Quantum Colliders

WQC Workshop on Quantum Entanglement of High Energy Particles

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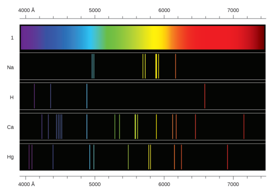
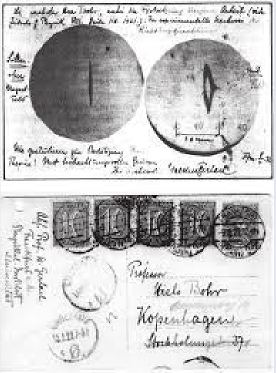
Wilczek Quantum Center, Shanghai, China, 21/07/2025

Part I: Quantum Theory

Quantum Theory: Quantization

- Quantum Mechanics was originally named after observation of quantized values:
  - Electromagnetic radiation (Black-body/Photoelectric effect)
  - Electron orbits (Atomic spectra)
  - Angular momentum (Stern-Gerlach)





Quantum Theory: Copenhagen Interpretation

- Copenhagen interpretation of Quantum Mechanics:
  1. Particles $\longleftrightarrow$ Waves $\rightarrow$ Superposition
  2. Outcomes of measurements: Observable eigenvalues $\rightarrow$ Quantization
  3. Probabilities of outcomes encoded in $|\Psi|^2$ $\rightarrow$ Interference

Quantum vs. Classical

- Quantum Mechanics: Superposition $\rightarrow$ *Fundamental* probabilistic description of measurements.
- Classical Mechanics: Random outputs using classical probability distributions resulting from *ignorance* (noise, experimental variations...)
- Is God *just* playing dice with the Universe? $\rightarrow$

Quantum vs. Classical

- Quantum Mechanics: Superposition $\rightarrow$ *Fundamental* probabilistic description of measurements.
- Classical Mechanics: Random outputs using classical probability distributions resulting from *ignorance* (noise, experimental variations...)
- Is God *just* playing dice with the Universe? $\rightarrow$ God is well beyond a mere croupier!
- Quantum Correlations=Correlations not accounted by classical probabilistic theories.

Quantum State

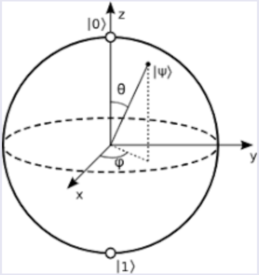
- **Pure state** $\rightarrow$ Wave function $|\Psi\rangle$
  1. $|\Psi\rangle = \sum_n \alpha_n \cdot |\phi_n\rangle\,,\ \langle\Psi|\Psi\rangle = \sum_n |\alpha_n|^2 = 1$
  2. Coherent mixture of quantum states $\rightarrow$ α_n are complex amplitudes
  3. Expectation values: $\langle A\rangle = \langle\Psi|A|\Psi\rangle = \sum_{n,m} \alpha_m^* \alpha_n \langle\phi_m|A|\phi_n\rangle$
- **Mixed state** $\rightarrow$ Generalization to density matrix ρ
  1. $\rho = \sum_n p_n \cdot |\phi_n\rangle\langle\phi_n|\,,\ \mathrm{tr}\rho = \sum_n p_n = 1,\ p_n \geq 0$
  2. Incoherent mixture of quantum states $\rightarrow$ p_n are probabilities
  3. Expectation values: $\langle A\rangle = \mathrm{tr}(\rho A) = \sum_n p_n \langle\phi_n|A|\phi_n\rangle$

Qubits: Pure state

- Qubit: Two-level quantum system $|0\rangle, |1\rangle \rightarrow$ Most simple!
- Paradigmatic example: Spin-1/2 particle. $|0\rangle \equiv |+\rangle$, $|1\rangle \equiv |-\rangle$
- General wave function: $|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\phi}|1\rangle \equiv |\hat{\mathbf{n}}\rangle \rightarrow$ Eigenstate of spin projection: $\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}|\hat{\mathbf{n}}\rangle = |\hat{\mathbf{n}}\rangle$
- Projective measurements with ± 1 outcome:

$$\Pi_{\pm\hat{\mathbf{n}}} = |\pm\hat{\mathbf{n}}\rangle\langle\pm\hat{\mathbf{n}}| = \frac{1 \pm \boldsymbol{\sigma}\cdot\hat{\mathbf{n}}}{2}$$

- Unit vector $\hat{\mathbf{n}} = [\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta] \rightarrow$ Surface of Bloch sphere.



Qubits: Density matrix

- General density matrix (2×2) for 1 qubit $\rightarrow$ 3 parameters B_i:

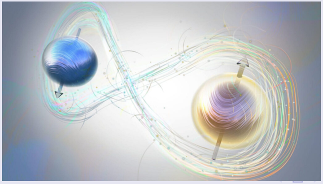
$$\rho = \frac{1 + \sum_i B_i \sigma^i}{2}, \ \mathbf{B} = \mathrm{tr}[\boldsymbol{\sigma}\rho], \ |\mathbf{B}| \leq 1$$

- Two qubits $\rightarrow$ Most simple example of quantum correlations.
- General density matrix (4×4) for 2 qubits $\rightarrow$ 15 parameters $B_i^{\pm}, C_{ij}$

$$\rho = \frac{\mathbb{1} + \sum_i \left(B_i^+ \sigma^i \otimes \mathbb{1} + B_i^- \mathbb{1} \otimes \sigma^i\right) + \sum_{i,j} C_{ij} \sigma^i \otimes \sigma^j}{4}$$

- Polarization vectors $\mathbf{B}^{\pm}$ and correlation matrix $\mathbf{C}$:

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle, \ B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle, \ C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$



Quantum Discord

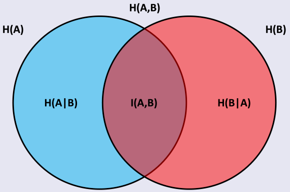
- Classically, two equivalent expressions for mutual information of bipartite system A and B (Alice and Bob):

$$\begin{aligned} I(A,B) &= H(A)+H(B)-H(A,B) = H(A)-H(A|B) \\ H(A,B) &= -\sum_{x,y} p(x,y)\log_2 p(x,y) \\ H(A|B) &= \sum_{y} p(y) H(A|B=y) \end{aligned}$$

- Quantum mechanics can introduce a "discord" between both expressions:

PRL 88, 017901 (2001)

$$\mathcal{D}(A,B) \equiv H(B) - H(A,B) + H(A|B) \neq 0$$



- Most basic form of quantum correlations!
- Quantum Discord is asymmetric $\mathcal{D}(A,B) \neq \mathcal{D}(B,A)$

Quantum Discord: Two qubits

- How do we translate classical into quantum?
- Shannon entropy $\rightarrow$ Von Neumann entropy ($p_n \geq 0$, ρ eigenvalues)

$$
\begin{aligned}
H(A,B) &\rightarrow H(\rho) = -\sum_n p_n \log_2 p_n \\
H(A) &\rightarrow H(\rho_A),\ H(B) \rightarrow H(\rho_B),\ \rho_{A,B} = \mathrm{Tr}_{B,A}\rho
\end{aligned}
$$

- Conditional probability $\rightarrow$ Conditional state $\rho_{A|B}$ = One-qubit state after Bob's spin measurement along $\hat{\mathbf{n}}$:

$$
H(A|B) = p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}})
$$

$$
\rho_{\hat{\mathbf{n}}} = \frac{\Pi^B_{\hat{\mathbf{n}}} \rho \Pi^B_{\hat{\mathbf{n}}}}{p_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^+_{\hat{\mathbf{n}}} \cdot \sigma}{2},\ \mathbf{B}^+_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^+ + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-},\ p_{\hat{\mathbf{n}}} = \frac{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^-}{2}
$$

- Genuine quantumness $\rightarrow$ Minimization over all spin directions to exclude quantization effects:

$$
\mathcal{D}(A,B) = H(\rho_B) - H(\rho) + \min_{\hat{\mathbf{n}}} p_{\hat{\mathbf{n}}} H(\rho_{\hat{\mathbf{n}}}) + p_{-\hat{\mathbf{n}}} H(\rho_{-\hat{\mathbf{n}}}) \neq 0
$$

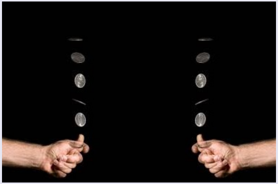
Quantum Discord: Classical states

- OK...but where is the Physics here? $\rightarrow$ Only classical states have zero discord!

$$\rho_{\text{class}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right|$$

- $\left|n\right\rangle, \left|m\right\rangle$ form an *orthonormal* basis for A, B
- $p_{n,m}$: classical probability of being $\left|n\right\rangle \otimes \left|m\right\rangle$
- Qubits $\rightarrow$ Tails and heads between two coins!

$$\begin{aligned} \rho_{\text{class}} &= p_{++} \left|++\right\rangle \left\langle ++\right| + p_{+-} \left|+-\right\rangle \left\langle +-\right| \\ &+ p_{-+} \left|-+\right\rangle \left\langle -+\right| + p_{--} \left|--\right\rangle \left\langle --\right| \end{aligned}$$



- Minimization=Search for correct basis $\left|\pm\right\rangle$ for Alice, Bob

Entanglement

- What if we generalize the previous idea? $\rightarrow$ Separability:

$$\rho_{\text{sep}} = \sum_{n,m} p_{n,m} \left|n\right\rangle \otimes \left|m\right\rangle \left\langle n\right| \otimes \left\langle m\right| = \sum_k p_k \rho_k^{(A)} \otimes \rho_k^{(B)}$$

- $\left|n\right\rangle, \left|m\right\rangle$ not necessarily orthonormal basis now $\rightarrow$ $p_{n,m}$ are *quasi-probabilities* (not disjoint events)
- Any classically correlated state (classical probability) is separable.
- **Entanglement**: Non-separability of a bipartite quantum state.

Entanglement: Two qubits

- Two qubits: Separability=$\exists$ Positive P-representation $P(\mathbf{n}_A, \mathbf{n}_B) \geq 0$:

$$\rho = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) \left|\mathbf{n}_A \mathbf{n}_B\right\rangle \left\langle \mathbf{n}_A \mathbf{n}_B\right|, \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- $P(\mathbf{n}_A, \mathbf{n}_B)$ is a quasi-probability: Overlap $|\langle \mathbf{n}_A | \mathbf{n}_B \rangle|^2 \neq 0$
- Separability=Purely classical spins pointing at directions $\mathbf{n}_A, \mathbf{n}_B$

$$C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle = \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B \, P(\mathbf{n}_A, \mathbf{n}_B) n_A^i n_B^j$$

- Entanglement=NO positive P-representation $\rightarrow$ Genuine non-classical!

Entanglement: Two qubits

- Practical entanglement criteria:
  - Peres-Horodecki criterion: ρ separable $\rightarrow$ Partial tranpose $\rho^{T_B} = \sum_n p_n \rho_n^A \otimes (\rho_n^B)^T$ non-negative. ρ^{T_B} not non-negative $\rightarrow$ ρ entangled!
    Very powerful: 2×2, 2×3 $\rightarrow$ Peres-Horodecki criterion=Entanglement
    Peres, PRL 77, 1413 (1996), Horodecki, PLA 232, 333 (1997).
  - Concurrence $0 \leq C[\rho] \leq 1$, $C[\rho] > 0$ iff ρ entangled. Wootters, PRL 80, 2245 (1998)
  - Entanglement witness: Not non-negative bservable W such that $\langle W\rangle_{\mathrm{sep}} = \mathrm{Tr}[W\rho_{\mathrm{sep}}] \geq 0 \rightarrow \langle W\rangle < 0$ implies entanglement. Terhal, PLA 271, 319 (2000)


Steering: Two qubits

- Steering: Original conception of Schrödinger of EPR paradox $\rightarrow$ Only well-defined in 2007! (Wiseman, Jones, Doherty, PRL 98, 140402 (2007))
- *Local* Quantum Mechanics: Alice post-measurement (un-normalized) state described by local-hidden states:

$$\tilde{\rho}_{\hat{\mathbf{n}}} = \Pi^B_{\hat{\mathbf{n}}} \rho \Pi^B_{\hat{\mathbf{n}}} = \int d\lambda \; p(1|\hat{\mathbf{n}}\lambda) p(\lambda) \rho_B(\lambda)$$

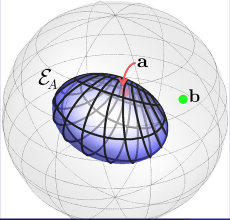
- If not, Bob measurements can "steer" Alice quantum state $\rightarrow$ **Steerability**.

Steering: Two qubits

- Alice post-measurement state: same as for quantum discord.

$$\rho_{\hat{\mathbf{n}}} = \frac{\tilde{\rho}_{\hat{\mathbf{n}}}}{\mathrm{Tr}\tilde{\rho}_{\hat{\mathbf{n}}}} = \frac{1 + \mathbf{B}^{+}_{\hat{\mathbf{n}}} \cdot \sigma}{2}, \quad \mathbf{B}^{+}_{\hat{\mathbf{n}}} = \frac{\mathbf{B}^{+} + \mathbf{C} \cdot \hat{\mathbf{n}}}{1 + \hat{\mathbf{n}} \cdot \mathbf{B}^{-}}$$

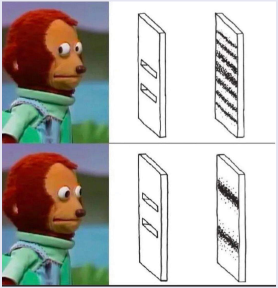
- Set of conditional polarizations $\mathbf{B}^{+}_{\hat{\mathbf{n}}}$ describes an ellipsoid.
- Steering ellipsoid: Fundamental QI object, containing all information about the system.
- Similar for Bob $\rightarrow$ Steering: also asymmetric between Alice and Bob.
- First (and sole) experimental reconstruction ever at Hefei University (10^7 repetitions!!!) Zhang et al. PRL 122, 070402 (2019)

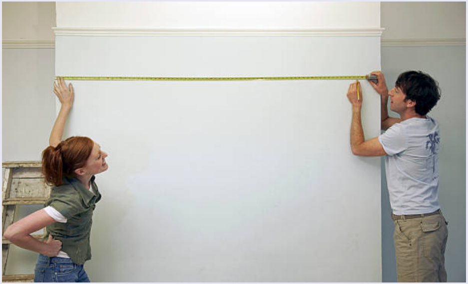


Jevtic, Pusey, Jennings, Rudolph PRL 113, 020402 (2014)

Bell nonlocality

- EPR Paradox: Quantum Mechanics challenges local realism!
- Bell test: Joint Alice and Bob perform causally disconnected measurements in alternate freely chosen setups M_A, M_B
- Bell Theorem: Local realistic theories satisfy certain (Bell) inequalities
- Beauty of Bell Theorem: NO underlying model, just two people enjoying their measurements





Bell nonlocality: Two qubits

- Two qubits: M_A, M_B are projective spin measurements along axes $\mathbf{a}_i, \mathbf{b}_i$

$$C(M_A, M_B) = \sum_{a,b=\pm 1} (a \cdot b) p(a, b|M_A M_B)$$

- Local realism $\rightarrow$ Local hidden-variable model (LHVM)

$$p(a, b|M_A M_B) = \int d\lambda \; p(a|M_A\lambda) p(b|M_B\lambda) p(\lambda)$$

- Bell inequality $\rightarrow$ **CHSH inequality**:

$$|C(M_A, M_B) - C(M_A, M'_B) + C(M'_A, M_B) + C(M'_A, M'_B)| \leq 2$$

- Theoretical prediction from Quantum Mechanics:

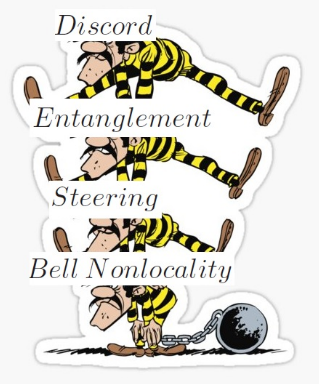
$$|\mathbf{a}_1^{\mathrm{T}} \mathbf{C} (\mathbf{b}_1 - \mathbf{b}_2) + \mathbf{a}_2^{\mathrm{T}} \mathbf{C} (\mathbf{b}_1 + \mathbf{b}_2)| \leq 2$$

- CHSH violation *iff* $\sqrt{\mu_1 + \mu_2} > 1$, $\mu_{1,2}$ largest eigenvalues of $\mathbf{C}^{\mathrm{T}}\mathbf{C}$
- Bell nonlocal states: Quantum states able to violate CHSH inequality

Hierarchy of Quantum Correlations

- Steering and Discord can be asymmetric between Alice and Bob.
- Bell Nonlocality and Entanglement are always symmetric.
- Quantum Hierarchy:

$$\textit{Bell Nonlocality} \subset \textit{Steering} \subset \textit{Entanglement} \subset \textit{Discord}$$



Quantum Tomography: Two qubits

- **Quantum Tomography**: Reconstruction of quantum state from measurement of a set of observables.
- Quantum tomography $\rightarrow$ Characterization of ALL quantum correlations.

- One-qubit tomography=3 parameters: polarization vector **B**: $B_i = \langle \sigma^i \rangle$
- Two-qubit tomography=15 parameters: polarization vectors $\mathbf{B}^{\pm}$ and correlation matrix **C**:

$$B_i^+ = \langle \sigma^i \otimes \mathbb{1} \rangle \,,\; B_i^- = \langle \mathbb{1} \otimes \sigma^i \rangle \,,\; C_{ij} = \langle \sigma^i \otimes \sigma^j \rangle$$

Part II: Quantum Fields

Quantum Optics

- Light has motivated the major advancements in modern Physics:
  1. EM radiation: Maxwell equations
  2. Special relativity: Michelson-Morley experiment
  3. Quantum mechanics: Black-body spectrum & Photoelectric effect
  4. Quantum field theory: First quantized field

... and God said: "let there be light" ...

Quantum field theory

- Set of classical fields $\{\phi_A\}$ described by Lagrangian density $\mathcal{L} : (\phi_A, \partial_\mu \phi_A, x)$

$$L = \int \mathrm{d}\mathbf{x}\ \mathcal{L}(\phi_A, \partial_\mu \phi_A, x),\ S = \int \mathrm{d}t\ L = \int \mathrm{d}^4 x\ \mathcal{L}$$

- Equations of motion: $\partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \phi_A} = \frac{\delta \mathcal{L}}{\delta \phi_A}$
- Canonical momentum: $\Pi_A(x) \equiv \frac{\delta \mathcal{L}}{\delta \partial_t \phi_A}$
- Poisson brackets from Hamiltonian theory:

$$\{Q, P\} = \sum_A \int \mathrm{d}\mathbf{x}\ \frac{\delta Q}{\delta \phi_A(\mathbf{x})} \frac{\delta P}{\delta \Pi_A(\mathbf{x})} - \frac{\delta Q}{\delta \Pi_A(\mathbf{x})} \frac{\delta P}{\delta \phi_A(\mathbf{x})}$$

- Quantization à la Dirac: promotion of magnitudes Q, P to operators $\hat{Q}, \hat{P}$ satisfying commutation relations:

$$[\hat{Q}, \hat{P}] = i\hbar \{Q, P\}$$

Quantum KG field

- Simplest example: massless Hermitian scalar field $\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi$.
- Field $\phi \to$ Doublet $\Phi = \begin{bmatrix} \phi \\ i\Pi \end{bmatrix}$
- KG equation $\Box\phi = \partial_\mu\partial^\mu\phi = 0 \to$ Linear equation $i\hbar\partial_t\Phi = M\Phi$:
  - Expansion of the field in terms of eigenmodes: $M\Phi_n = \epsilon_n\Phi_n$
  - Conjugate solutions: $\bar{\Phi}_n = \sigma_z\Phi_n^*$
  - Conserved inner product:

$$(\Phi_n|\Phi_m) \equiv i\int \mathrm{d}^3\mathbf{x}\,(\phi_n^*\Pi_m - \phi_m\Pi_n^*) = \langle\Phi_n|\sigma_x|\Phi_m\rangle$$

- Careful: both M and inner product are not positive definite!

$$M\bar{\Phi}_n = -\epsilon_n^*\bar{\Phi}_n,\ (\Phi_n|\Phi_m) = -(\bar{\Phi}_m|\bar{\Phi}_n)$$

- Plane waves provide complete orthonormal basis with positive/negative frequency (norm)

$$M\Phi_{\mathbf{k}} = \hbar\omega_{\mathbf{k}}\Phi_{\mathbf{k}},\ \Phi_{\mathbf{k}}(\mathbf{x}) = \frac{e^{i\mathbf{kx}}}{\sqrt{2\omega_{\mathbf{k}}(2\pi)^3}}\begin{bmatrix} 1 \\ \omega_{\mathbf{k}} \end{bmatrix},\ M\bar{\Phi}_{\mathbf{k}} = -\hbar\omega_{\mathbf{k}}\bar{\Phi}_{\mathbf{k}}$$

Harmonic oscillators

- Fourier amplitudes promoted to operators: $\hat{a}(\mathbf{k}) = (\Phi_{\mathbf{k}}|\hat{\Phi}) \rightarrow$

$$\hat{\Phi}(\mathbf{x}) = \sqrt{\hbar}\int \mathrm{d}^3\mathbf{k}\ [\hat{a}(\mathbf{k})\Phi_{\mathbf{k}}(\mathbf{x}) + \hat{a}^{\dagger}(\mathbf{k})\Phi^{*}_{\mathbf{k}}(\mathbf{x})]$$

- Canonical commutation rules $\rightarrow [\hat{a}(\mathbf{k}), \hat{a}^{\dagger}(\mathbf{k}')] = (\Phi_{\mathbf{k}}|\Phi_{\mathbf{k}'}) = \delta(\mathbf{k}-\mathbf{k}')$
- QFT described in terms of harmonic oscillators $\rightarrow$ We can import all our knowledge from textbooks:
  - Phase space variables: $\hat{X} = \frac{\hat{a}+\hat{a}^{\dagger}}{\sqrt{2}}$, $\hat{P} = i\frac{\hat{a}^{\dagger}-\hat{a}}{\sqrt{2}}$
  - Number states: $\hat{a}^{\dagger}\hat{a}\,|n\rangle = n\,|n\rangle$
  - Coherent states: $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$, $|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$, $\alpha \in \mathbb{C}$
  - Squeezed coherent states: Coherent states of Bogoliubov transformation

$$U = e^{\lambda[(\hat{a}^{\dagger})^2-\hat{a}^2]},\ \hat{b} = U^{\dagger}\hat{a}U = (\cosh\lambda)\hat{a} + (\sinh\lambda)\hat{a}^{\dagger}$$

Fock, P and Wigner representations

- Number states form complete orthonormal basis $I = \sum_n |n\rangle\langle n|$:

$$\rho = \sum_{n,m} \rho_{nm} |n\rangle\langle m|$$

- Glauber-Sudarshan P-representation:

$$\rho = \int \mathrm{d}^2\alpha \; P(\alpha) |\alpha\rangle\langle\alpha|, \; \int \mathrm{d}^2\alpha \; P(\alpha) = 1,$$

- Normal-ordered expectation values $\int \mathrm{d}^2\alpha \; |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a}\rangle$
- Wigner representation ($\alpha = \frac{X+iP}{\sqrt{2}}$) $\rightarrow$ Quantum Boltzmann distribution:

$$W(X,P) = \frac{1}{2\pi}\int \mathrm{d}\Delta X \; \langle X + \frac{\Delta X}{2}|\rho|X - \frac{\Delta X}{2}\rangle \, e^{-iP\Delta X}$$

- *Symmetric* expectation values: $\int \mathrm{d}^2\alpha \; |\alpha|^2 W(\alpha) = \langle \frac{\hat{a}^\dagger \hat{a} + \hat{a}\hat{a}^\dagger}{2}\rangle$
- *Quasiprobabily* distributions: Coherent states form overcomplete basis

$$I = \frac{1}{\pi}\int \mathrm{d}^2\alpha \; |\alpha\rangle\langle\alpha|, \; |\langle\alpha|\beta\rangle|^2 \neq 0$$

Quantum Correlations

- General quantum state of EM field:

$$\rho = \int \mathrm{d}^2\{\alpha_{\mathbf{k}}\}\ P(\{\alpha_{\mathbf{k}}\}) \prod_{\mathbf{k}} \otimes \left|\alpha_{\mathbf{k}}\right\rangle \left\langle\alpha_{\mathbf{k}}\right|$$

- Measurements=Normal-ordered expectation values $\rightarrow$ Positive P-representation yields classical states of light $\rightarrow$ Quantum correlations require negative P-representations!
- Classical states $\sim |\alpha_{\mathbf{k}}| \gg 1$, $|\Delta\alpha_{\mathbf{k}}/\alpha_{\mathbf{k}}| \ll 1$
- Bipartite Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$: Product space of Fock spaces of two modes $\hat{a}_i, \hat{a}_j \rightarrow$ Infinite dimension!

$$\rho = \int \mathrm{d}^2\alpha_i \mathrm{d}^2\alpha_j\ P(\alpha_i, \alpha_j) \left|\alpha_i\alpha_j\right\rangle \left\langle\alpha_i\alpha_j\right|$$

Continuous Variable Entanglement

- Continuous system: Covariance matrix M must be positive if the density matrix is physical (alternative expression of uncertainty principle)

$$
\begin{aligned}
M_{\alpha\beta} &= \langle \Delta\xi_\alpha \Delta\xi_\beta \rangle = \frac{\langle\{\Delta\xi_\alpha \Delta\xi_\beta\}\rangle}{2} + \frac{\langle[\Delta\xi_\alpha \Delta\xi_\beta]\rangle}{2} \equiv V_{\alpha\beta} + i\frac{\Omega_{\alpha\beta}}{2} \\
\Delta\xi_\alpha &= \xi_\alpha - \langle\xi_\alpha\rangle\,,\ \xi_\alpha = [X_1, X_2, P_1, P_2],\ \alpha = \frac{X + iP}{\sqrt{2}}
\end{aligned}
$$

- Transpose operation via Wigner representation is time inversion!

$$
\begin{aligned}
\rho \quad &: \quad W(X_1, X_2, P_1, P_2) \rightarrow \rho^{T_2} :\ W(X_1, X_2, P_1, -P_2) \\
&\rightarrow \quad V' = \Lambda V \Lambda,\ \Lambda \equiv \mathrm{diag}[1, 1, 1, -1]
\end{aligned}
$$

- Peres-Horodecki: ρ^{T_2} covariance matrix $M' = V' + i\frac{\Omega}{2}$ not non-negative $\rightarrow$ Entanglement!
- Gaussian state$\rightarrow$ Peres-Horodecki$=$Entanglement

Simon, PRL 84, 2726 (2000)

Cauchy-Schwarz violation

- Classical light (non-negative P-representation) satisfies

$$|\langle \hat{a}^2 \rangle| = \left| \int \mathrm{d}^2\alpha \; P(\alpha)\alpha^2 \right| \leq \int \mathrm{d}^2\alpha \; P(\alpha)|\alpha|^2 = \langle \hat{a}^\dagger \hat{a} \rangle$$

- Proof explicitly based on positivity of P $\rightarrow$ Effective scalar product $\rightarrow$ Cauchy-Schwarz inequalities!
- Operators A, B do satisfy mathematical Cauchy-Schwarz inequality associated to trace scalar product

$$\langle A|B \rangle \equiv \mathrm{tr}(\rho A^\dagger B) = \langle A^\dagger B \rangle$$

- *Quantum* CS inequality: $|\langle \hat{a}^2 \rangle| \leq \langle \hat{a}\hat{a}^\dagger \rangle \langle \hat{a}^\dagger \hat{a} \rangle = \langle \hat{a}^\dagger \hat{a} \rangle (\langle \hat{a}^\dagger \hat{a} \rangle + 1)$
- There is room for violation of *classical* CS inequality!
- Explicit example: Squeezed vacuum state $|\lambda\rangle = e^{\lambda[(\hat{a}^\dagger)^2 - \hat{a}^2]} |0\rangle$:

$$|\langle \hat{a}^2 \rangle| = \cosh\lambda |\sinh\lambda| > \sinh^2\lambda = \langle \hat{a}^\dagger \hat{a} \rangle \,, \;\; \lambda \neq 0 \in \mathbb{R}$$

Cauchy-Schwarz violation: Bipartite case

- Bipartite quantum correlations tested via first, second-order correlations:

$$g_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j \rangle \,, \ c_{ij} = \langle \hat{a}_i \hat{a}_j \rangle \,, \ \Gamma_{ij} = \langle \hat{a}_i^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_i \rangle$$

- Classical vs. Quantum CS inequality
  - $|c_{ij}|^2 \leq g_{ii} g_{jj}$ vs. $|c_{ij}|^2 \leq g_{ii}(g_{jj}+1)$
  - $\Gamma_{ij} \leq \sqrt{\Gamma_{ii}\Gamma_{jj}}$ vs. $\Gamma_{ij} \leq \sqrt{(\Gamma_{ii}+g_{ii})(\Gamma_{jj}+g_{jj})}$
- CS violation *independent* from entanglement $\rightarrow$ Complementary test of quantum behavior!
- Explicit example: Product number state $\rho = |nm\rangle \langle nm|$ with $n, m \neq 0$

$$\Gamma_{ij} = nm > \sqrt{n(n-1)m(m-1)} = \sqrt{\Gamma_{ii}\Gamma_{jj}}$$

- Number states do have negative Wigner representations, strongest requirement than negative P-representation $\rightarrow$ Entanglement is not the whole story...

Quantum Many-Body Field Theory

- Second-quantization many-body Hamiltonian (fermions and bosons):

$$\hat{H} = \int \mathrm{d}\mathbf{x}\, \hat{\Psi}^\dagger(\mathbf{x}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x})\right] \hat{\Psi}(\mathbf{x})$$
$$+ \frac{1}{2}\int \mathrm{d}\mathbf{x} \int \mathrm{d}\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}^\dagger(\mathbf{x}')V(\mathbf{x}-\mathbf{x}')\hat{\Psi}(\mathbf{x}')\hat{\Psi}(\mathbf{x})$$

- Field operator: $\hat{\Psi}(\mathbf{x}) = \sum_n \hat{a}_n \phi_n(\mathbf{x})$, $\hat{a}_n = \langle \phi_n | \hat{\Psi} \rangle$
- Fermionic or bosonic annihilation operators: $[\hat{a}_n, \hat{a}^\dagger_m]_\pm = \delta_{nm}$
- Heisenberg equation of motion of field operator:

$$i\hbar\partial_t \hat{\Psi}(\mathbf{x},t) = [\hat{\Psi}(\mathbf{x},t), \hat{H}]$$
$$= \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + \int \mathrm{d}\mathbf{x}' \hat{\Psi}^\dagger(\mathbf{x}',t)V(\mathbf{x}-\mathbf{x}')\hat{\Psi}(\mathbf{x}',t)\right] \hat{\Psi}(\mathbf{x},t)$$

- Same equation of motion derived from a classical Lagrangian:

$$L = \int \mathrm{d}\mathbf{x}\; i\Psi^*\partial_t\Psi - H$$

Non-relativistic vs. relativistic QFT

- Differences with respect relativistic QFT:
  - Not Lorentz invariant
  - Fixed cutoffs (IR $\rightarrow$ System size; UV $\rightarrow$ Microscopic physics)
  - Interactions are (typically) classical
  - Effective and approximate theories
  - Second-quantization is not fundamental (trick to rewrite symmetric/antisymmetric wavefunctions)
- Features:
  - Highly tunable (interactions, external potential, mass)
  - Non-trivial spatiotemporal dependence leads to rich physics (Analogue Gravity, Floquet systems)
  - Variety of internal degrees of freedom (high spin, isospin, mixtures)
  - Many-body systems lead to spontaneous symmetry breaking (supersolids, time crystals)
  - Topological physics and synthetic dimensions

Condensates: GP mean-field

- Bose-Einstein condensate close to $T = 0$
- Field operator: $\hat{\Psi}(\mathbf{x}, t) = \Psi(\mathbf{x}, t) + \hat{\varphi}(\mathbf{x}, t) \rightarrow$ Higgs-mechanism
- Contact interaction $V(\mathbf{x} - \mathbf{x}') = g\delta(\mathbf{x} - \mathbf{x}') \rightarrow$ Non-relativistic Higgs boson!

$$\mathcal{L} = i\Psi^*\partial_t\Psi - \frac{|\nabla\Psi|^2}{2m} - V(\mathbf{x})|\Psi|^2 - g\frac{|\Psi|^4}{2}$$

- Heisenberg e.o.m. $\rightarrow$ Gross-Pitaevskii equation:

$$i\hbar\partial_t\Psi(\mathbf{x}, t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g|\Psi(\mathbf{x}, t)|^2\right]\Psi(\mathbf{x}, t)$$

- Condensate wavefunction=Coherent state spontaneously breaking $U(1)$ symmetry

Condensates: Bogoliubov approximation

- Stationary condensate: $\hat{\Psi}(\mathbf{x},t)=[\Psi_0(\mathbf{x})+\hat{\varphi}(\mathbf{x},t)]e^{-i\mu t/\hbar}$+Quantum fluctuations at linear order $\rightarrow$ Bogoliubov-de Gennes equations:

$$M\hat{\Phi} = i\hbar\partial_t\hat{\Phi},\ M=\begin{bmatrix} N & A \\ -A^* & -N^* \end{bmatrix},\ \hat{\Phi}\equiv\begin{bmatrix} \hat{\varphi}(\mathbf{x},t) \\ \hat{\varphi}^\dagger(\mathbf{x},t) \end{bmatrix}$$

$$N = -\frac{\hbar^2\nabla^2}{2m}+V(\mathbf{x})+2g|\Psi_0(\mathbf{x})|^2-\mu,\ A=g\Psi_0^2(\mathbf{x})$$

- Eigenmodes: $Mz_n=\epsilon_n z_n,\ z_n=\begin{bmatrix} u_n \\ v_n \end{bmatrix},\ M\bar{z}_n=-\epsilon_n^*\bar{z}_n,\ \bar{z}_n=\sigma_x z_n^*$
- Conserved inner product:

$$(z_n|z_m)\equiv\langle z_n|\sigma_z|z_m\rangle=\int \mathrm{d}\mathbf{x}\ z_n^\dagger\sigma_z z_m=\int \mathrm{d}\mathbf{x}\ u_n^*u_m-v_n^*v_m$$

- Complete basis $(z_n|z_m)=\delta_{nm}\Longrightarrow$ Quantization mimics KG field:

$$\hat{\Phi}=\sum_n z_n\hat{b}_n+\bar{z}_n\hat{b}_n^\dagger$$

Bogoliubov vs. Higgs

- Amplitude-phase (Madelung) decomposition:

$$\hat{\Psi}(\mathbf{x},t)=\sqrt{\hat{n}(\mathbf{x},t)}e^{i\hat{\theta}(\mathbf{x},t)}=\sqrt{n(\mathbf{x},t)+\delta\hat{n}(\mathbf{x},t)}e^{i(\theta(\mathbf{x},t)+\delta\hat{\theta}(\mathbf{x},t))}$$

- Density/phase fluctuations=Higgs/Goldstone modes:

$$\begin{aligned}\delta\hat{n}(\mathbf{x},t) &= \Psi_0^*(\mathbf{x})\hat{\varphi}(\mathbf{x},t)+\Psi_0(\mathbf{x})\hat{\varphi}^\dagger(\mathbf{x},t)\\ \delta\hat{\theta}(\mathbf{x},t) &= i\frac{\Psi_0(\mathbf{x})\hat{\varphi}^\dagger(\mathbf{x},t)-\Psi_0^*(\mathbf{x})\hat{\varphi}(\mathbf{x},t)}{2|\Psi_0(\mathbf{x})|^2}\end{aligned}$$

- Homogeneous condensate: $\Psi_0(\mathbf{x})=\sqrt{n_0}\rightarrow$ Massless superluminal dispersion relation for both Higgs and Goldstone modes:

$$\omega^2=c^2k^2\left(1+\frac{k^2}{4\Lambda^2}\right),\ c^2=\frac{gn_0}{m},\ \Lambda=\frac{1}{\xi_0}=\frac{mc}{\hbar}$$

- Actual Higgs: $\omega^2=c^2k^2$ (Goldstone), $\omega^2=\frac{m_H^2c^4}{\hbar^2}+c^2k^2$ (Higgs)

Analogue gravity

- GP/Schrödinger equation similar to Euler equation for potential flow!

$$\partial_t n + \nabla(n\mathbf{v}) = 0,\ \ \mathbf{v}(\mathbf{x},t) = \frac{\hbar\nabla\theta(\mathbf{x},t)}{m} \rightarrow \text{Potential flow!}$$

$$\hbar\partial_t\theta = \frac{\hbar^2\nabla^2\sqrt{n}}{2m\sqrt{n}} - \frac{1}{2}m\mathbf{v}^2 - gn - V$$

- Neglecting quantum ($\hbar$) potential in smooth background $\rightarrow$ Ideal potential flow $\rightarrow$ Analogue gravity (massless Hermitian field in curved spacetime)

$$\begin{aligned}\Box\delta\hat{\theta} &\equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\delta\hat{\theta}) = 0\\ g_{\mu\nu}(x) &= \frac{n(x)}{c(x)}\begin{bmatrix} -[c^2(x)-v^2(x)] & -\mathbf{v}^T(x)\\ -\mathbf{v}(x) & \delta_{ij}\end{bmatrix},\end{aligned}$$

- Gravitational phenomena (e.g., Hawking radiation) in the lab!

W. G. Unruh, PRL 46, 1351 (1981)

L. J. Garay, J. R. Anglin, J. I. Cirac, P. Zoller, PRL 85, 4643 (2000)

Entanglement: practical resource

- PhD question: What makes Hawking radiation quantum? How to distinguish zero-point quantum effect from stimulated emission from thermal or coherent sources?
- Answer: Quantum correlations can be used as a smoking gun of genuine, quantum Hawking effect!

PHYSICAL REVIEW A **89**, 043808 (2014)

Violation of Cauchy-Schwarz inequalities by spontaneous Hawking radiation in resonant boson structures

J. R. M. de Nova, F. Sols, and I. Zapata
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(Received 7 November 2012; revised manuscript received 20 February 2014; published 7 April 2014)

New Journal of Physics
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Deutsche Physikalische Gesellschaft DPG
IOP Institute of Physics
Published in partnership with: Deutsche Physikalische Gesellschaft and the Institute of Physics

PAPER

Entanglement and violation of classical inequalities in the Hawking radiation of flowing atom condensates

J R M de Nova, F Sols and I Zapata
Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain

- Eventual experimental observations of Hawking radiation for the first (and sole!) time:

LETTER

https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

nature physics
ARTICLES
https://doi.org/10.1038/s41567-020-01076-0
Check for updates

Observation of stationary spontaneous Hawking radiation and the time evolution of an analogue black hole

Victor I. Kolobov, Katrine Golubkov, Juan Ramón Muñoz de Nova and Jeff Steinhauer

- These techniques can also characterize the quantum nature of gravitational waves/dark matter (Frank, Liantao talks)

Time crystals & Spontaneous Floquet states

- Time crystals were proposed by Frank Wilczek in 2012 as a (equilibrium) state spontaneously time-translation symmetry:

Quantum Time Crystals

Frank Wilczek
Center for Theoretical Physics Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
(Received 29 March 2012; published 15 October 2012)

- Exciting prospects to study Quantum Chronodynamics!

Simultaneous symmetry breaking in spontaneous Floquet states: Floquet-Nambu-Goldstone modes, Floquet thermodynamics, and the time operator

Juan Ramón Muñoz de Nova and Fernando Sols
Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain
(Dated: April 22, 2024)

Hawking time crystal

Juan Ramón Muñoz de Nova and Fernando Sols
Departamento de Física de Materiales, Universidad Complutense de Madrid, E-28040 Madrid, Spain
(Dated: July 16, 2025)

- Hawking time crystal: Time crystal spawned by the quantum spontaneous Floquet state achieved through self-amplification of spontaneous Hawking radiation
- Spontaneous Floquet states display unique properties:
  - Floquet-Nambu-Goldstone modes of zero-quasifrequency associated to temporal symmetry breaking
  - Tangible and unique realization of Time Operator in Quantum Mechanics

Part III: Quantum Colliders

How did I end up here????

- So many analogies...why not studying quantum problems in High-Energy Physics, a relativistic QFT?
- One day, after years of coffee breaks at the Technion with my friend Yoav Afik...

- Juan: Mmmm...Could you measure a CS violation in a collider? It is like entanglement but not the same and...
- Yoav: This Cauchy-Schwarz thing is weird...Entanglement is interesting. Tell me more. With fermions.
- Juan: Oh, well, then there is the spin, it is a qubit you see, there are products of Pauli matrices...
- Yoav: Better. Give me a sec.

Dramatization

Top Drosophila

- Yoav eventually came to my office and showed me one single equation on top quarks...

$$R^I_{\alpha_1\alpha_2,\ \beta_1\beta_2} = \overline{\sum} \langle t(k_1,\alpha_2), \bar{t}(k_2,\beta_2)|\mathcal{T}|a(p_1), b(p_2)\rangle^* \times \langle t(k_1,\alpha_1), \bar{t}(k_2,\beta_1)|\mathcal{T}|a(p_1), b(p_2)\rangle \quad (2.3)$$

where $I \equiv ab = gg, q\bar{q}$ and the bar denotes averaging over the spins and colors of I and summing over the colors of $t, \bar{t}$. Moreover, α, β are spin labels referring to t and $\bar{t}$, respectively. The matrices R^I can be decomposed in the spin spaces of t and $\bar{t}$ as follows:

$$R^I = f_I \left[A^I \mathbb{1} \otimes \mathbb{1} + \tilde{B}^{I+}_i \sigma^i \otimes \mathbb{1} + \tilde{B}^{I-}_i \mathbb{1} \otimes \sigma^i + \tilde{C}^I_{ij} \sigma^i \otimes \sigma^j \right], \quad (2.4)$$

$$f_{gg} = \frac{(4\pi\alpha_s)^2}{N_c(N_c^2-1)}, \qquad f_{q\bar{q}} = \frac{(N_c^2-1)(4\pi\alpha_s)^2}{N_c^2},$$

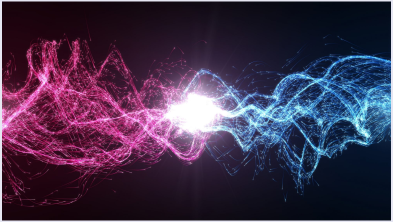
where N_c denotes the number of colors. The first (second) factor in the tensor products of the 2×2 unit matrix $\mathbb{1}$ and of the Pauli matrices σ^i refers to the t ($\bar{t}$) spin space.

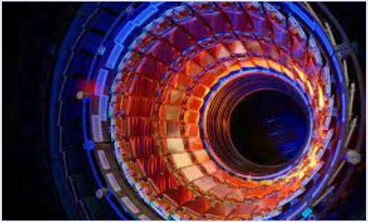
W. Bernreuther, D. Heisler, Z.-G. Si, JHEP 12, 026 (2015)

- Top quarks are a drosophila of relativistic two-qubit system $\rightarrow$ Huge potential!

Quantum High-Energy Colliders?

- Naively, Quantum Correlations should be easily studied in colliders...right? $\rightarrow$ Not so fast!
  - Momentum measurement $\rightarrow$ Decoherence
  - Lack of control of internal d.o.f. in initial state $\rightarrow$ Decoherence
  - Most relevant observables in colliders: cross-sections, lifetimes... $\rightarrow$ Classical probabilistic objects
  - QFT cares about expectation values, not quantum states!





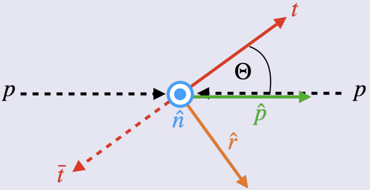
Top-pair production: Kinematics

- Kinematics determined by invariant mass $M_{t\bar{t}}$ and top direction $\hat{k}$ in c.m. frame

$$\begin{aligned} k^{\mu} &= (k^0, \mathbf{k}), \bar{k}^{\mu} = (k^0, -\mathbf{k}) \\ M_{t\bar{t}}^2 &\equiv s \equiv (k+\bar{k})^2 \end{aligned}$$

- Invariant mass M is simply related to top c. m. velocity β

$$M_{t\bar{t}} = \frac{2m_t}{\sqrt{1-\beta^2}} \to \beta = 0 \to M_{t\bar{t}} = 2m_t$$



Top-pair production: Quantum State

- Production process from initial state I with internal degrees of freedom λ: $|I\lambda\rangle \to t + \bar{t}$
- $t\bar{t}$ spins described by production spin density matrix:

$$R^{I\lambda}_{\alpha\beta,\alpha'\beta'}(M_{t\bar{t}},\hat{k}) \equiv \langle M_{t\bar{t}}\hat{k}\alpha\beta|\, T\, |I\lambda\rangle \langle I\lambda|\, T^{\dagger}\, |M_{t\bar{t}}\hat{k}\alpha'\beta'\rangle$$

- Experiment: Momentum measurements+Average over events $\to$ Genuine density-matrix description!

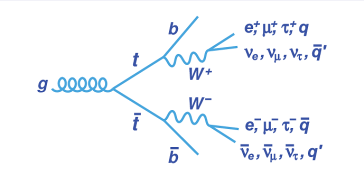
$$\begin{aligned} R^{I}(M_{t\bar{t}},\hat{k}) &= \frac{1}{N_\lambda}\sum_{\lambda} R^{I\lambda}(M_{t\bar{t}},\hat{k}) \\ \rho^{I}(M_{t\bar{t}},\hat{k}) &= \frac{R^{I}(M_{t\bar{t}},\hat{k})}{\mathrm{tr}\left[R^{I}(M_{t\bar{t}},\hat{k})\right]} \end{aligned}$$

Quantum Tomography: Two qubits, two tops

- Top quarks: Spin polarizations $\mathbf{B}^{\pm}$ and spin correlation matrix $\mathbf{C}$ extracted from cross-section $\sigma_{\ell\bar{\ell}}$ of dileptonic decay

$$\frac{1}{\sigma_{\ell\bar{\ell}}}\frac{d\sigma_{\ell\bar{\ell}}}{d\Omega_+ d\Omega_-} = \frac{1}{(4\pi)^2}\left[1 + \mathbf{B}^+ \cdot \hat{\ell}_+ - \mathbf{B}^- \cdot \hat{\ell}_- - \hat{\ell}_+ \cdot \mathbf{C} \cdot \hat{\ell}_-\right]$$

- $\hat{\ell}_{\pm}$: antilepton (lepton) directions in top (antitop) rest frames → Spin well-defined in parent rest frame
- Whole spin-density matrix can be reconstructed! → Quantum tomography in a high-energy collider!



Cauchy-Schwarz violation in spin systems

- Simple criterion of entanglement: Cauchy-Schwarz violation

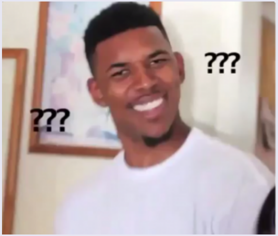
$$|\mathrm{tr}\ \mathbf{C}| = |\langle \boldsymbol{\sigma} \cdot \boldsymbol{\sigma} \rangle| = \left| \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) \mathbf{n}_A \cdot \mathbf{n}_B \right|$$

$$\leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) \left| \mathbf{n}_A \cdot \mathbf{n}_B \right| \leq \int \mathrm{d}\Omega_A \mathrm{d}\Omega_B\ P(\mathbf{n}_A, \mathbf{n}_B) = 1$$

- Wait a minute...A cosine average larger than one??? $\rightarrow$ CS Violation $\rightarrow$ Entanglement
  - Directly measurable entanglement witness:

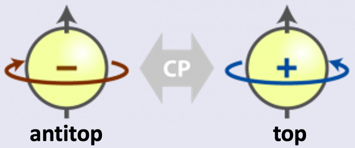
$$\frac{1}{\sigma} \frac{d\sigma}{d \cos \varphi} = \frac{1 - D \cos \varphi}{2}$$

  - D=Quantum observable with a genuine quantum range of values $-1 \leq D < -1/3$



New Physics Witnesses

- Approximate CP-invariance of Standard Model $\rightarrow$ $\mathbf{C} = \mathbf{C}^{\mathrm{T}}$, $\mathbf{B}^{+} = \mathbf{B}^{-}$ $\rightarrow$ Symmetric Discord and Steering!
- Therefore: Discord and/or Steering asymmetry $\rightarrow$ New Physics!
- New physics witnesses: Symmetry protected observables by SM, only non-zero for New Physics:
  - $\Delta\mathcal{D}_{t\bar{t}} \equiv \mathcal{D}_t - \mathcal{D}_{\bar{t}}$
  - Asymmetries in steering measurements.
- No SM contribution to New Physics witnesses!



Y. Afik, JRMdN, PRL 130, 221801 (2023)

Thank You

- And now...the real story...

Backup

Quantum Theory: Measurement

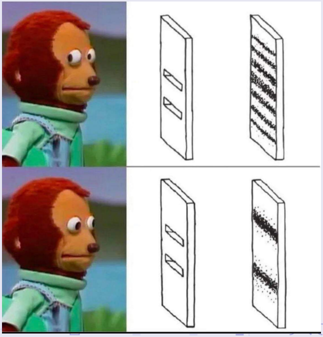
- Copenhagen interpretation of Quantum Mechanics:
  - Wavefunction collapses by measurement process: Quantum state projected to measured state.
- Simple model of measurement: System+Apparatus get entangled

$$|\Psi\rangle \otimes |A\rangle \rightarrow \sum_{n} c_n |\phi_n\rangle \otimes |A_n\rangle \rightarrow \rho = \sum_{n,n} c_n c_m^* |\phi_n\rangle \otimes |A_n\rangle \langle\phi_m| \otimes \langle A_m|$$

- Decoherence: Tracing out environment $\rightarrow$

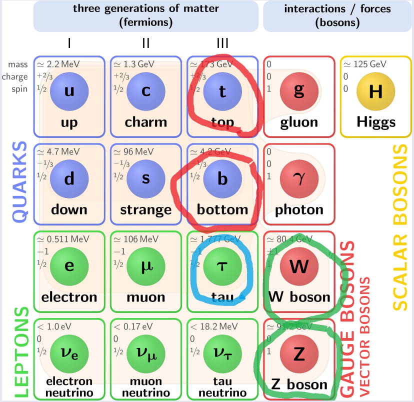
$$\rho_S = \mathrm{tr}_E(\rho) = \sum_{n} |c_n|^2 |\phi_n\rangle \otimes |A_n\rangle \langle\phi_n| \otimes \langle A_n|$$

- Probabilities according to Born rule!
- Collapse is irreversible and... who is irreversible??? Exactly!
- CAUTION: Still not solved



Alternative and complementary approaches

- Quantum spin-correlations in alternative qubits and qutrits:



1. t (EPJ Plus 136, 907 (2021))
2. b (arXiv 2406.04402 (2024))
3. τ (EPJC 83, 162 (2023))
4. $W^{\pm}$(PLB 825, 136866 (2022))
5. Z^0(PRD 107, 016012 (2023)).

- Complementary approaches QI-HEP:
  1. Flavor entanglement in mesons: PRL 99, 131802 (2007)
  2. Neutrino oscillations: PRL 117, 050402 (2016)
  3. QI techniques to study QCD interactions: PRL 124, 062001 (2020)

Quantum optics representations

- Quantum state completely determined by characteristic function

$$\chi(\eta) \equiv \langle e^{\eta\hat{a}^\dagger - \eta^* \hat{a}} \rangle = \mathrm{Tr}[e^{\eta\hat{a}^\dagger - \eta^* \hat{a}} \hat{\rho}].$$

- Normal and antinormal versions

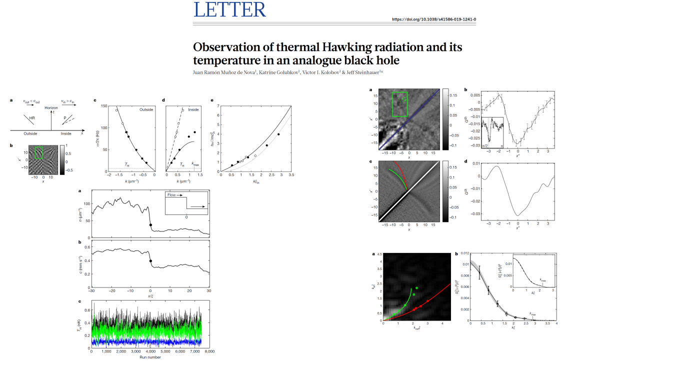
$$\begin{aligned} \chi_N(\eta) &\equiv \langle e^{\eta\hat{a}^\dagger} e^{-\eta^* \hat{a}} \rangle \,, \\ \chi_A(\eta) &\equiv \langle e^{-\eta^* \hat{a}} e^{\eta\hat{a}^\dagger} \rangle \,. \end{aligned}$$

- Fourier transforms $\rightarrow P, Q, W$ representations for computing normal, antinormal or symmetric expectation values:

$$\begin{aligned} P(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_N(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 P(\alpha) = \langle \hat{a}^\dagger \hat{a} \rangle \\ Q(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} e^{(\eta^*\alpha - \eta\alpha^*)} \chi_A(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 Q(\alpha) = \langle \hat{a} \hat{a}^\dagger \rangle \\ W(\alpha) &\equiv \int \frac{\mathrm{d}^2\eta}{\pi^2} \; e^{(\eta^*\alpha - \eta\alpha^*)} \chi(\eta) \rightarrow \int \mathrm{d}^2\alpha \; |\alpha|^2 W(\alpha) = \langle \frac{\hat{a}^\dagger \hat{a} + \hat{a} \hat{a}^\dagger}{2} \rangle \end{aligned}$$

2019 Hawking radiation+Hawking Temperature

- Improved experiment after years of exclusive dedication:
  - Reduced noise in magnetic field.
  - Recorrection of trap center every 200 runs.
  - Improved mechanical and optical stability
  - Redesigned optics for waterfall potential
  - Mirror for translating waterfall.


LETTER

https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

- Experiment extended (97000 repetitions over 124 days!) to observe life of black hole and birth of inner horizon.