Ergotropy of Quantum Many-body Scars

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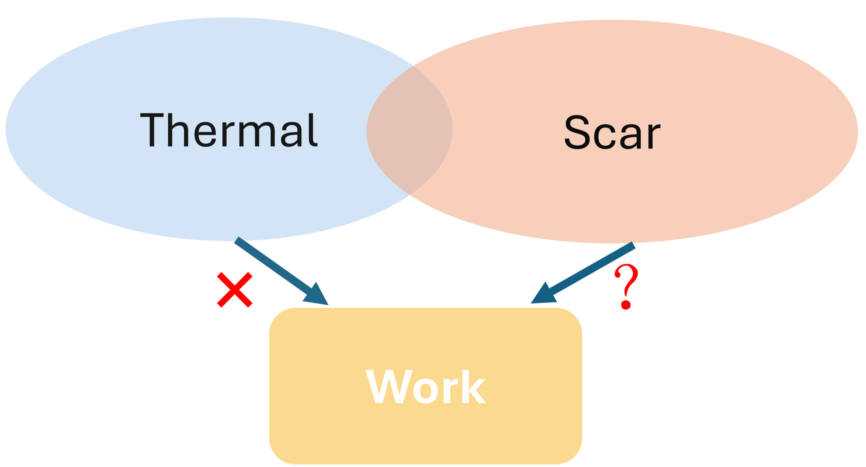
Introduction

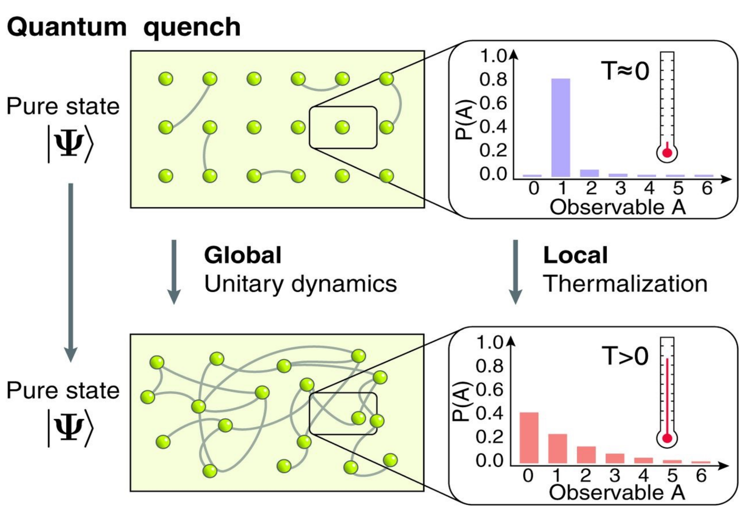
Thermalization and Violation of Thermalization



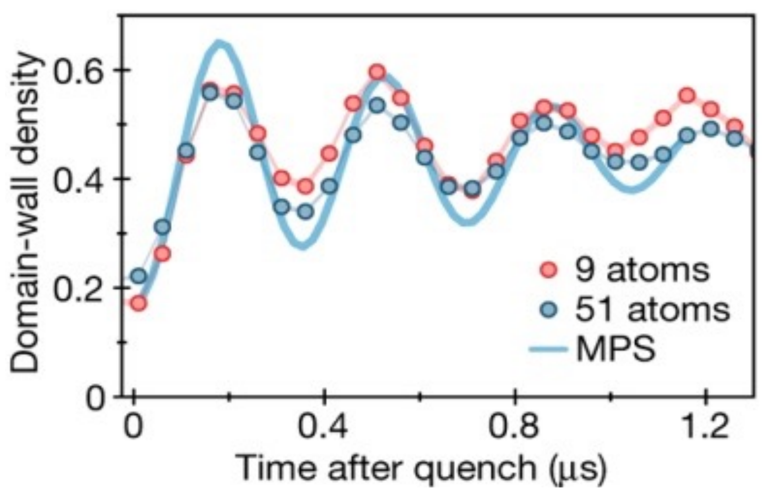
Thermalization: systems ρ **locally** behaves like canonical ensemble, $\rho_A \sim \frac{1}{Z} e^{-\beta H}$

ρ_{passive} no extractable energy

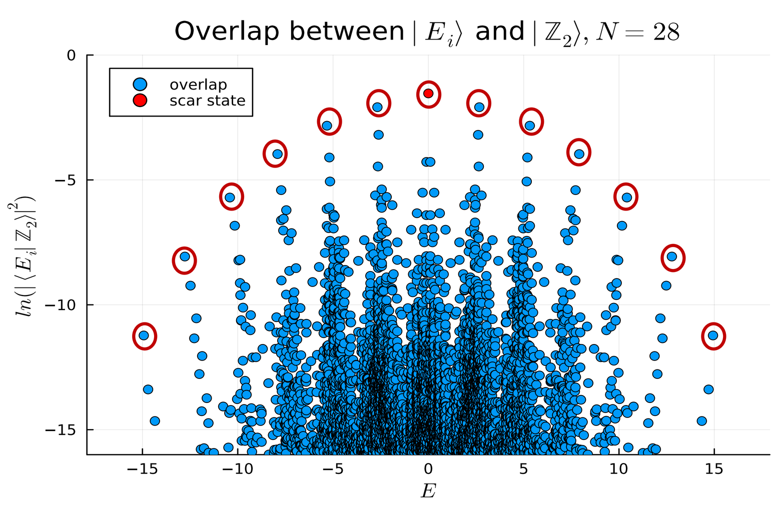
$\rho_{\text{thermal}} \subset \rho_{\text{passive}}$



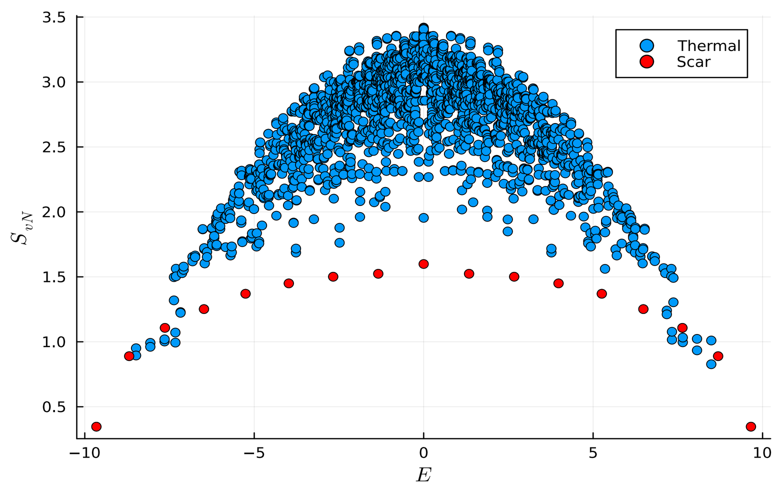
Thermalization: Thermal



Violation of Thermalization: Scar

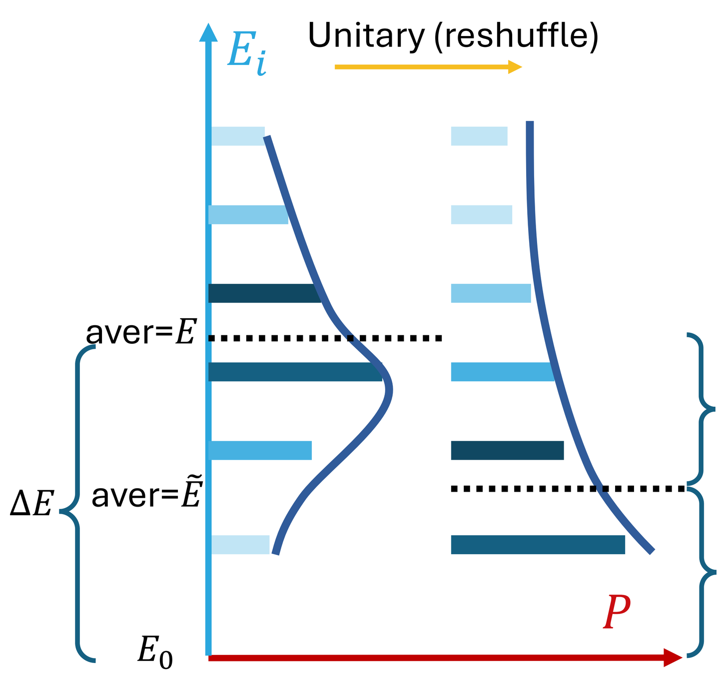


Scar: High energy density



Scar: Low entanglement

Ergotropy and Entanglement



E_i is the energy spectrum of H

P is the entanglement spectrum of state ρ

$E = \text{Tr}(\rho H) \qquad \tilde{E} = \text{Tr}(\rho_{\text{reshuffle}} H)$

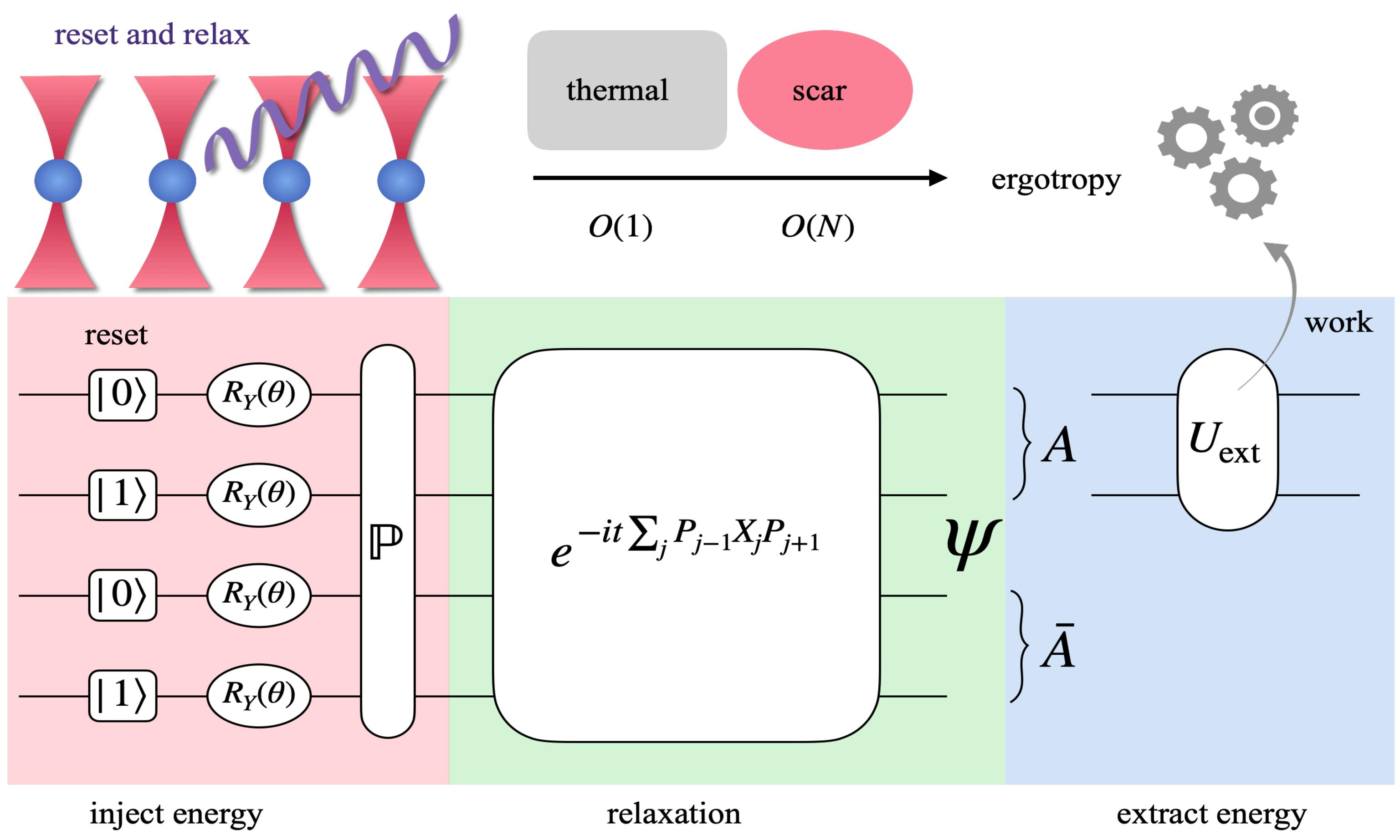
Maximal Extracble Work: Ergotropy $\boldsymbol{W = E - \tilde{E}}$

For non-interacting critical state, $Q \sim S_{vN}^2/L$
L is the system size

High entanglement ⟷ Low ergotropy

Q: How much energy can we extract from quantum many-body scar (QMBS)?

Rydberg atom arrays protocol



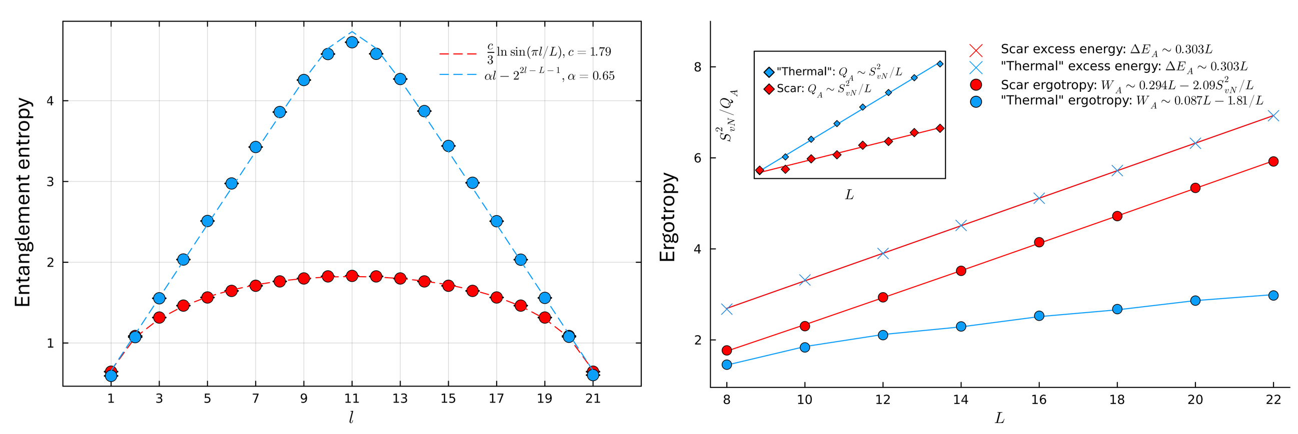
Divide into subsystem A and environment $\bar{A}$, $H = H_A \otimes H_{\bar{A}}$
consider subsystem mixed state ρ

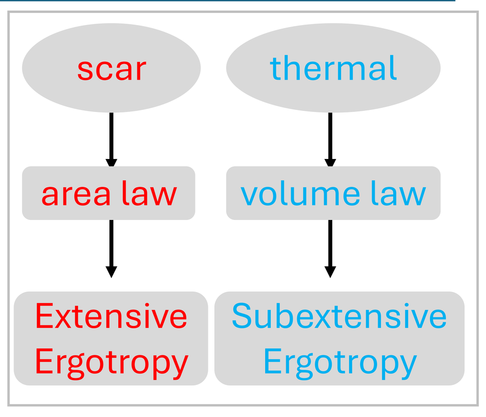
$$H = \sum_i P_{i-1}X_iP_{i+1}, P_i = (1 - Z_i)/2 \quad \text{PXP model}$$

Dims of Constrained Hilbert space $D_L \sim F_{L+2}$,
$\mathcal{H}_{\text{total}} = \mathcal{H}_{\text{FSA}} \oplus \mathcal{H}_{\text{thermal}}$, host scar and thermal
Diagonalize $\boldsymbol{P_{E=0}\, P_{\text{FSA}}\, P_{E=0}}$ to separate
degenerated $|\textbf{scar}\rangle$ and $|\textbf{thermal}\rangle$ in $\mathcal{H}_{E=0}$

Results

Eigenstates: Ergotropy crossover

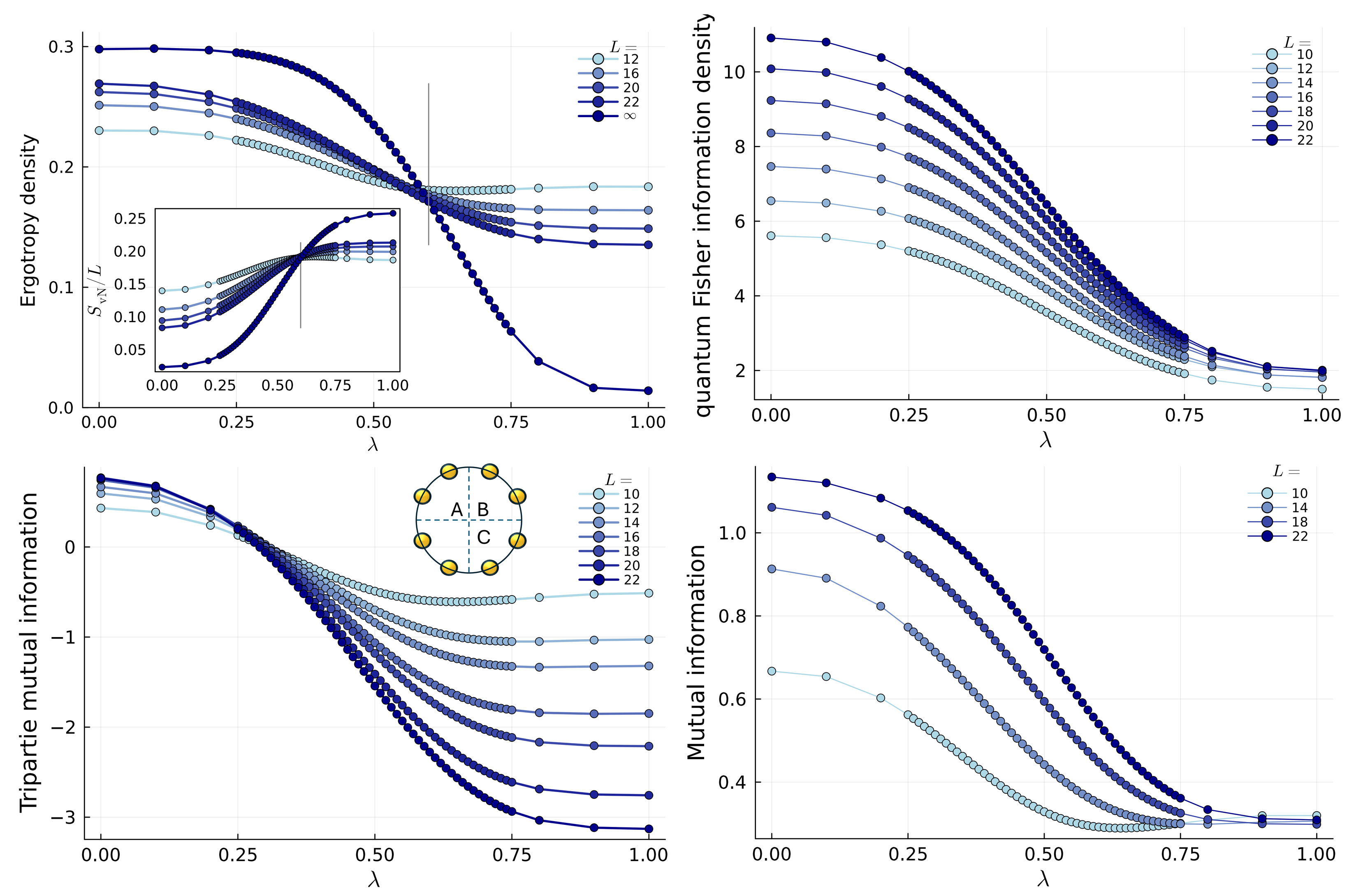




Relation between ergotropy and entanglement entropy

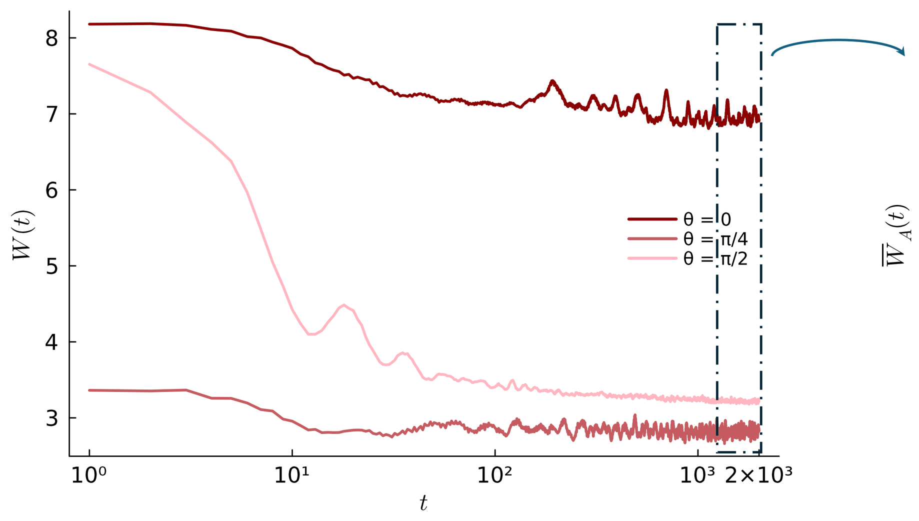
$|\psi(\lambda)\rangle = (1 - \lambda)|\text{scar}\rangle + \lambda|\text{thermal}_n\rangle$ Thermal ensemble

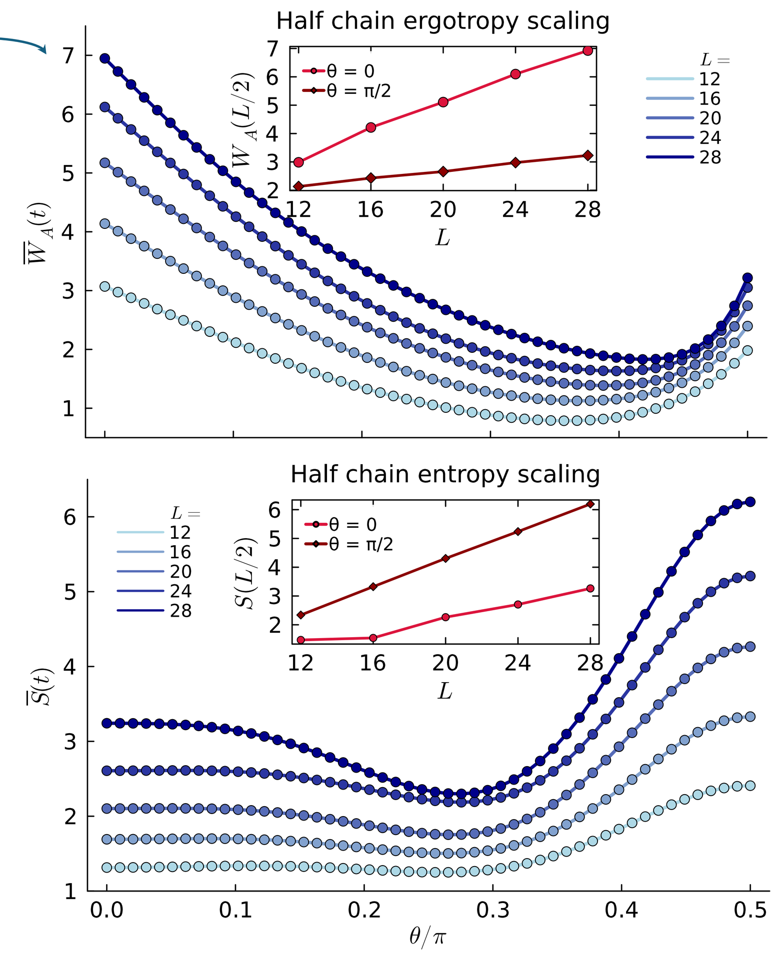
Global Rotation $R(\lambda)$



Ergotropy phase transition where **ergotropy** W_A from extensive to sub extensive, by tuning entanglement from area-law scar state to the volume-law thermal state, observed from **bipartie entanglement entropy density** S_{vN}/L, **tripartie mutual information** (TMI) $I_{ABC} = S_A + S_B + S_C - S_{AB} - S_{AC} - S_{BC} + S_{ABC}$, multipartie entanglement the **quantum Fisher information** (QFI) density $f_Q(|E_i\rangle)$ with repsect to the antiferromagnetic order operator $\sum_i (-1)^{i+1} Z_i$ and **mutual information** (MI). Not perfect seperated thermal has non-zero ergotropy.

Quantum quench dynamics ergotropy





$|\psi(\theta, t)\rangle = U(t)P\Pi_i R_i^y(\theta)|\mathbb{Z}_2\rangle$

$|\psi(0)\rangle = |\text{scar}\rangle, |\psi(\pi/2)\rangle = |\text{thermal}\rangle$

Local Rotation $R^y(\theta)$

Global projector P: Rydberg blockade

Steady state

Summary

Bound energy $Q_A \sim \frac{S_A^2}{N}$

Scar $\quad S \sim O(\log N)$, Ergotropy $W_A \sim O(N)$

Thermal $\quad S \sim O(N)$, Ergotropy $W_A \sim O(1/N)$

Ergotropy crossover exists for states with finite overlap with scar states
Ergotropy crossover can be observed at experiment accessible states.
Outlook: unitary + measurement -> Maxwell's demon

Reference

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