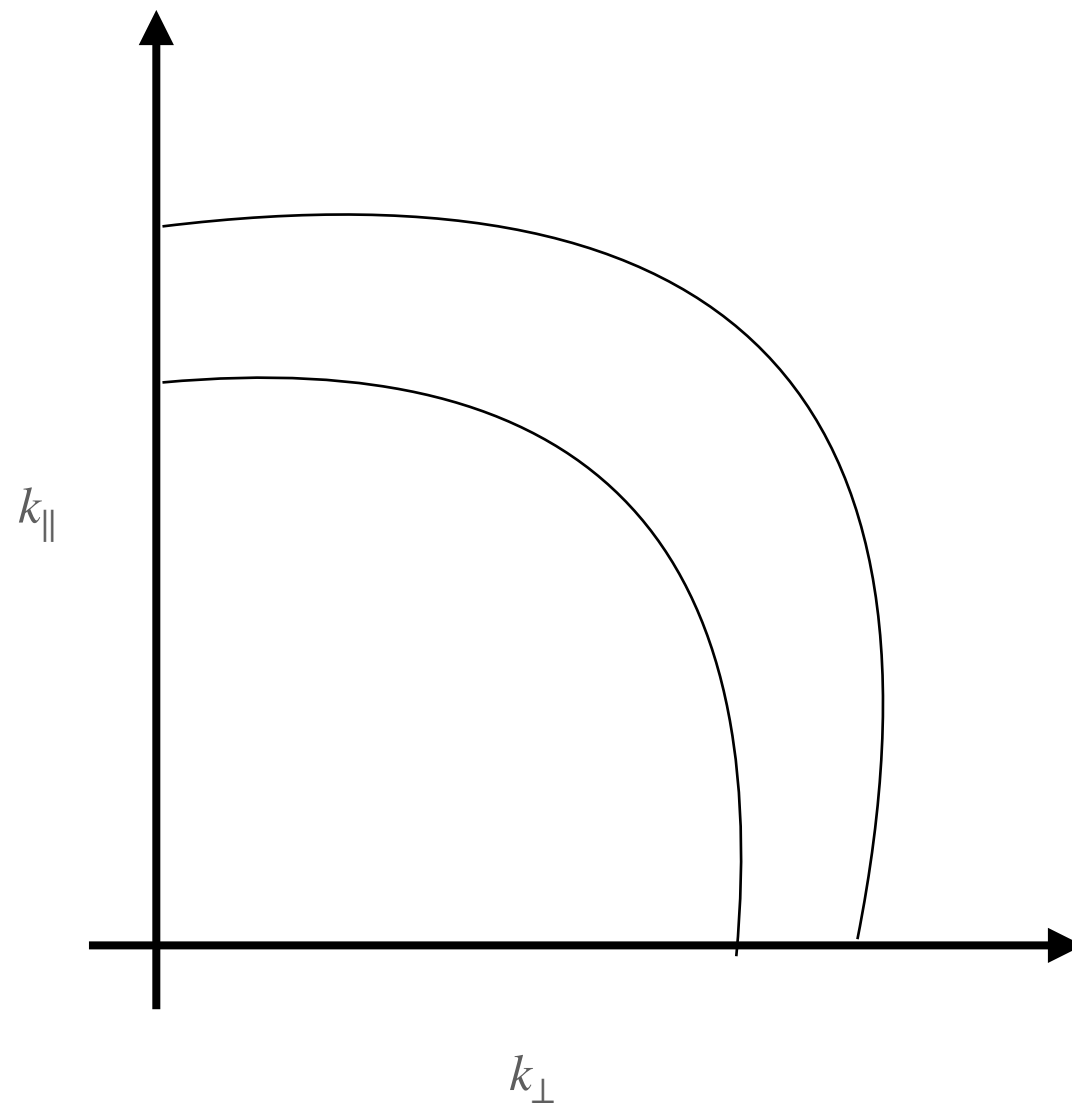
$$P(\mathbf{k}) = \text{Real} [\delta(\mathbf{k})\delta^*(\mathbf{k})]$$

$$P(k_\perp, k_\parallel) = \frac{1}{N_m} \sum_{k_x, k_y} P(k_x, k_y, k_z)$$



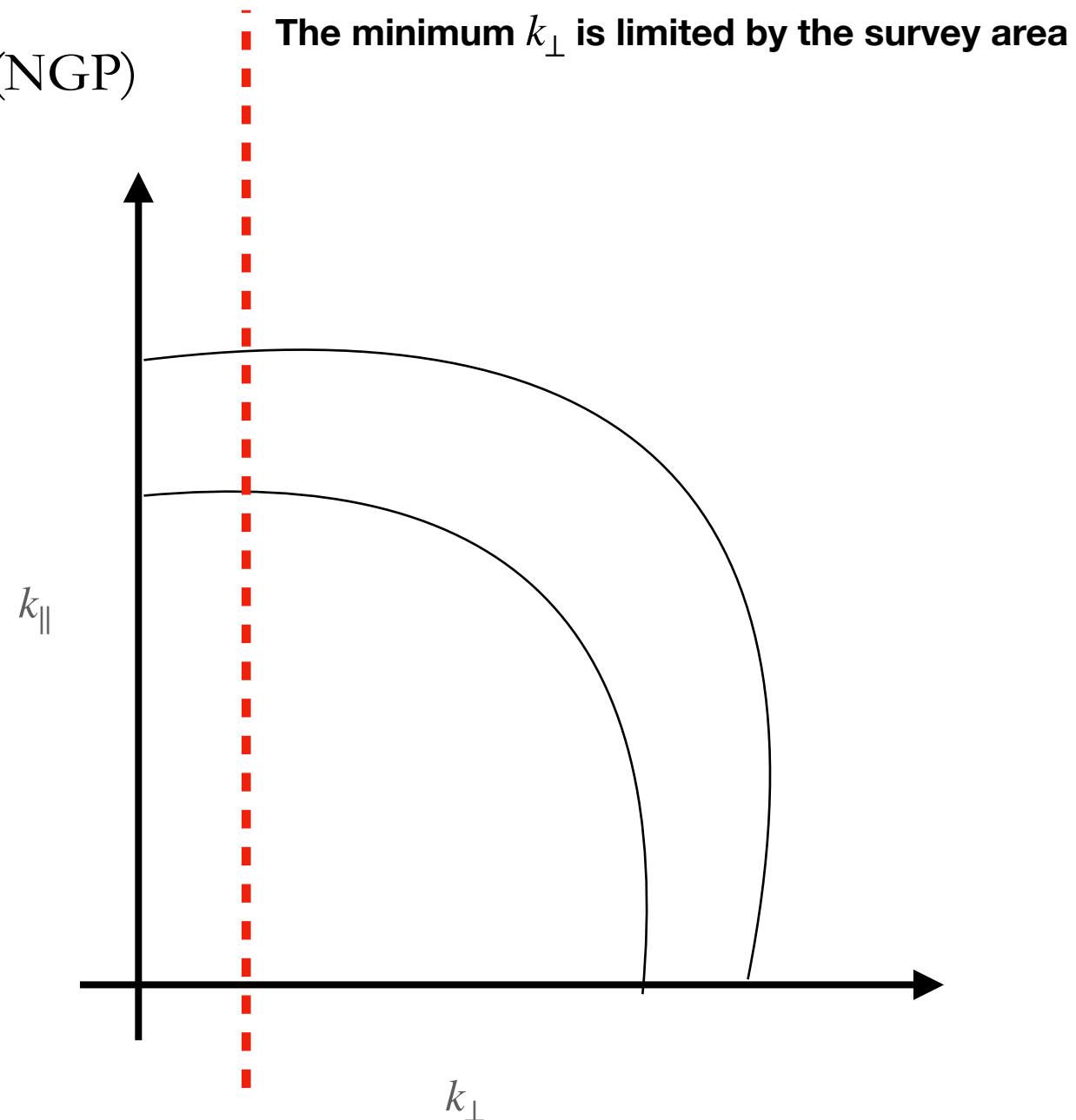
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



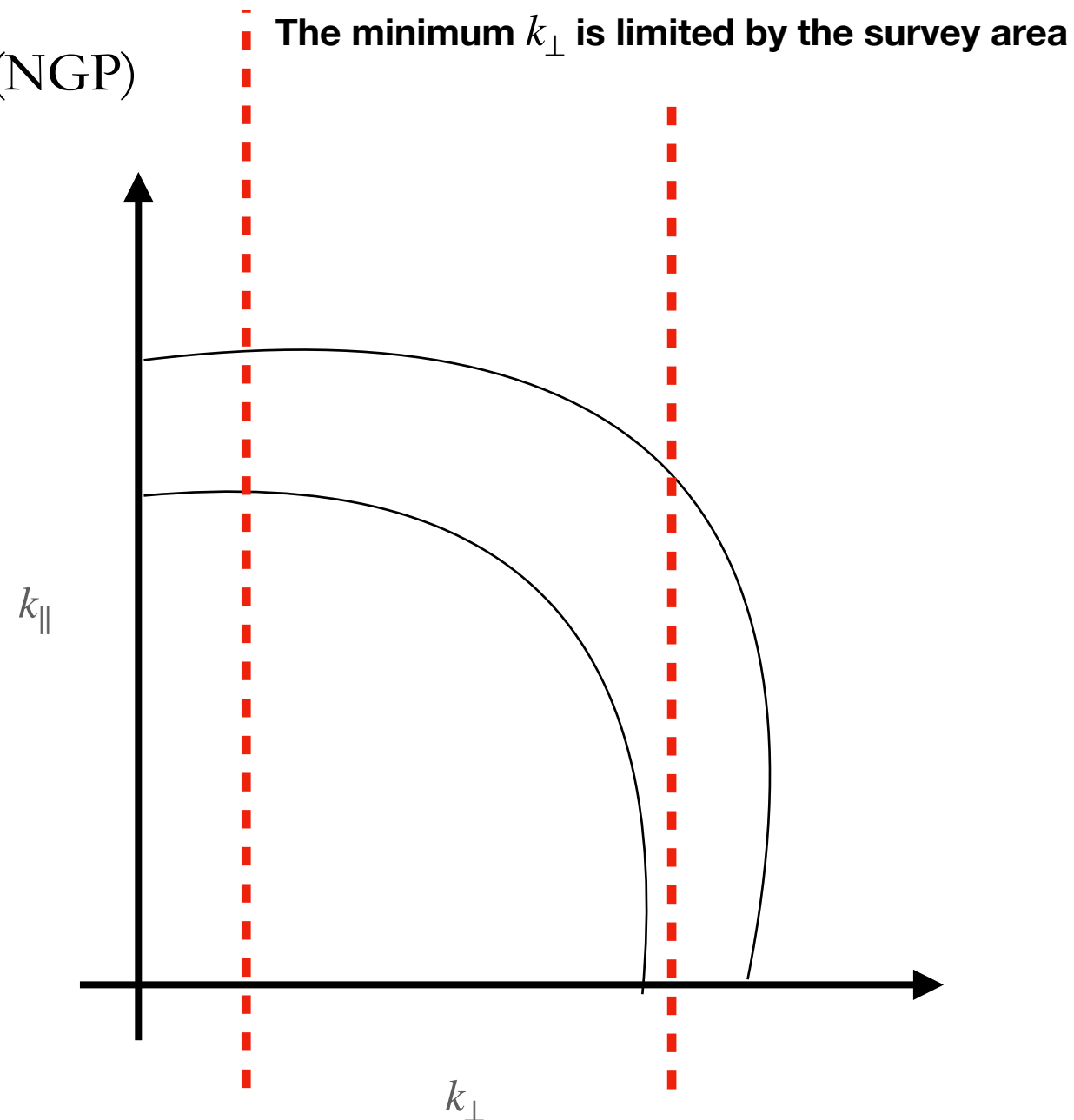
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



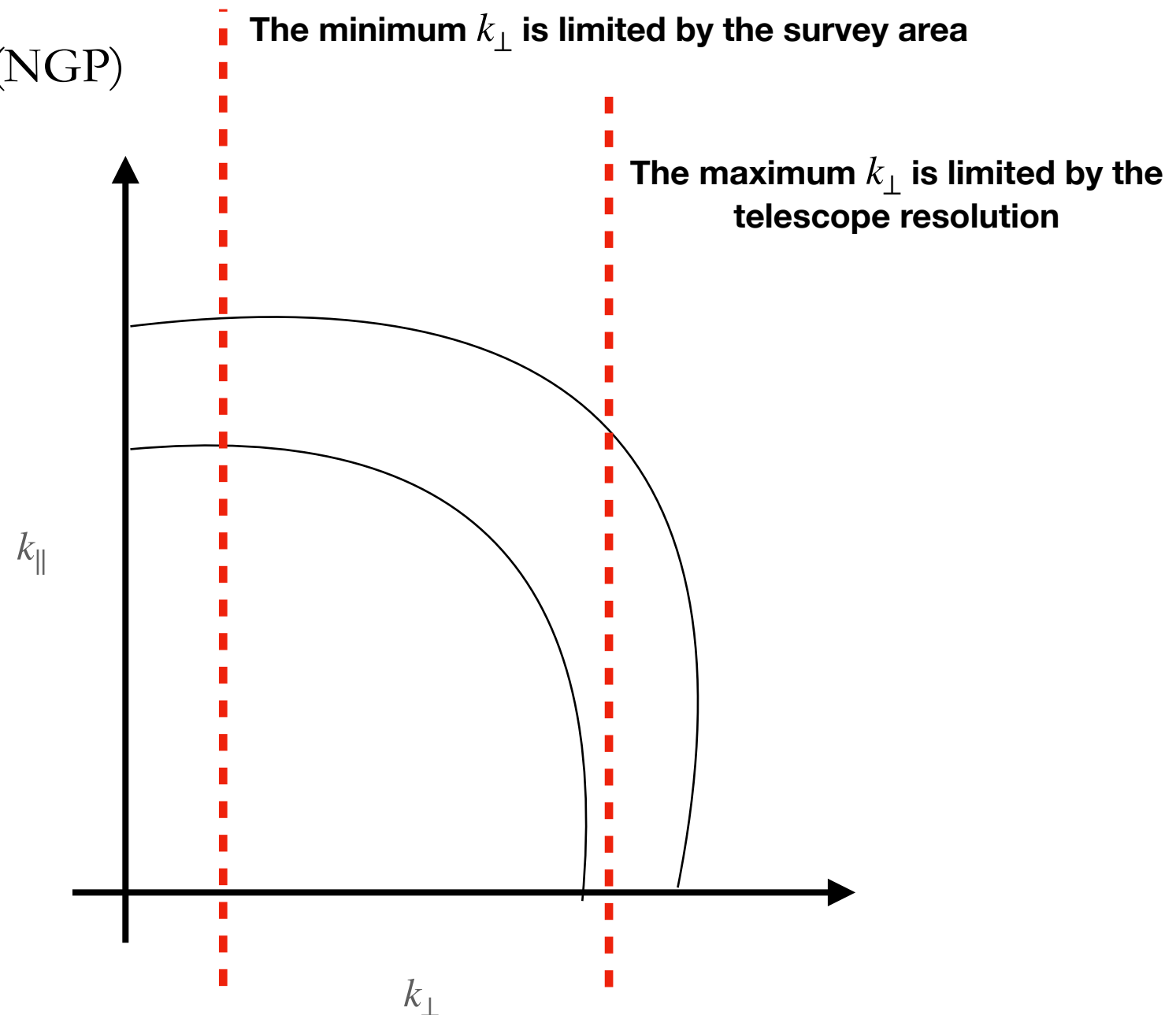
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



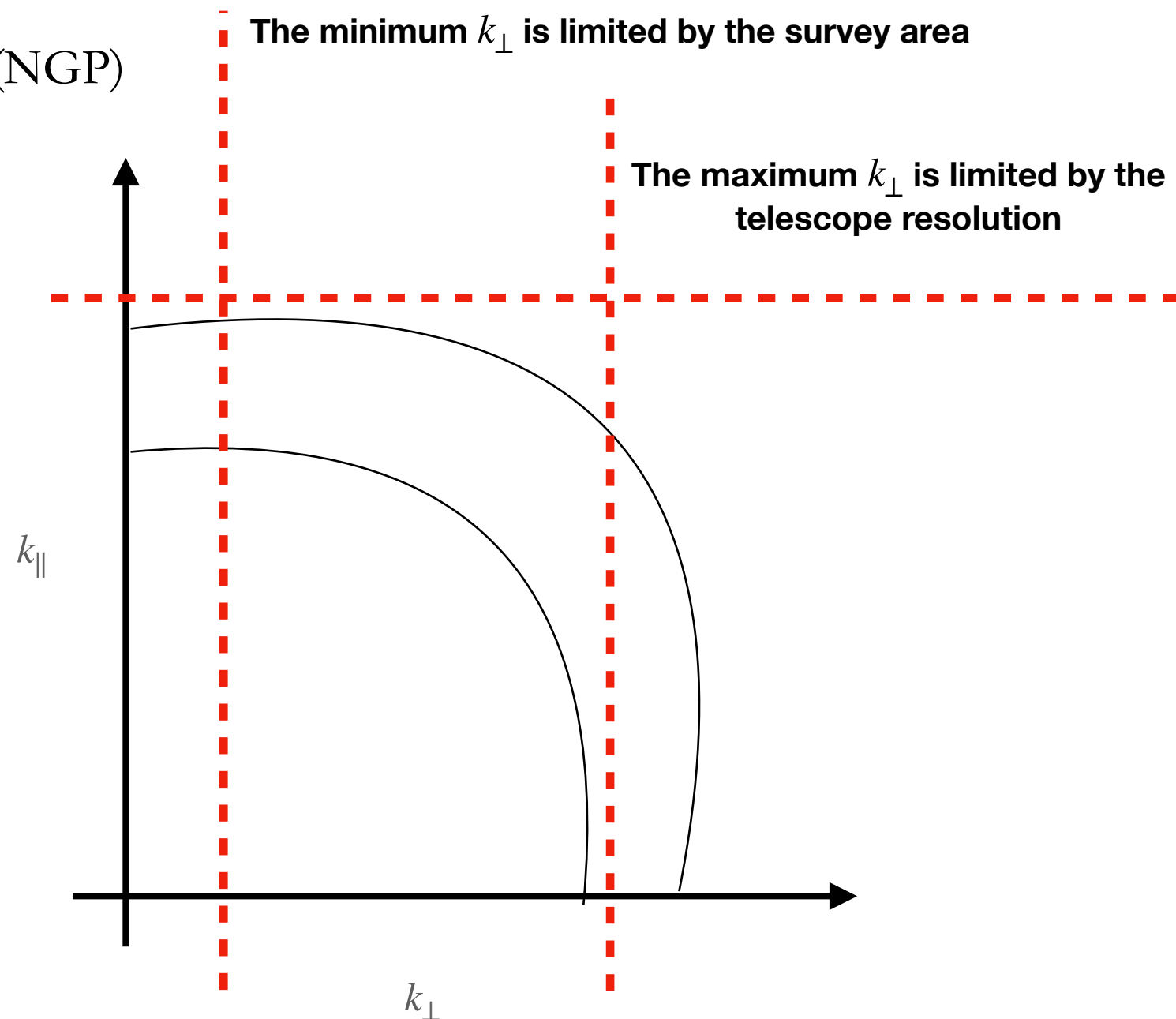
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



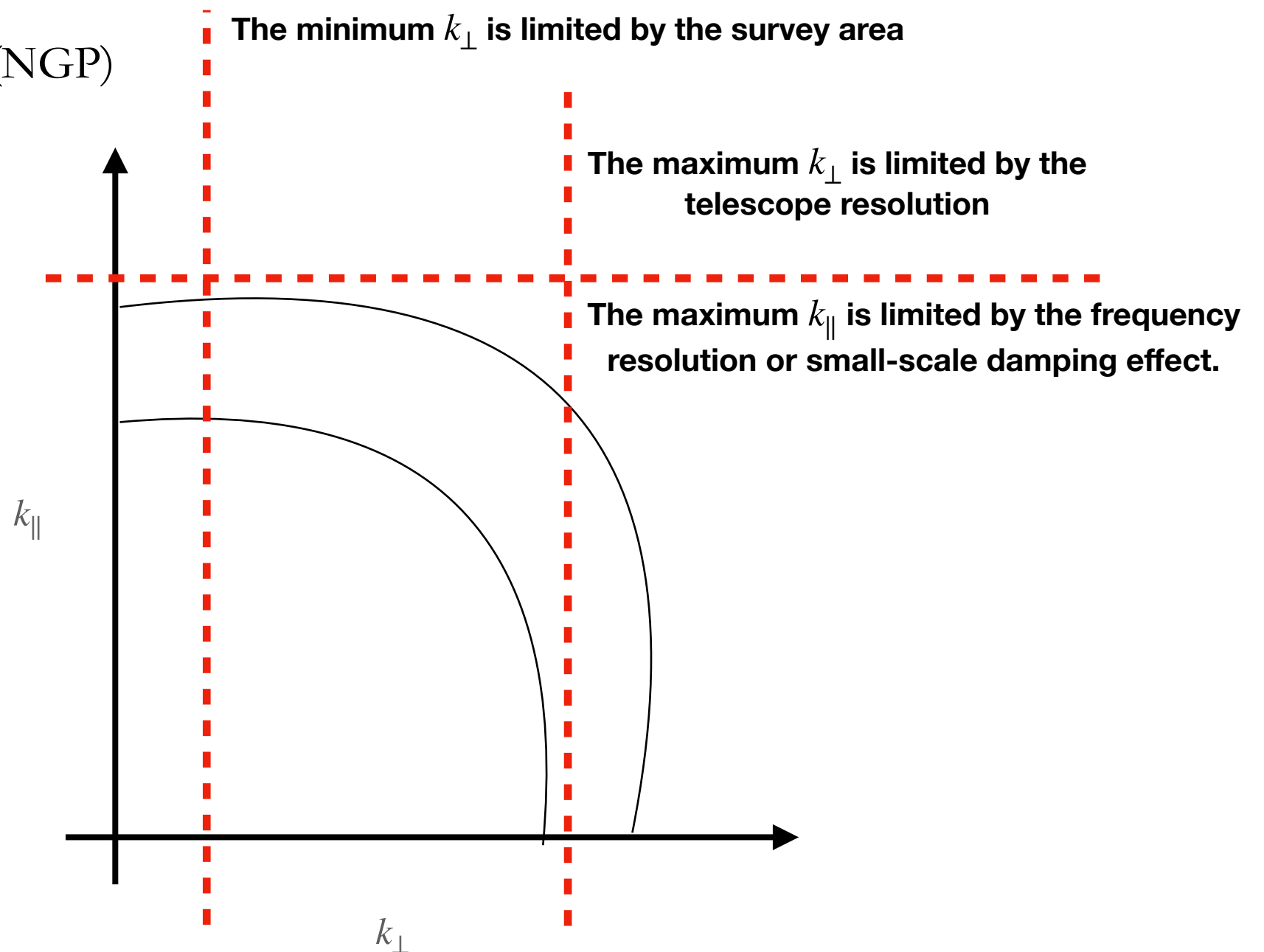
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



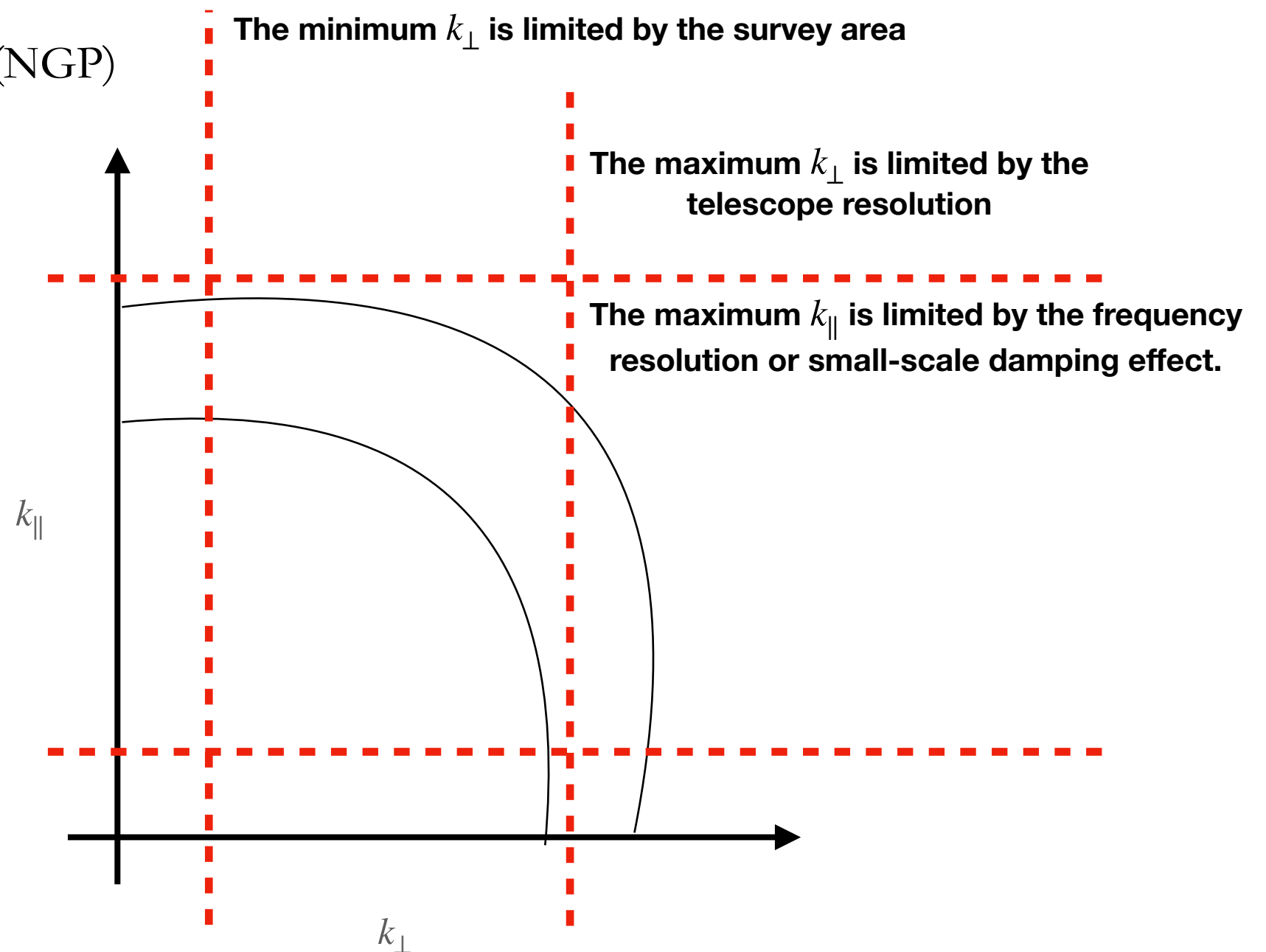
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



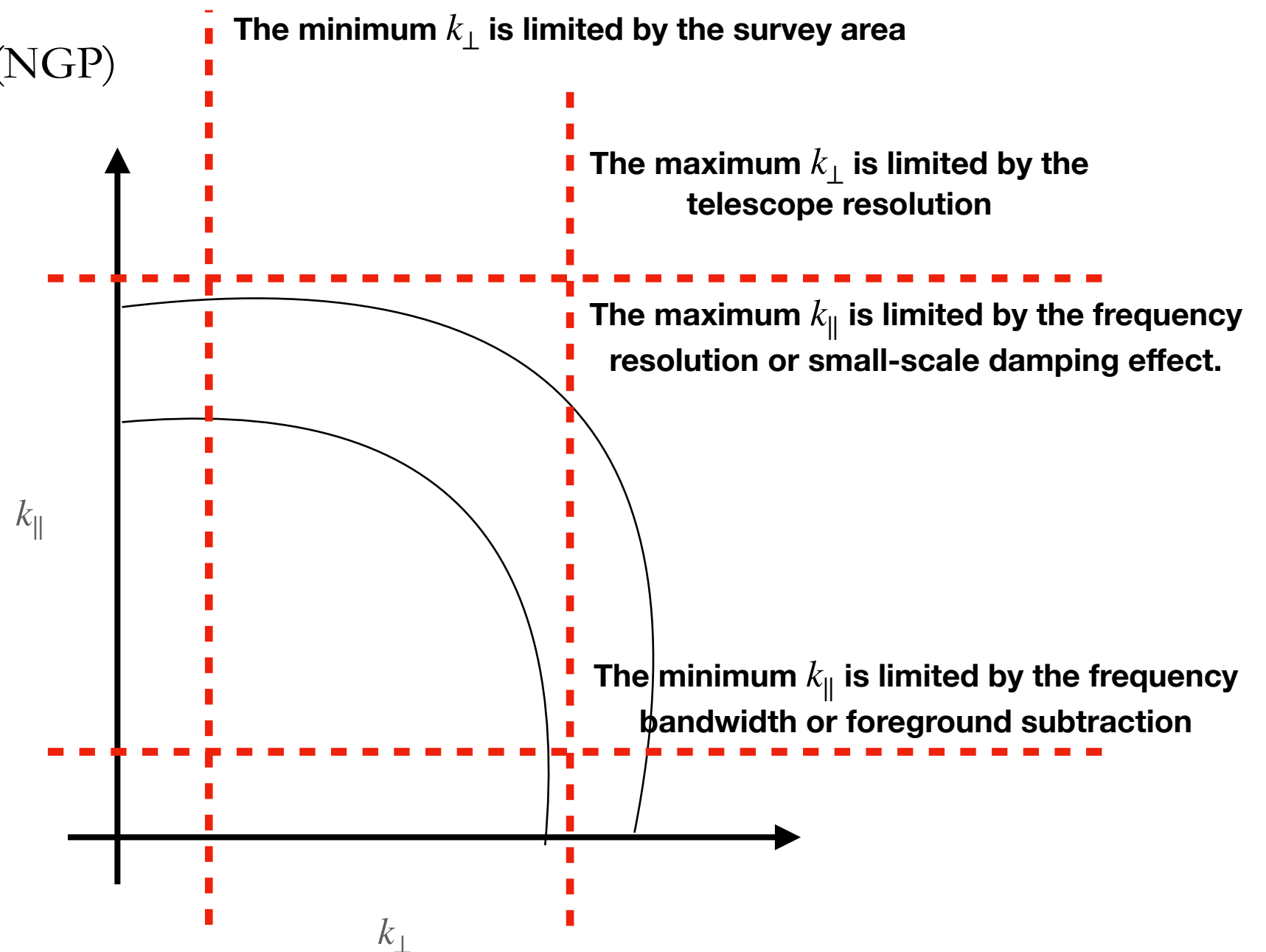
Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



Power Spectrum Estimation with HI IM

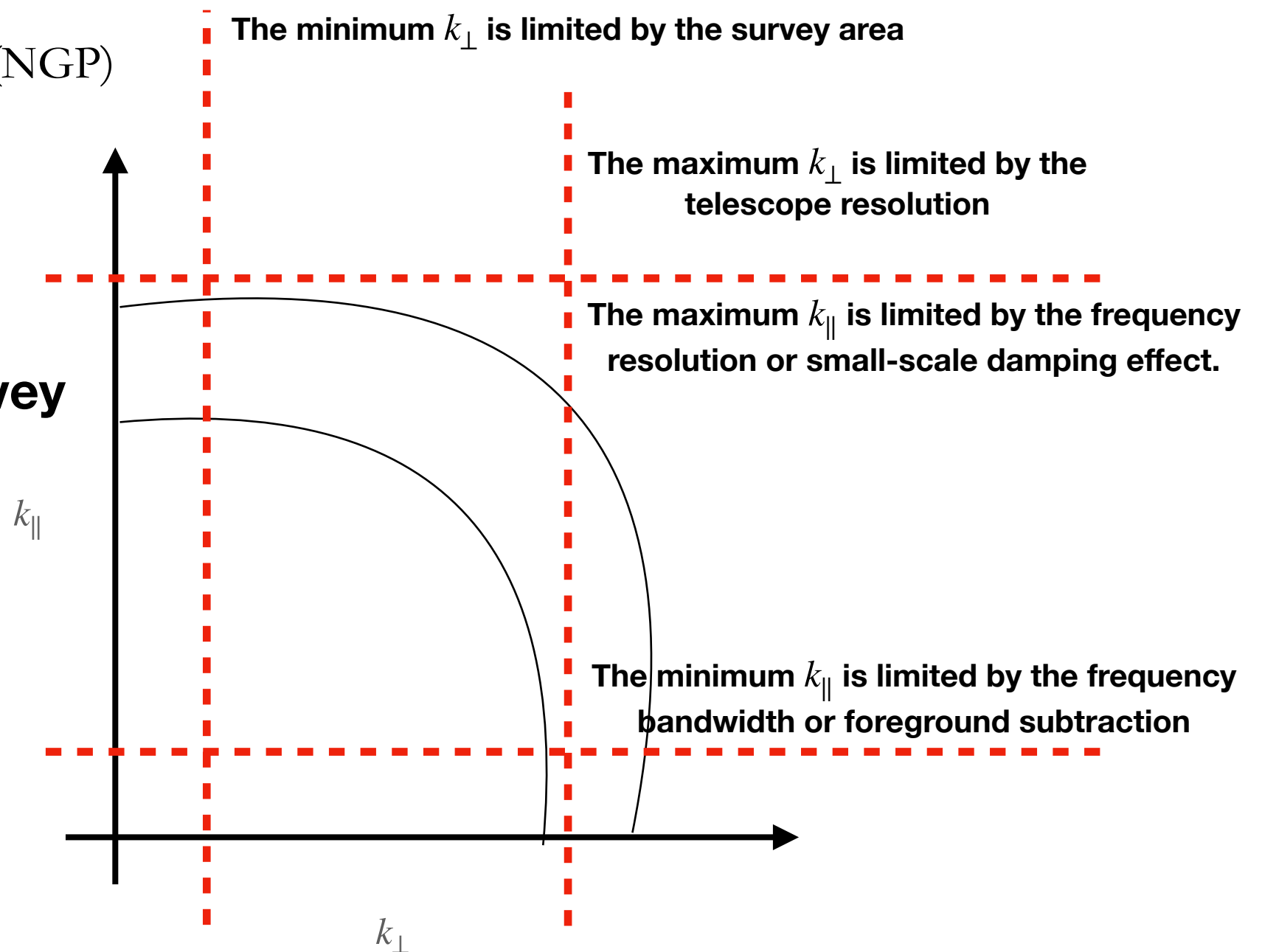
- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT



Power Spectrum Estimation with HI IM

- The HI IM maps
- Map to Cube
 - Nearest-Grid-Point (NGP)
 - Clouds-in-cell (CIC)
- FFT

Single Dish HI IM Survey



Cross-correlation Power Spectrum

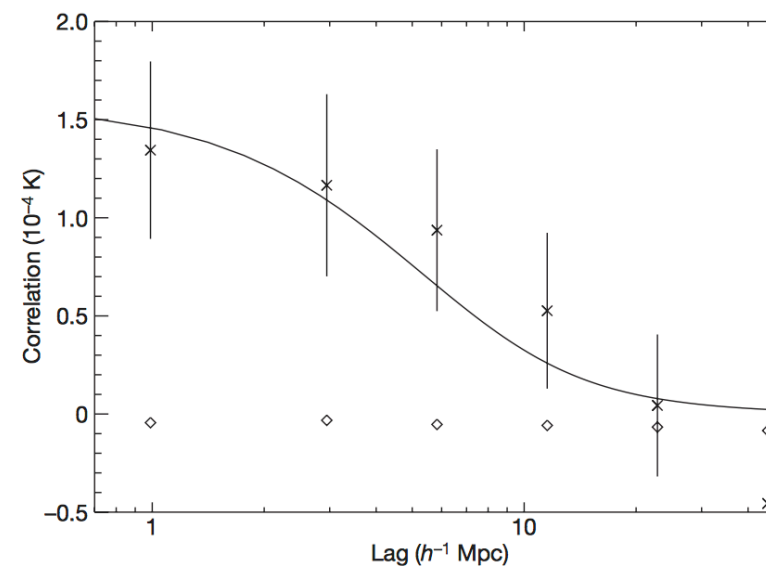
- Cross-correlation between different tracers
 - To get an unbiased PS from the foreground residual
 - HI x Optical Galaxy Sample

$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

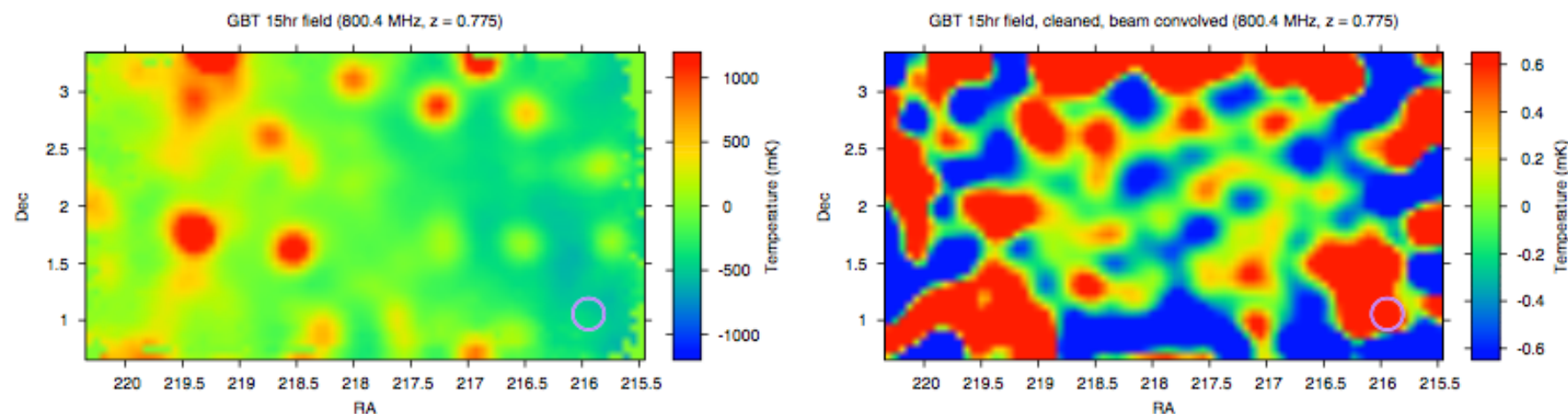
$$P(\mathbf{k}) = \text{Real} \left[\delta_A(\mathbf{k}) \delta_B^*(\mathbf{k}) \right]$$

Cross-correlation Power Spectrum

- Cross-correlation between different tracers
 - To get an unbiased PS from the foreground residual
 - HI x Optical Galaxy Sample



Cross-correlation function with GBT HIIM x DEEP2
T.-C. Chang et al. 2010 Nature Vol 466

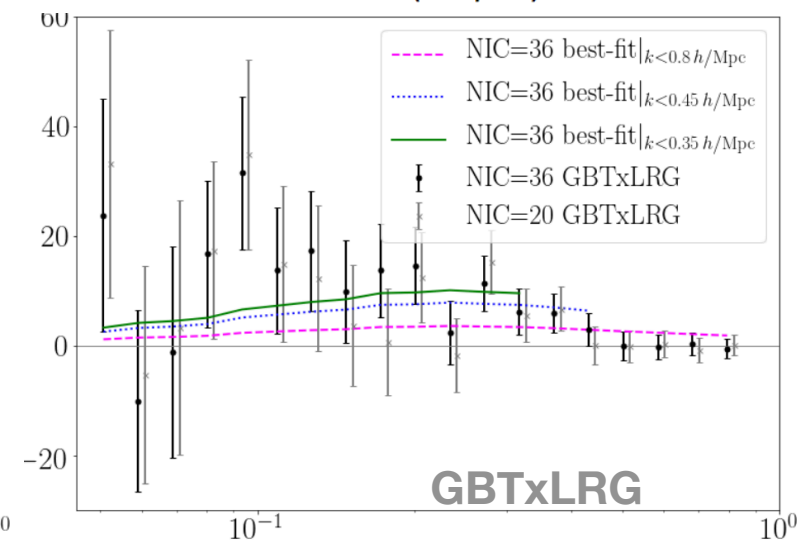
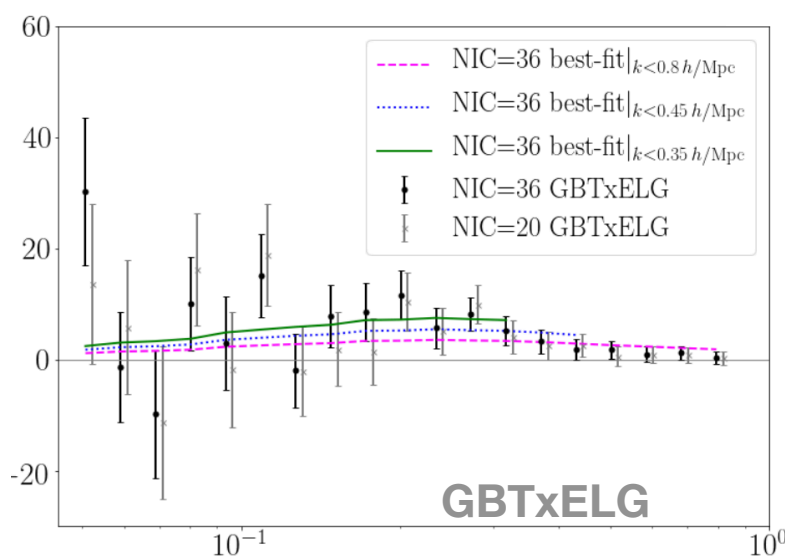
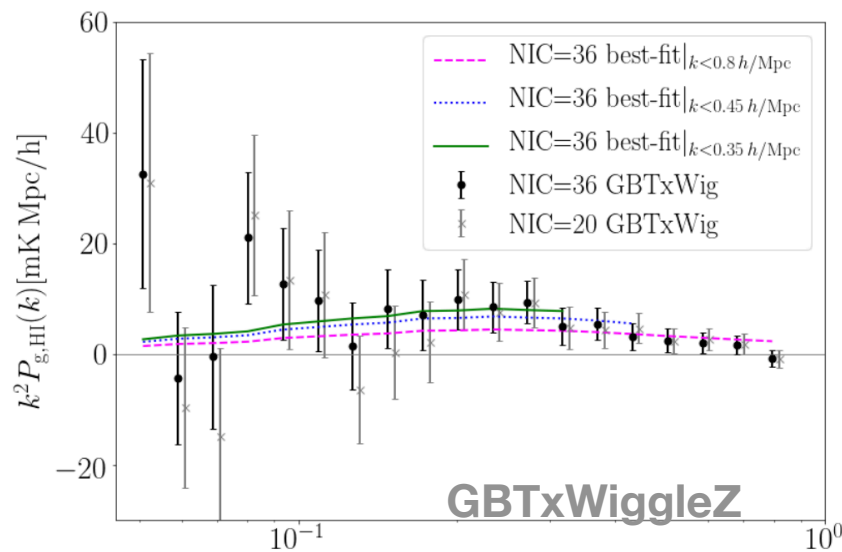
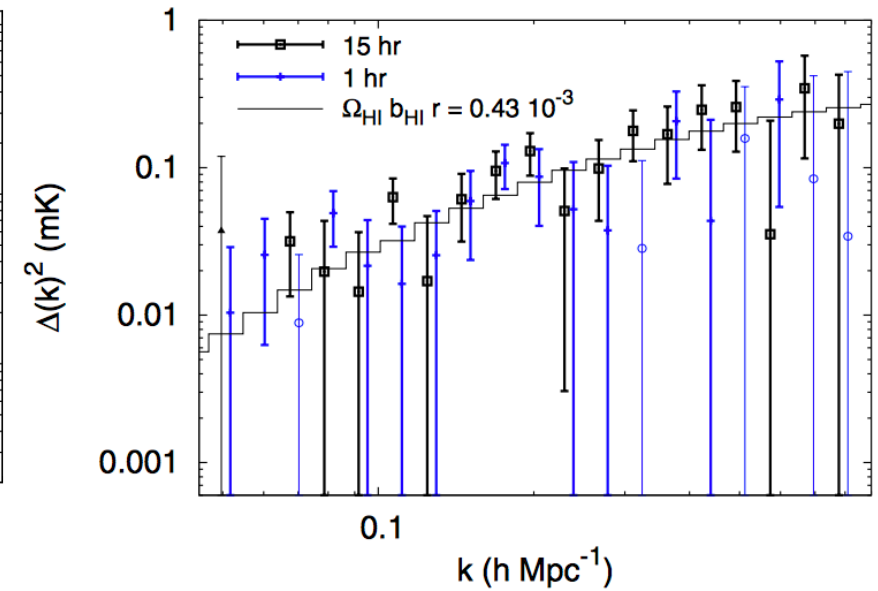
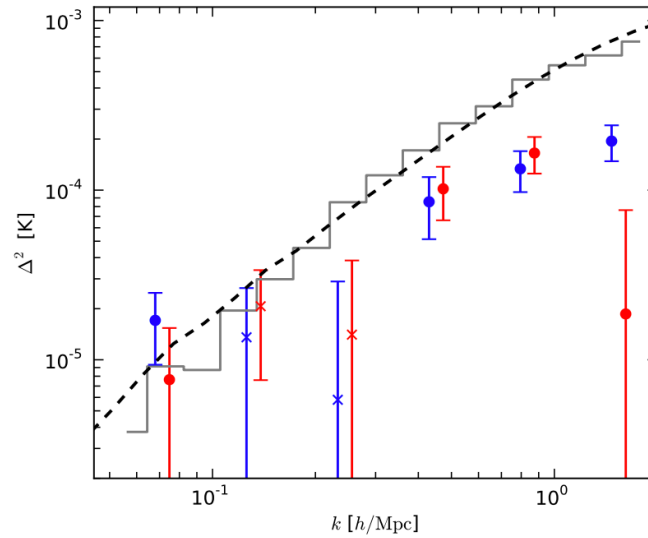
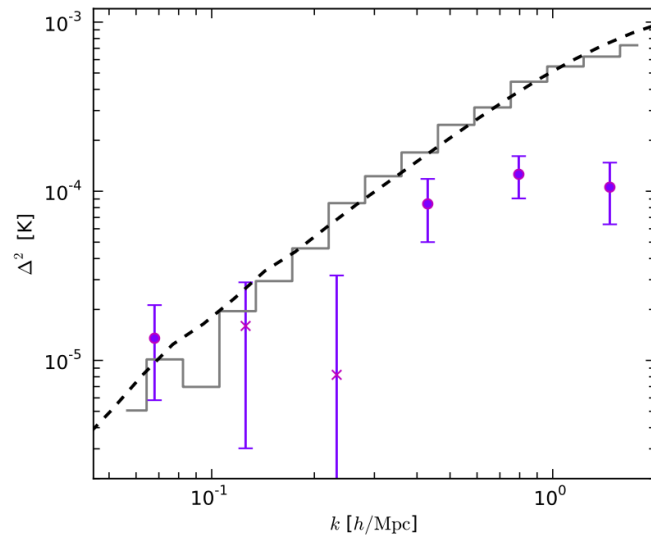


Cross PS GBT HIIM x WiggleZ K. Masui et al 2013 ApJ 763L 20M

Cross-correlation Power Spectrum

- Cross-correlation between different tracers
 - To get an unbiased PS from the foreground residual
 - HI x Optical Galaxy Sample

Cross PS PKS x 2dF
C. Anderson, N. J.
Luciw, Y. Li et, al
1710.00424



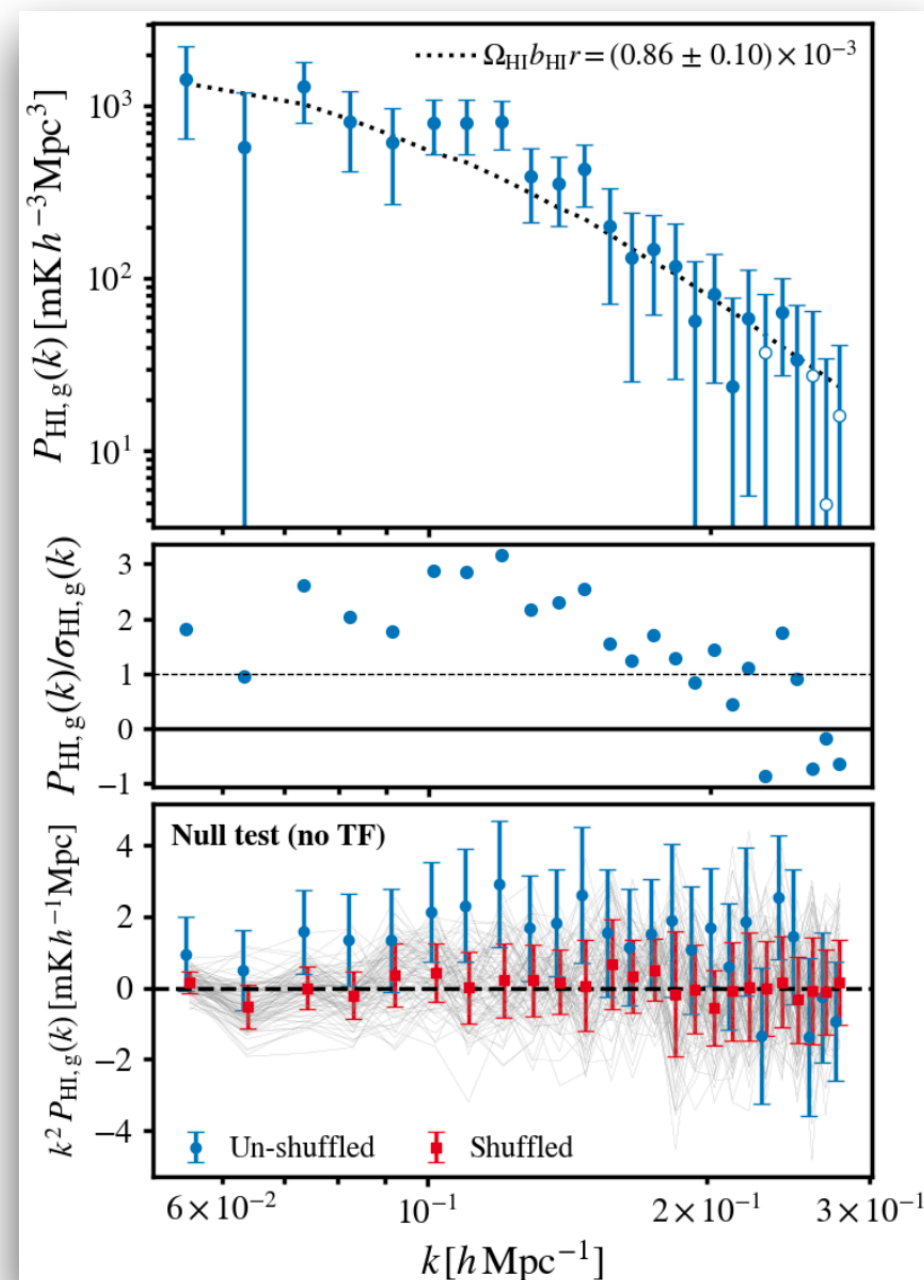
Cross-correlation Power Spectrum

- Cross-correlation between different tracers

- To get an unbiased PS from the foreground residual
- HI x Optical Galaxy Sample

- 10.5 hour HI IM observation using MeerKAT
64 dishes, we achieve 7.7σ detection of the
cross-correlation power spectrum;

Cunnington S., Li Y., Santos M.~G., et al.
arXiv: 2206.01579



Cross-correlation Power Spectrum

- Cross-correlation between different tracers
 - To get an unbiased PS from the foreground residual
 - HI x Optical Galaxy Sample
- Cross-correlation between different observation epochs
 - To get an unbiased PS from the noise power spectrum
 - HI (Day 1) x HI (Day 2)
 - HI (FAST) x HI (MeerKAT)

$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

$$P(\mathbf{k}) = \text{Real} \left[\delta_A(\mathbf{k}) \delta_B^*(\mathbf{k}) \right]$$

Cross-correlation Power Spectrum

- Cross-correlation between different tracers
 - To get an unbiased PS from the foreground residual
 - HI x Optical Galaxy Sample
- Cross-correlation between different observation epochs
 - To get an unbiased PS from the noise power spectrum
 - HI (Day 1) x HI (Day 2)
 - HI (FAST) x HI (MeerKAT)

- The Error for Cross-correlation PS

$$\delta(\mathbf{k}) = L^{3/2} \sum_i^{K^3} \delta(\mathbf{x}_i) e^{-i\mathbf{k} \cdot \mathbf{x}}, \quad \mathbf{k} \in (n_x, n_y, n_z)k_F, \quad k_F = 2\pi/L$$

$$P(\mathbf{k}) = \text{Real} \left[\delta_A(\mathbf{k}) \delta_B^*(\mathbf{k}) \right]$$

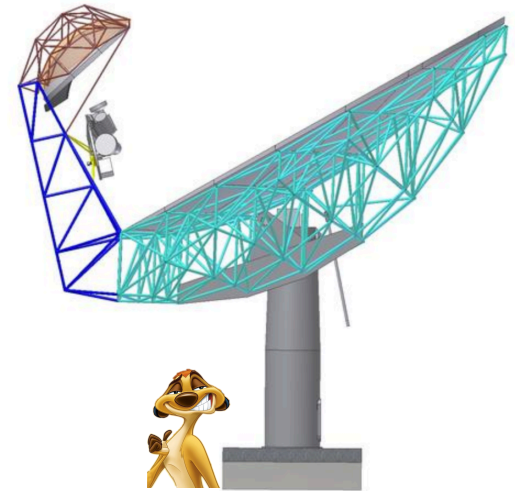
$$\left(\frac{\Delta P_a}{P_a} \right)^2 = \left(\frac{1}{2} V_{\text{bin}} \int \frac{k^2 d\mu dk}{(2\pi)^2} \frac{(P^{\text{HI,g}})^2}{(P^{\text{HI,g}})^2 + (P^{\text{HI}}(k, \mu) + N(k, \mu)) \left(P^{\text{g}}(k, \mu) + \frac{1}{\bar{n}} \right)} \right)^{-1}$$

Single Dish v.s. Interferometer HI IM

CHIME



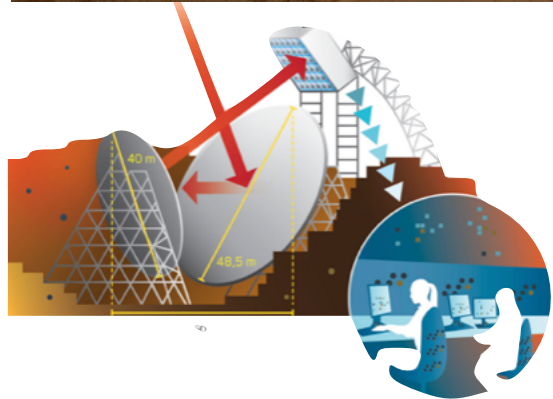
Tianlai (天籟)



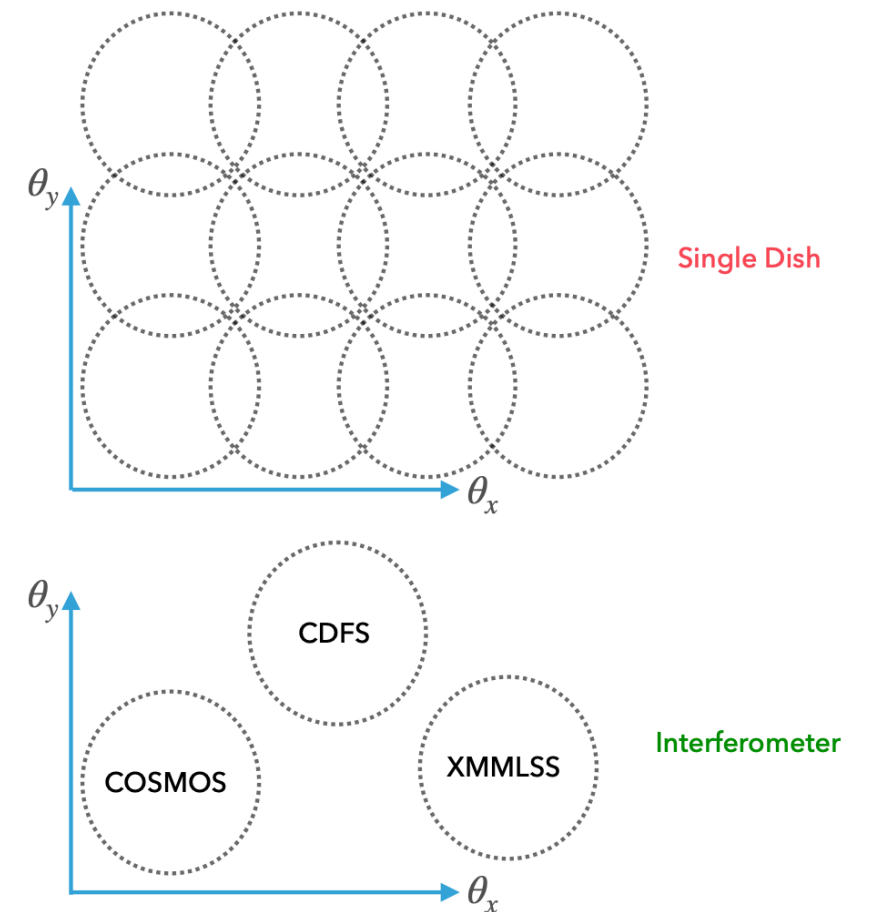
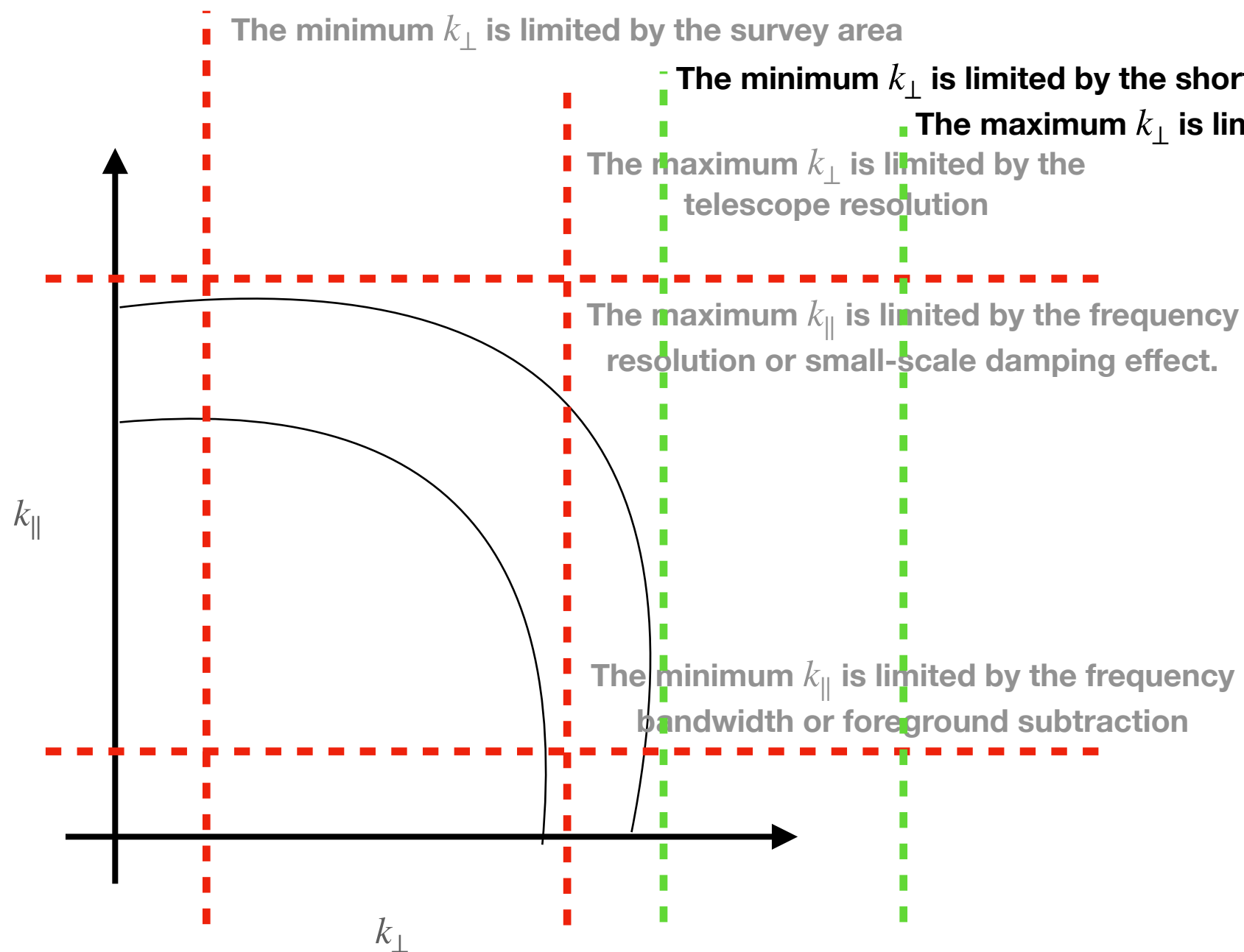
FAST



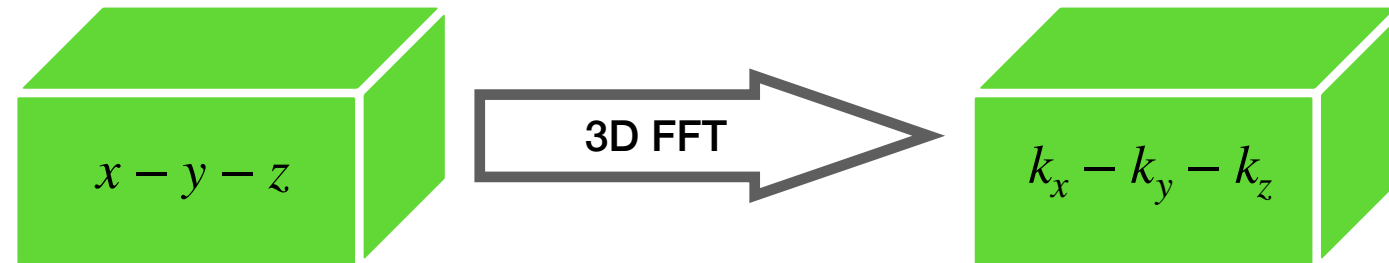
HIRAX



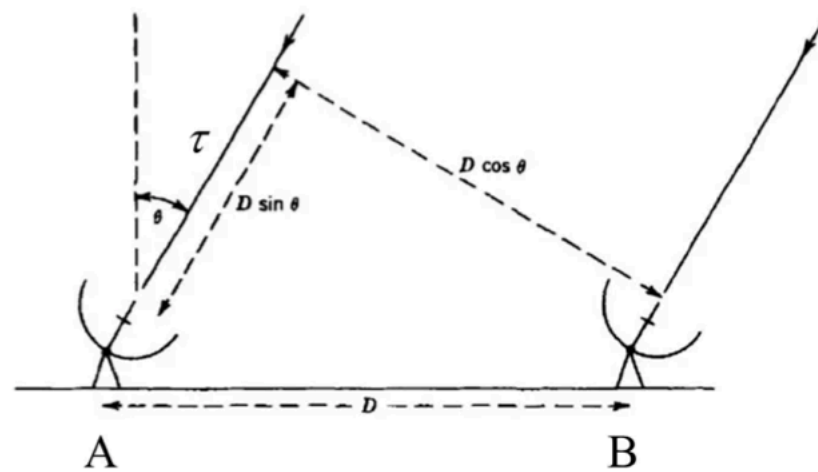
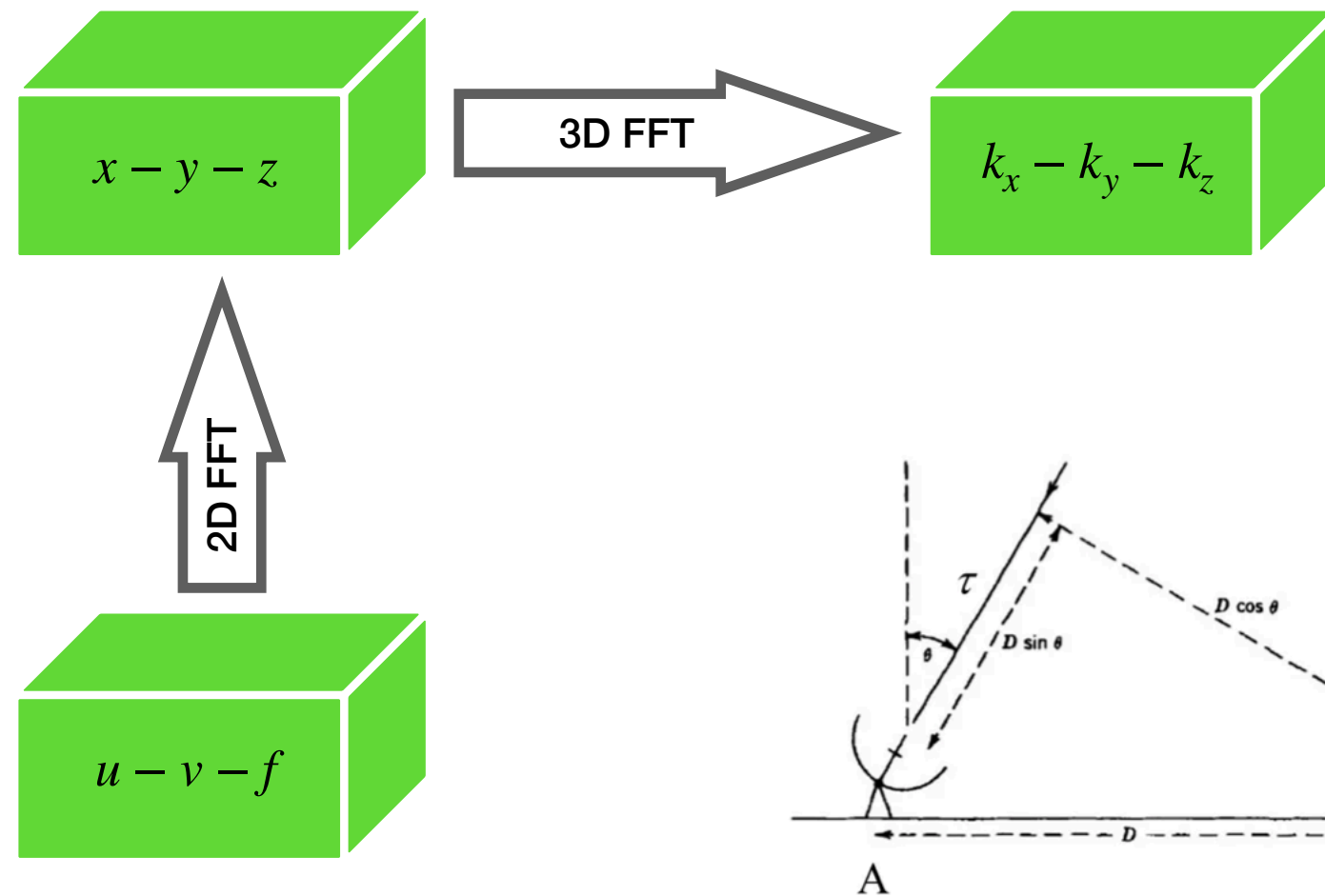
Single Dish v.s. Interferometer HI IM



Interformeter Delay Power Spectrum

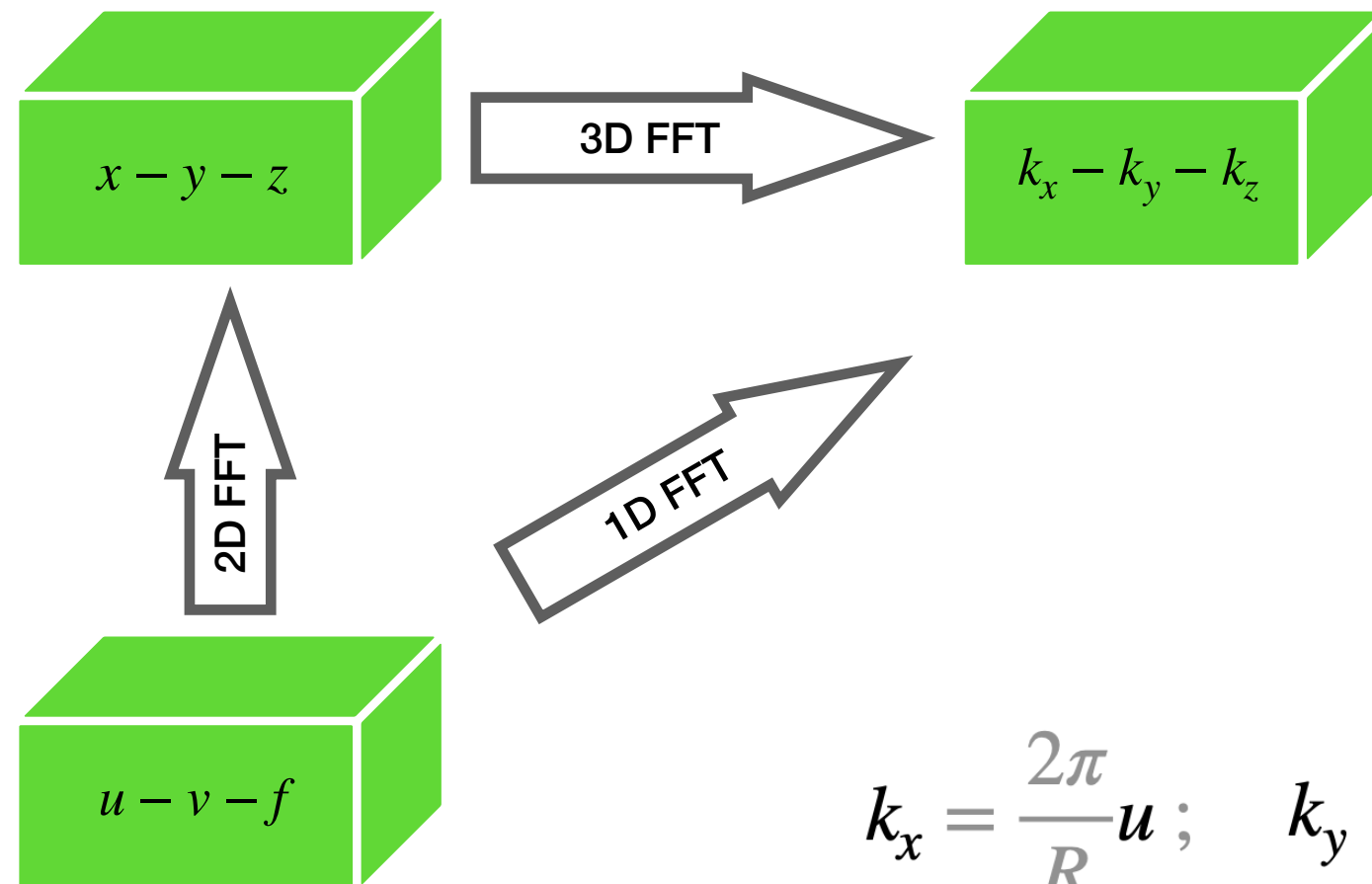


Interformeter Delay Power Spectrum



$$I(l, m) = \iint V(u, v) e^{-2\pi i(ul+vm)} du dv$$

Interformeter Delay Power Spectrum

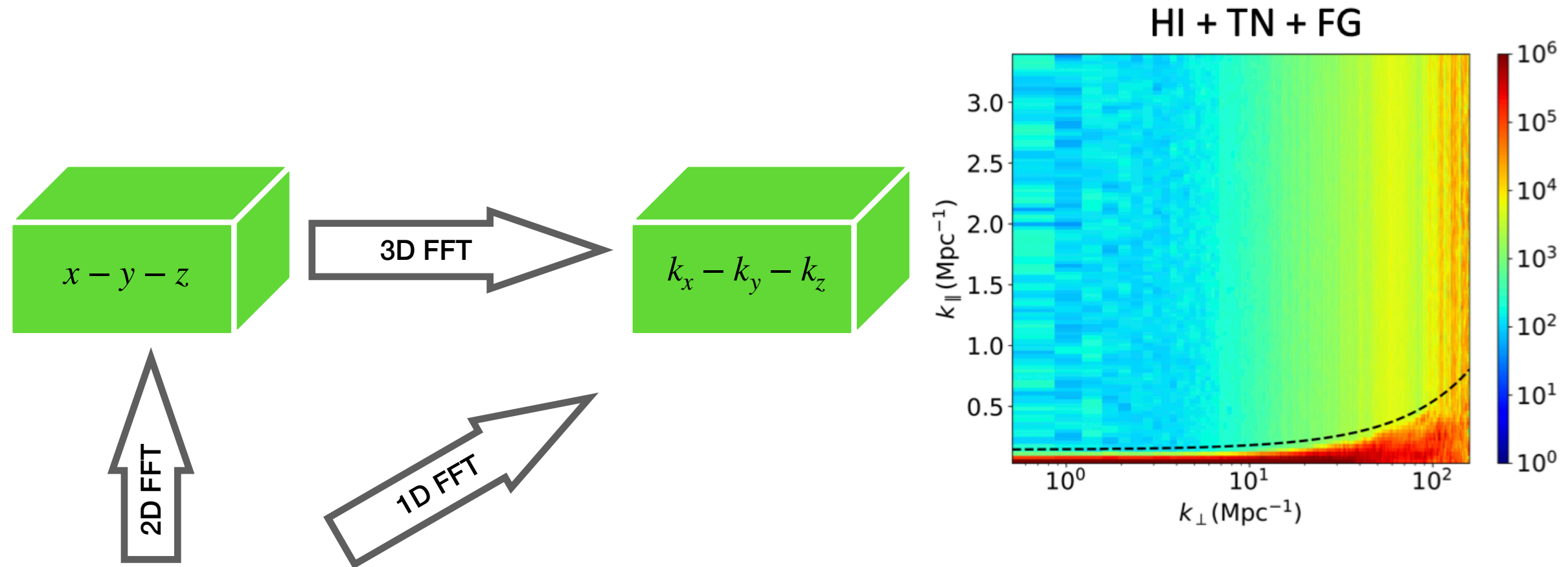


$$k_x = \frac{2\pi}{R}u ; \quad k_y = \frac{2\pi}{R}v ; \quad k_{\parallel} = \frac{2\pi\nu_{21}H_0E(z)}{c(1+z)^2}\tau$$

Delay PS

$$P(k_{\perp}, k_{\parallel}) \equiv \frac{A_e}{\lambda^2 B} \frac{R^2 \Delta R}{B} |V(u, v, \tau)|^2 \left(\frac{\lambda^2}{2k_B} \right)^2$$

Interferometer Delay Power Spectrum



$$k_x = \frac{2\pi}{R}u ; \quad k_y = \frac{2\pi}{R}v ; \quad k_{\parallel} = \frac{2\pi\nu_{21}H_0E(z)}{c(1+z)^2}\tau$$

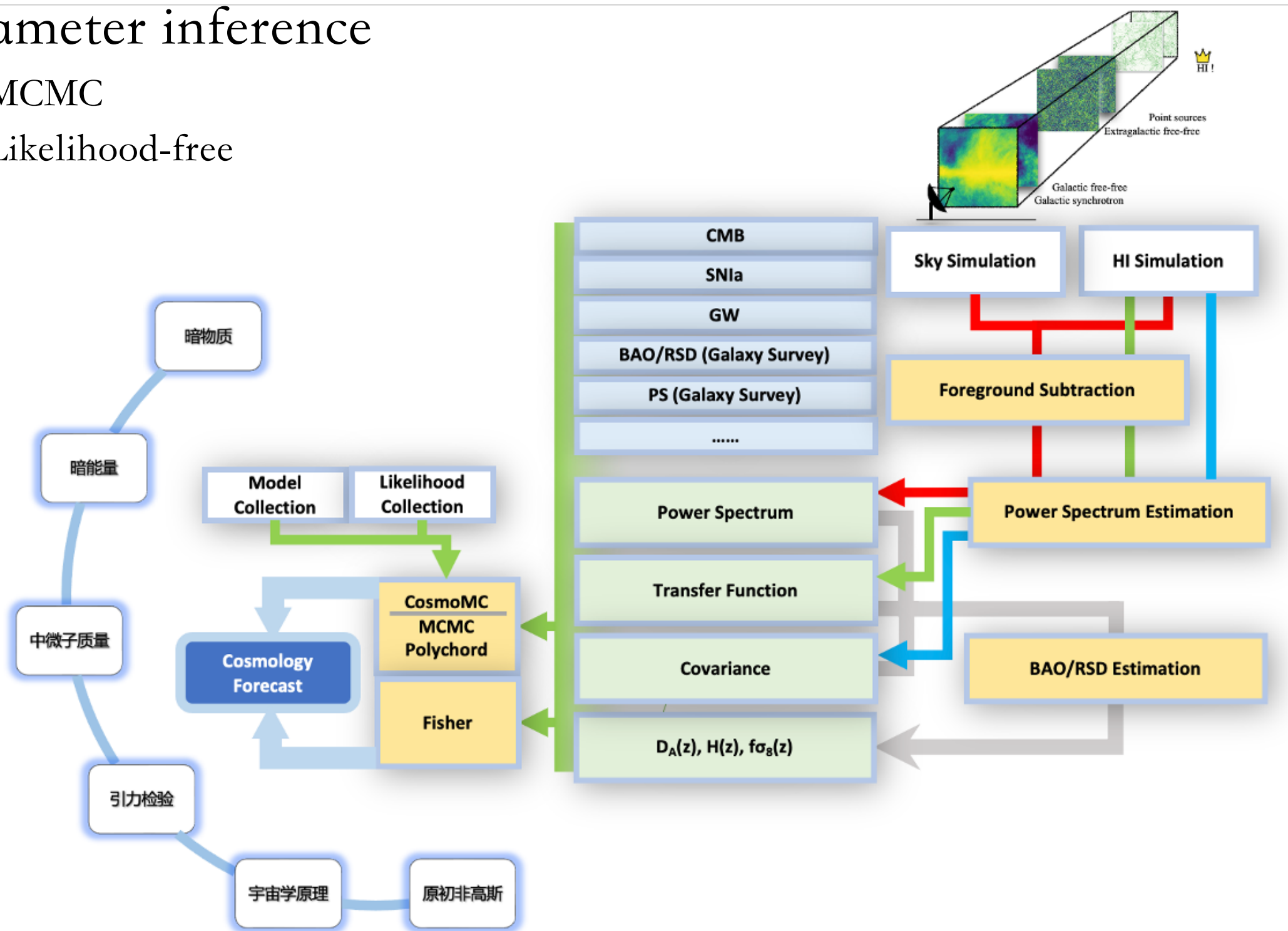
Delay PS

$$P(k_{\perp}, k_{\parallel}) \equiv \frac{A_e}{\lambda^2 B} \frac{R^2 \Delta R}{B} |V(u, v, \tau)|^2 \left(\frac{\lambda^2}{2k_B} \right)^2$$

From measurements to cosmological models

- Parameter inference

- MCMC
- Likelihood-free



- Powerspectrum Estimation Basic
 - Definition of correlation function
 - Definition of power spectrum
 - Relation between correlation function and power spectrum
- Galaxy Survey
 - Galaxy sample
 - Selection function
 - Window function
 - Error estimation
 - Correction of Alias Effects
- HI Intensity Mapping Survey
 - Fisher Information Matrix
 - FFT
 - Cross-correlation
 - Delay power spectrum

Many Thanks !!

The summer school for 21cm cosmology

July 2nd - 15th, 2025 Xi'an, P. R. China

liyichao@mail.neu.edu.cn

Yichao LI (李毅超) from Northeastern University

