An Introduction to Radio Interferometry

線上英語課程(https://github.com/baobabyoo/interferometryEMI)
(https://baobabyoo.github.io/pages/lectures.html#radio-interferometry)
(https://baobabyoo.github.io/pages/students_topics/UsingTelescope.html)

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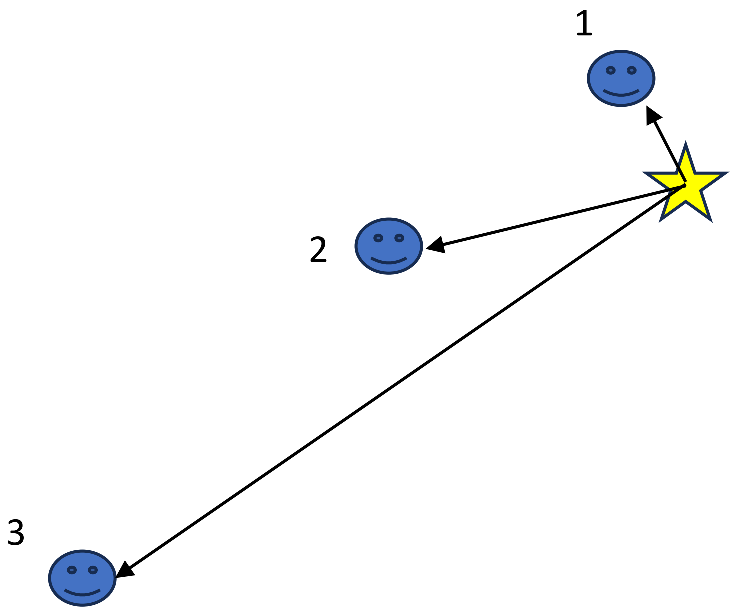
Emergence of electric (and magnetic field) in the observational devices.



Classically, light can be described as electromagnetic waves (EM waves):

(i) linear perturbations of electric and magnetic fields

(ii) propagation obeys Maxwell's equations

Electric field

Electric charge density

Gauss law ⇨ $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$

Magnetic field

Gauss law ⇨ $\nabla \cdot \vec{B} = 0$

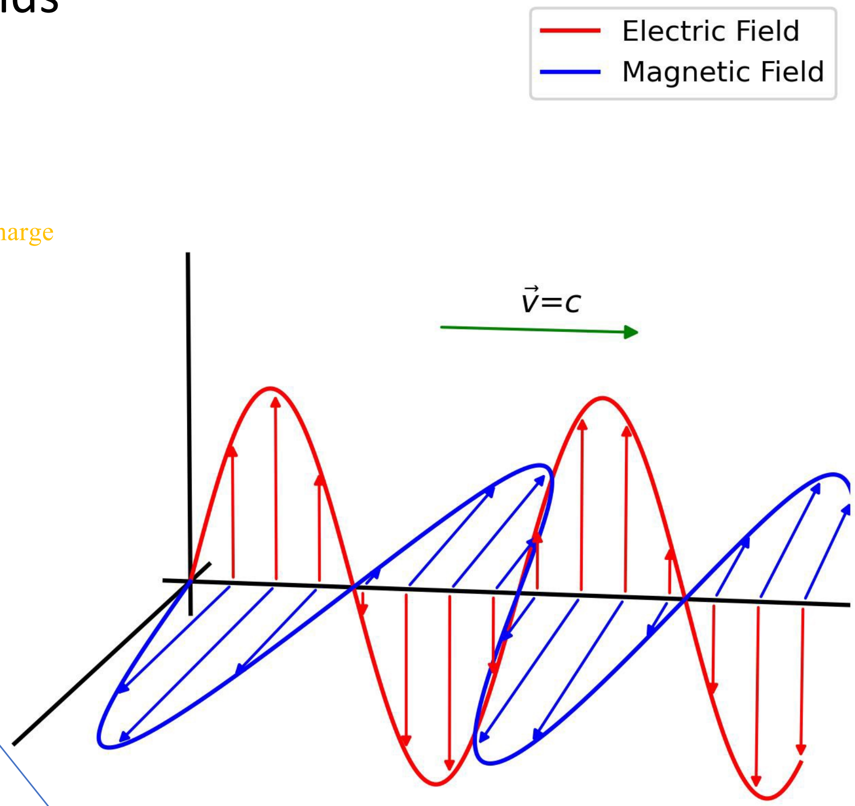
Faraday's law ⇨ $\nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$

Ampere's law ⇨ $\nabla \times \vec{B} - \mu_0 \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} \right) = \mu_0 \vec{J}$

(with Maxwell's correction)

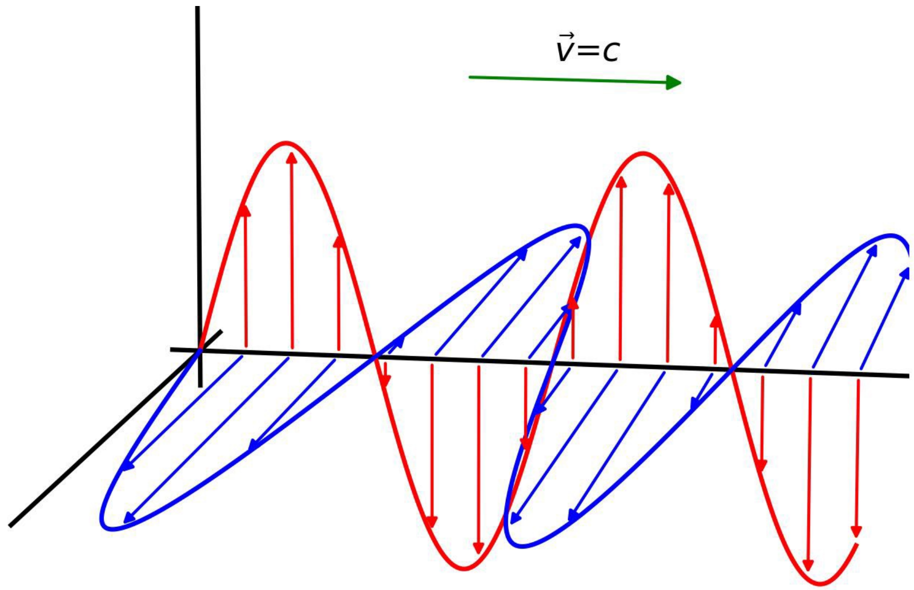
Vacuum permeability and permittivity

Current density



Monochromatic plane wave

$$
\begin{cases}
E = E_m \sin(kx - \omega t + \phi) \\
B = B_m \sin(kx - \omega t + \phi)
\end{cases}
$$

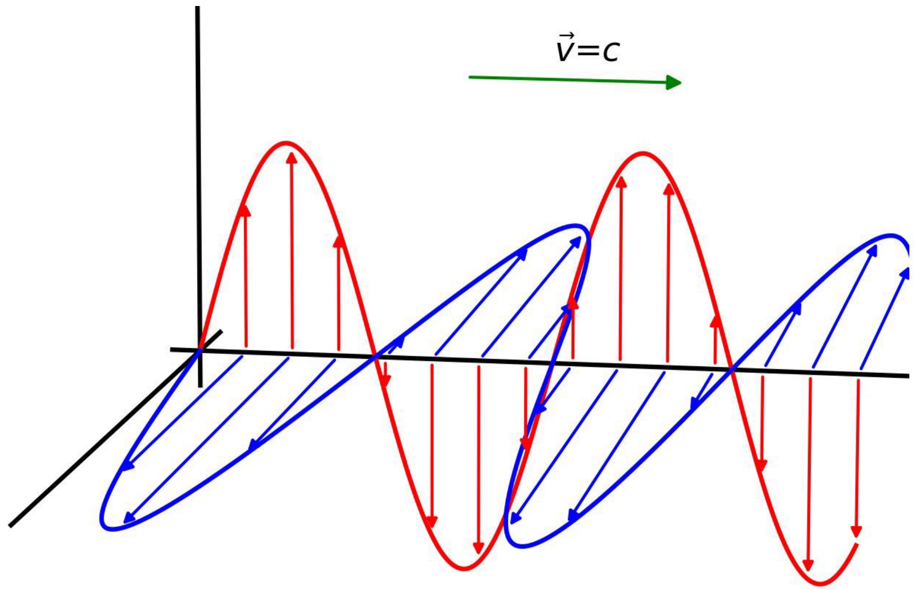


$$\omega = 2\pi\nu = \frac{2\pi}{T}$$

Monochromatic plane wave

$$\begin{cases} E = E_m \sin(kx - \omega t + \phi) \\ B = B_m \sin(kx - \omega t + \phi) \end{cases}$$

$$k = \frac{2\pi}{\lambda}$$



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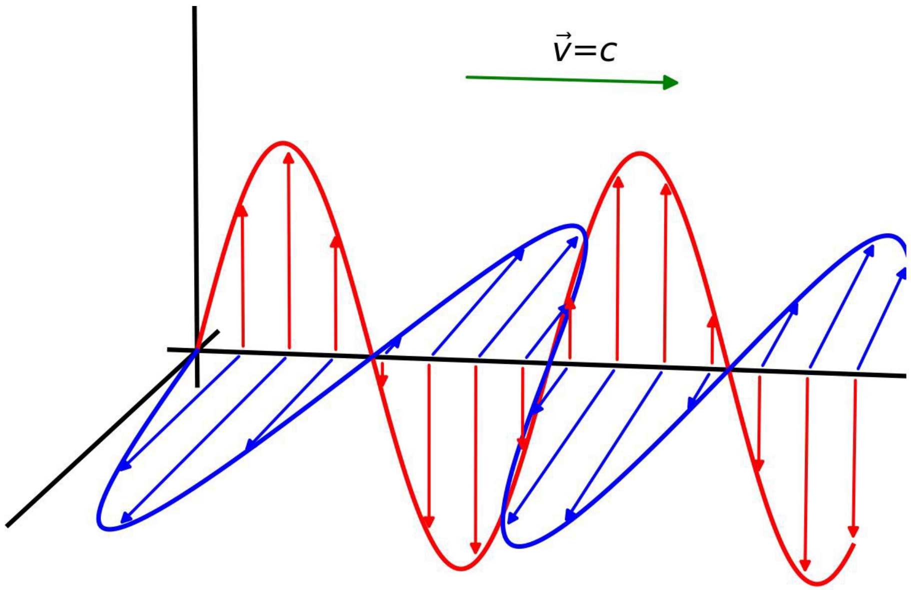
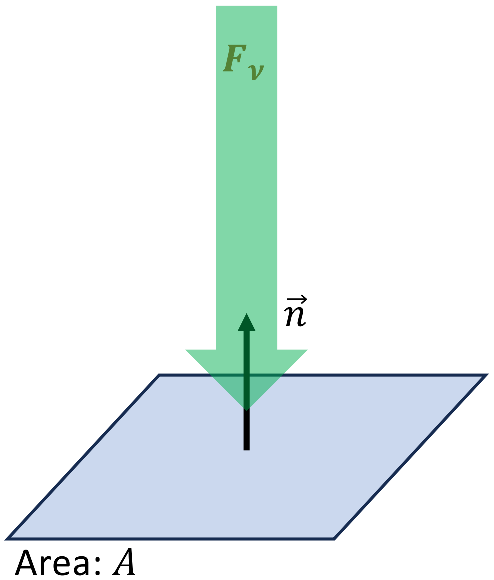
(time-averaged) Flux density:

$$F = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}|E_m|^2$$



Flux density as a vector field

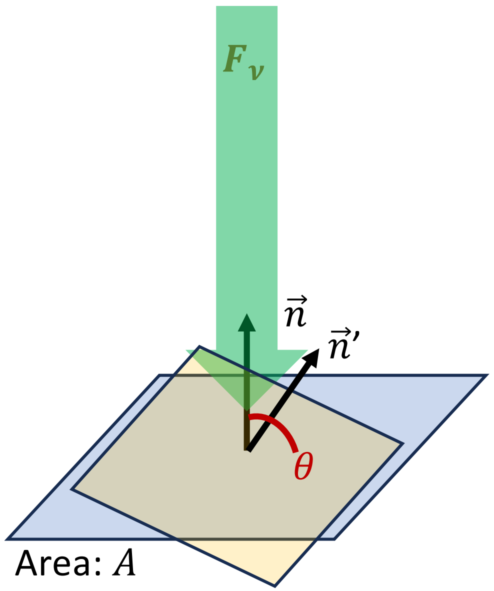


Normal incidence

Energy passing a CCD pixel that has area A in a unit of time

$$F_\nu A$$

Flux density as a vector field



Normal incidence

Energy passing a CCD pixel that has area A in a unit of time

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Inclined CCD pixel

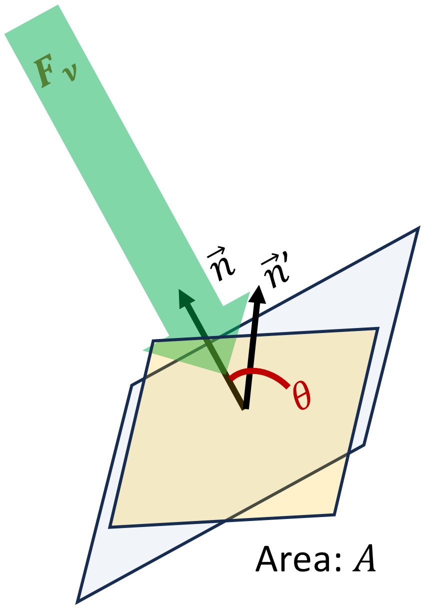
Energy passing a CCD pixel that has area A in a unit of time

$$F_\nu A\left(\vec{n}\cdot\overrightarrow{n'}\right) = F_\nu A\cos\theta$$

$$\vec{n}\cdot\overrightarrow{n'} = \cos\theta$$

Flux density as a vector field

Inclined incident photon streams



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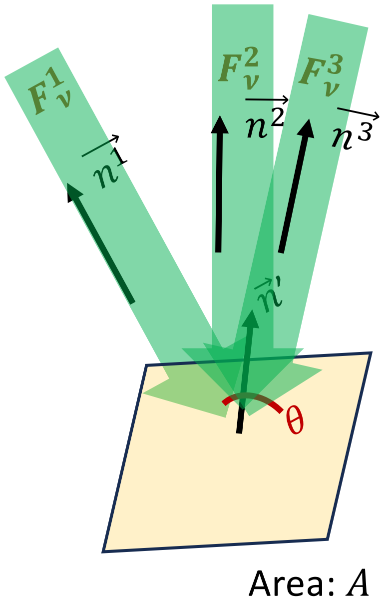
Inclined, discrete incident photon streams

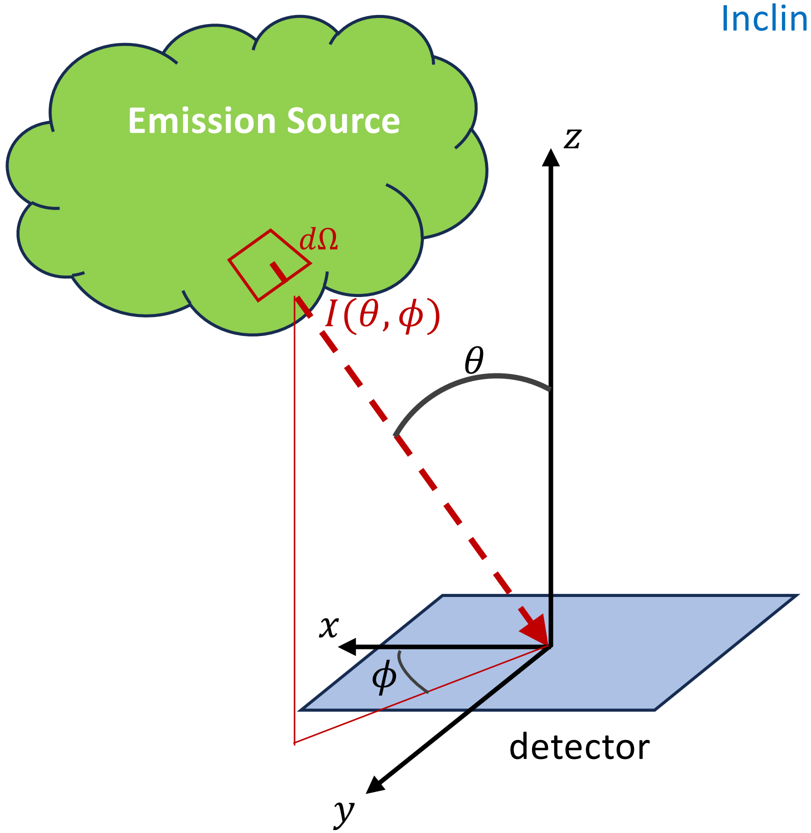


Energy passing a CCD pixel that has area A in a unit of time

$$\sum_i F_\nu^i A\left(\overrightarrow{n^i}\cdot\overrightarrow{n'}\right) = \sum_i F_\nu^i\, A\cos\theta^i$$

$$\overrightarrow{n^i}\cdot\overrightarrow{n'} = \cos\theta^i$$

Continuous incident light



Inclined, discrete incident photon streams

Energy passing a CCD pixel that has area A in a unit of time

$$\sum_i F_\nu^i A \left(\overrightarrow{n^i} \cdot \overrightarrow{n'}\right) = \sum_i F_\nu^i A \cos\theta^i$$

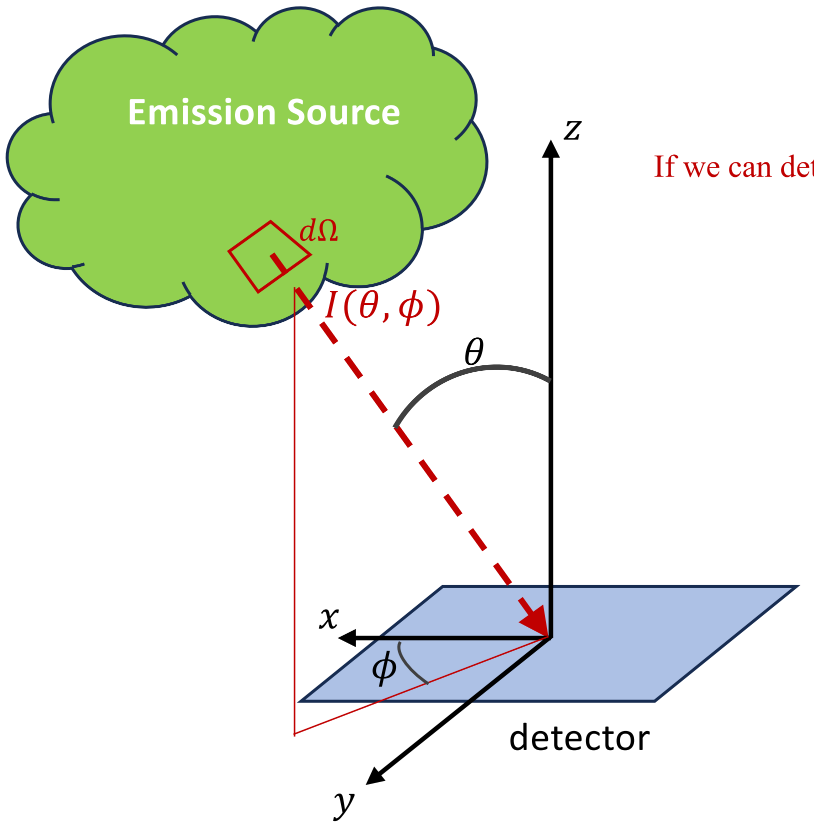
infinitesimal solid angle: $d\Omega \equiv \sin\theta \, d\theta d\phi$

Intensity $I(\theta, \phi)$: the amount of energy through a unit of area A in a unit of time t from a unit solid angle $d\Omega$ around the direction (θ, ϕ) [i.e., flux density per unit solid angle]

SI Unit: Joul s^{-1} m^{-2} Hz^{-1} Sr^{-1}

Flux density normal to our CCD pixel:

$$\int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$



Flux density (incoming energy per unit time, frequency, and area) received by a CCD pixel:

$$F_\nu = \frac{1}{2} \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

If we can detect one of the two polarizations

Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.

Flux density (流量) F_ν : 單位時間、單位頻率、單位探測器面積通過的能量。

SI Unit: Joul s^{-1} m^{-2} Hz^{-1}=W m^{-2} Hz^{-1}

天文常用單位: 1 Jy = 10^{-26} Joul s^{-1} m^{-2} Hz^{-1} = 10^{-26} W m^{-2} Hz^{-1}

F_ν正比於電磁波電場振幅平方$|E_m|^2$

單位時間、單位頻率、單位探測器面積通過的光子數等於 $F_\nu/h\nu$

Intensity I_ν : 單位時間、單位頻率、單位天體張角、單位探測器面積通過的能量。

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天文常用單位: 1 Jy/beam

單位時間、單位頻率內通過 單位探測器面積的能量: $\int I_\nu(\theta, \phi) \cos\theta \, d\Omega$

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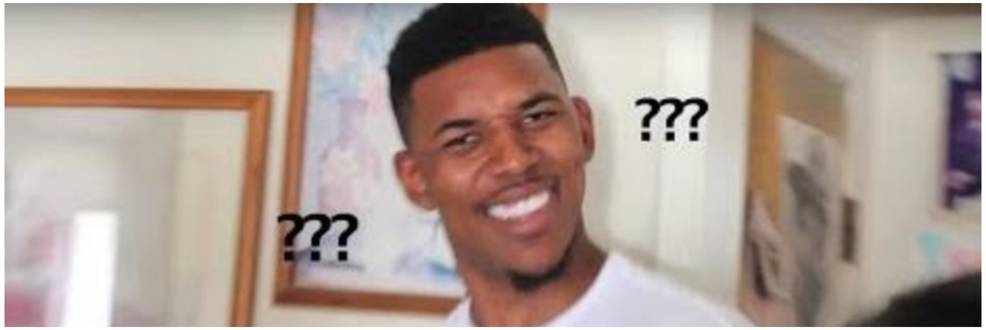
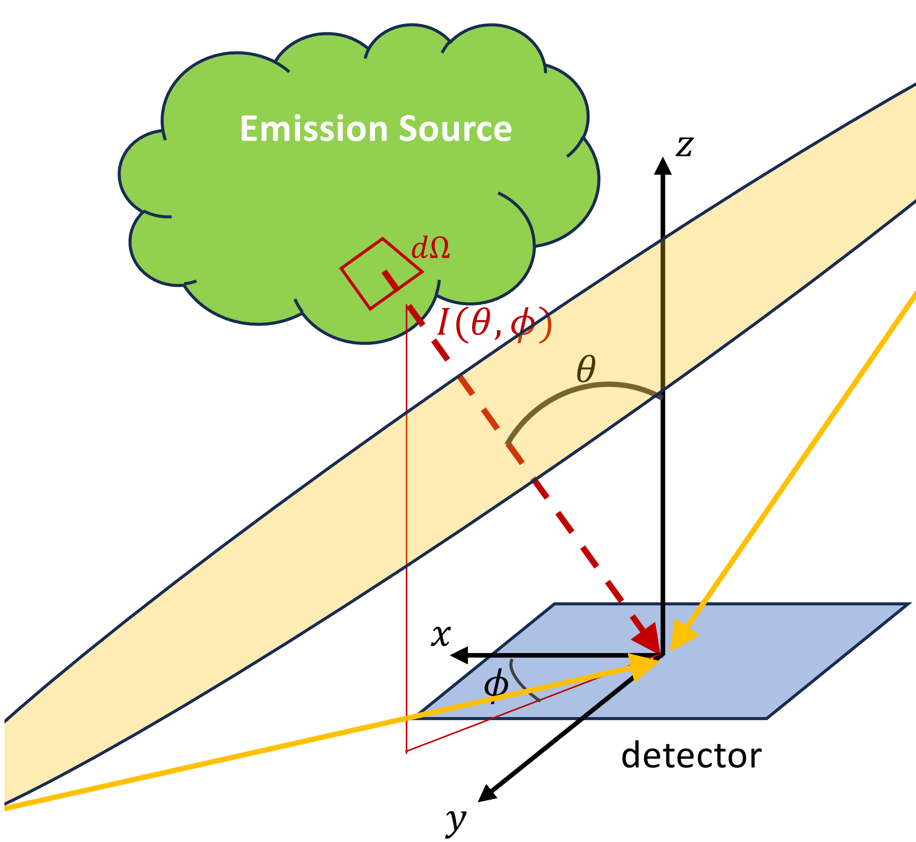


單位時間、單位頻率內通過 單位探測器面積的能量: $\int I_\nu(\theta,\phi)\cos\theta\, d\Omega$

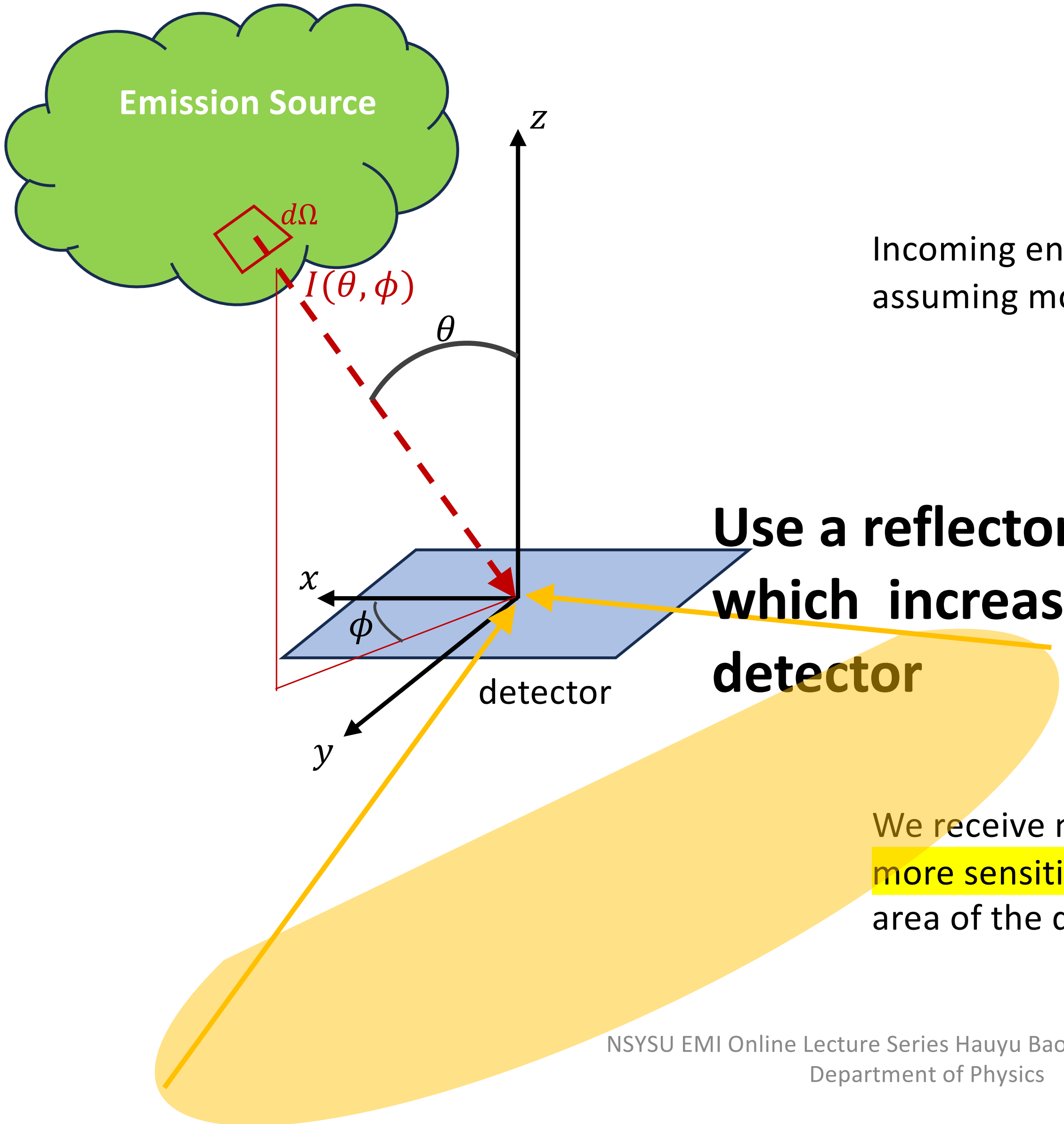


Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A_e \int I_\nu(\theta, \phi) \cos\theta \, d\Omega$$

Use a lense to collect light from a larger area, which increases the effective surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.



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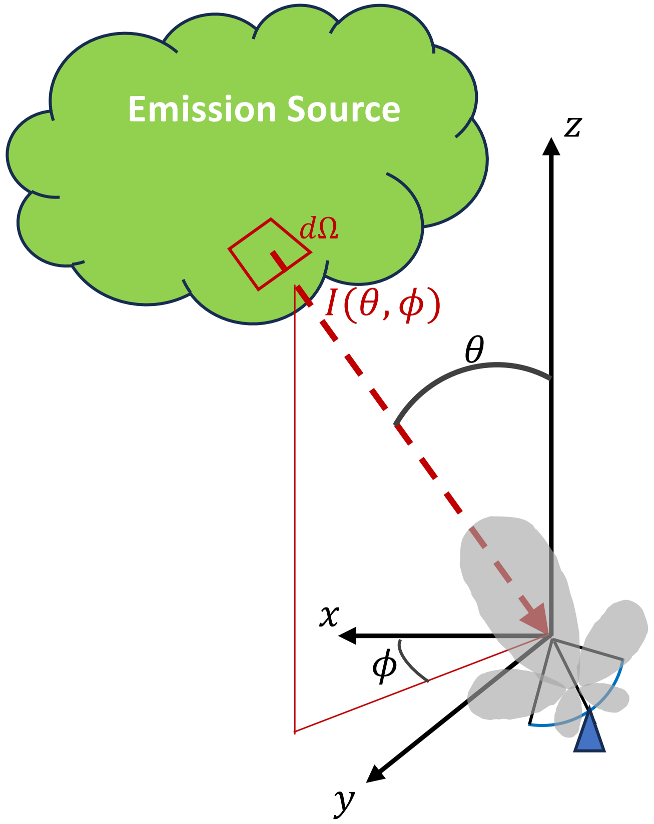
Use a reflector to collect light from a larger area, which increases the effective surface area of the detector

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = \frac{1}{2} A_e \int I_\nu(\theta, \phi) \cos\theta \underbrace{P(\theta, \phi)}\, d\Omega$$

Response function of the observing device.
Normalization: peak = 1.0 (i.e., 100% of incoming intensity can be detected.)

The optics determines the response function. With a telescope, we:

(1) Receive light from smaller solid angle(s) (usually, provides better angular resolution)

(2) Have a bigger collecting area

We receive more power in a unit of time (i.e., our device is more sensitive to weak signals) when the effective surface area of the detector is larger.

Example: 2D Gaussian beam

with Full width at half maximum (FWHM) θ_a and θ_b in the major and minor axes

(the relation between Gaussian standard deviation σ ang FWHM θ

$\theta = 2\sqrt{2ln2}\sigma \sim 2.35\sigma$)

$$P = e^{\left(\frac{-\xi^2}{2\sigma_a^2}+\frac{-\eta^2}{2\sigma_b^2}\right)} = e^{\left[4ln2\left(\frac{-\xi^2}{(2\sqrt{ln2}\sigma_a)^2}+\frac{-\eta^2}{(2\sqrt{ln2}\sigma_b)^2}\right)\right]} = e^{\left[4ln2\left(\frac{-\xi^2}{(\theta_a)^2}+\frac{-\eta^2}{(\theta_b)^2}\right)\right]}$$

$$\Omega_B = \int e^{\left[4ln2\left(\frac{-\xi^2}{(\theta_a)^2}+\frac{-\eta^2}{(\theta_b)^2}\right)\right]} d\xi d\eta = \frac{\pi\theta_a\theta_b}{4ln2}$$

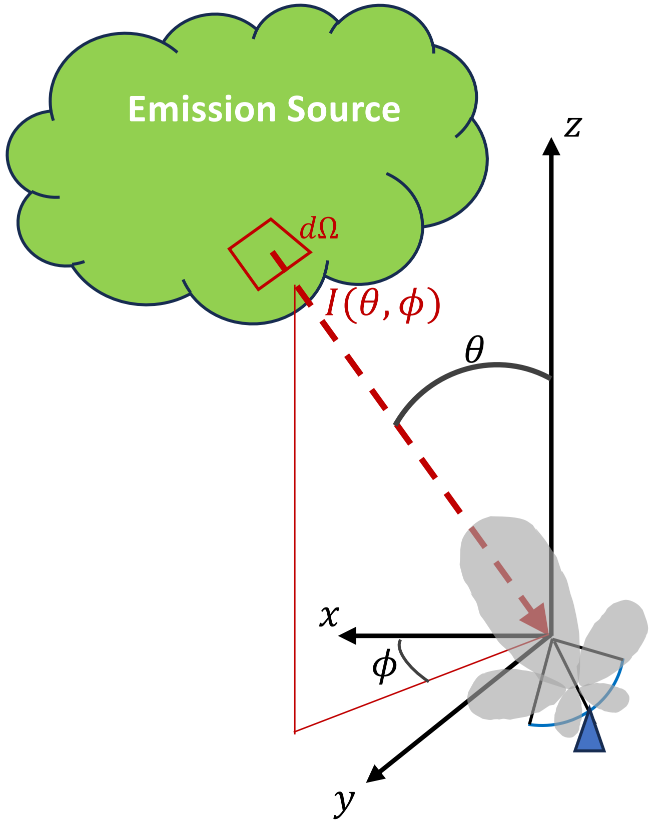
$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

Beam solid angle:

$$\Omega_B = \int_0^{4\pi} P(\theta,\phi) d\Omega = \int_0^{4\pi} P(\theta,\phi) \sin\theta \, d\theta d\phi$$

Primary beam solid angle:

$$\Omega_{PB} = \int_{Primary\ beam} P(\theta,\phi) \sin\theta \, d\theta d\phi$$



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Response function of the observing device.
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$$\frac{P_\nu}{A_e} = \int I_\nu(\theta,\phi)\cos\theta\, P(\theta,\phi)\, d\Omega \underset{\theta\to 0 \Longrightarrow \cos\theta\to 1}{\sim} \int I_\nu(\theta,\phi)\, P(\theta,\phi)\, d\Omega$$

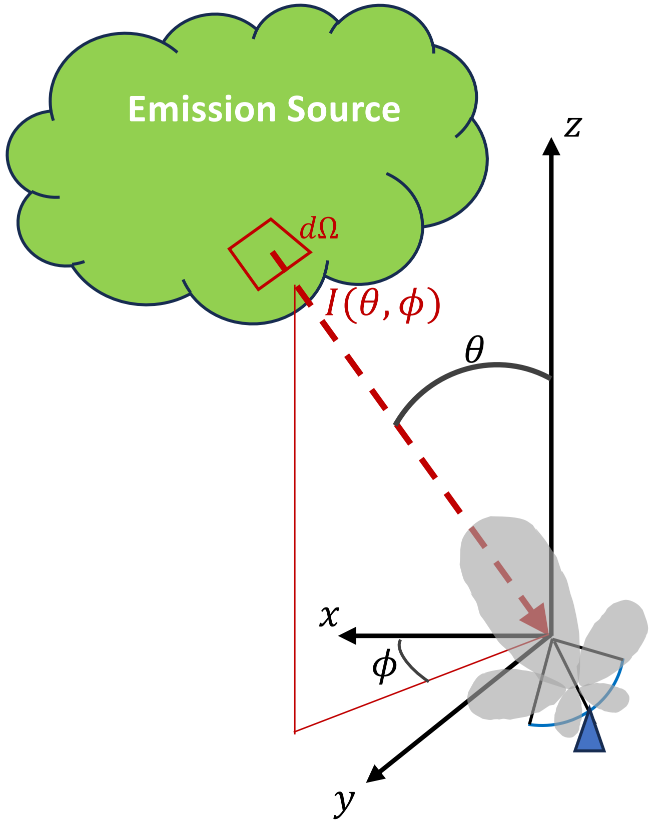
If $I_\nu(\theta,\phi)$ is approximately constant over the beam

Solid angle in Sr unit

$$\underbrace{\frac{P_\nu}{A_e}} = \sim I_\nu \int P(\theta,\phi)\, d\Omega = \underbrace{I_\nu}\overbrace{\Omega_B} = 1\, Jy$$

Flux density

Intensity in Jy Sr^{-1} unit



Incoming energy per unit time and frequency (i.e., power if assuming monochromatic light) received by a CCD pixel:

$$P_\nu = A_e \int I_\nu(\theta,\phi)\cos\theta\, \underbrace{P(\theta,\phi)}\, d\Omega \equiv k T_A$$

Boltzmann constant

Antenna temperature

Response function of the observing device.
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$\theta \to 0 \Longrightarrow \cos\theta \to 1$

If $I_\nu(\theta,\phi)$ is approximately constant over the beam

$$\underbrace{\frac{P_\nu}{A_e}} = \sim I_\nu \int P(\theta,\phi)\, d\Omega = \underbrace{I_\nu}\overbrace{\Omega_B} = \underbrace{I_\nu^{Jy/beam}}\times\overbrace{1} = 1\ Jy$$

Solid angle in Sr unit

Solid angle in beam unit

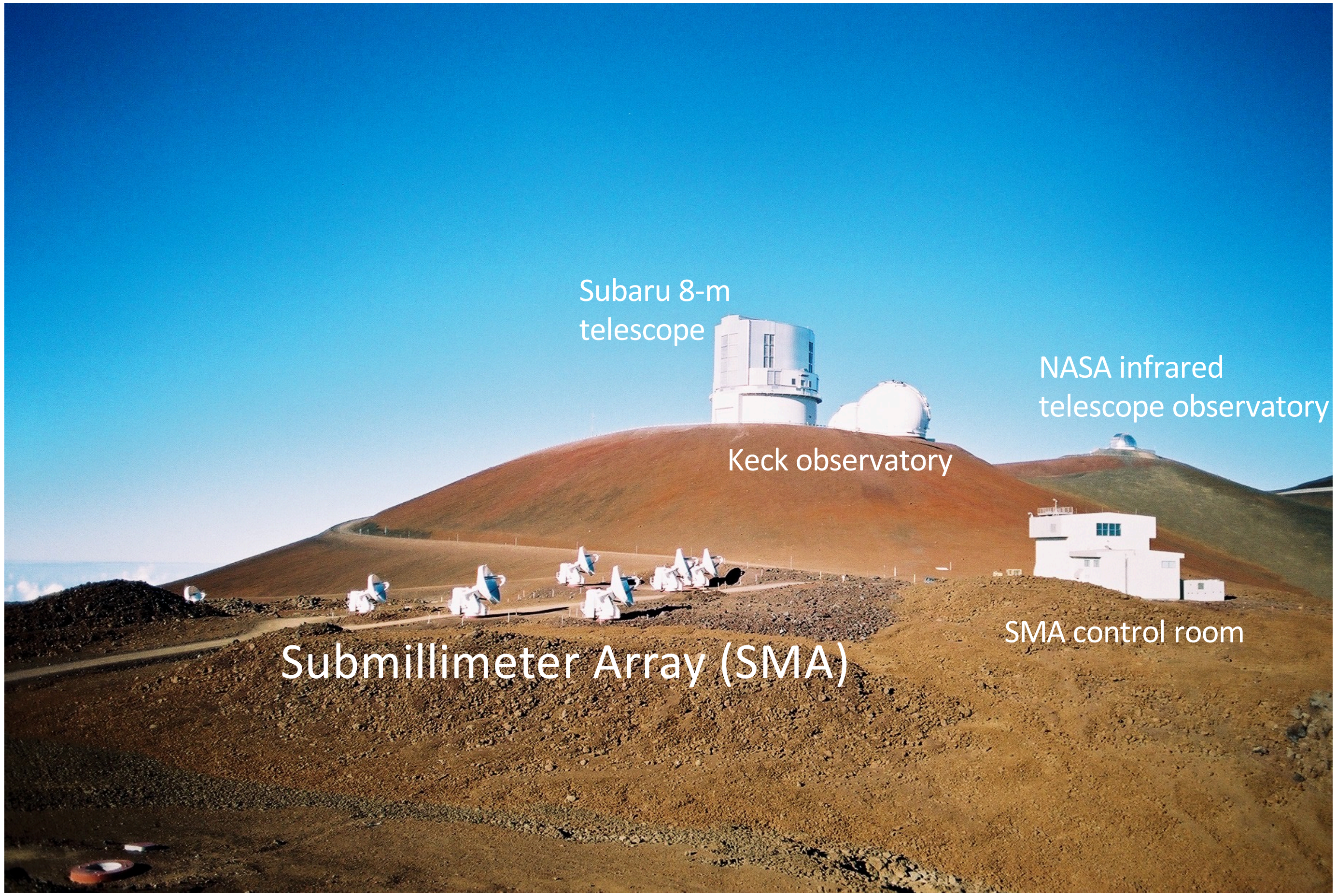
Flux density

Intensity in Jy Sr^{-1} unit

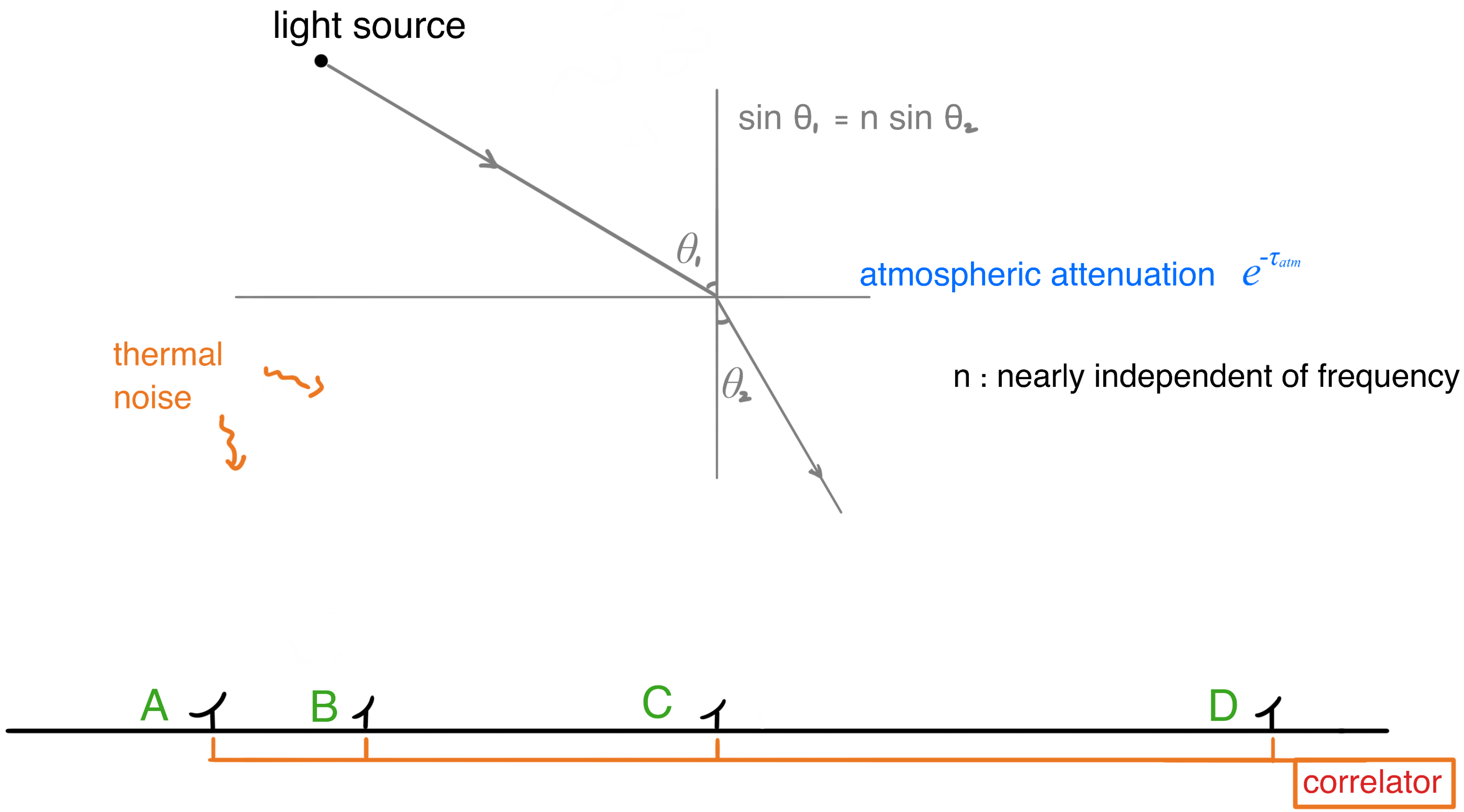
changing unit

Intensity in Jy beam^{-1} unit

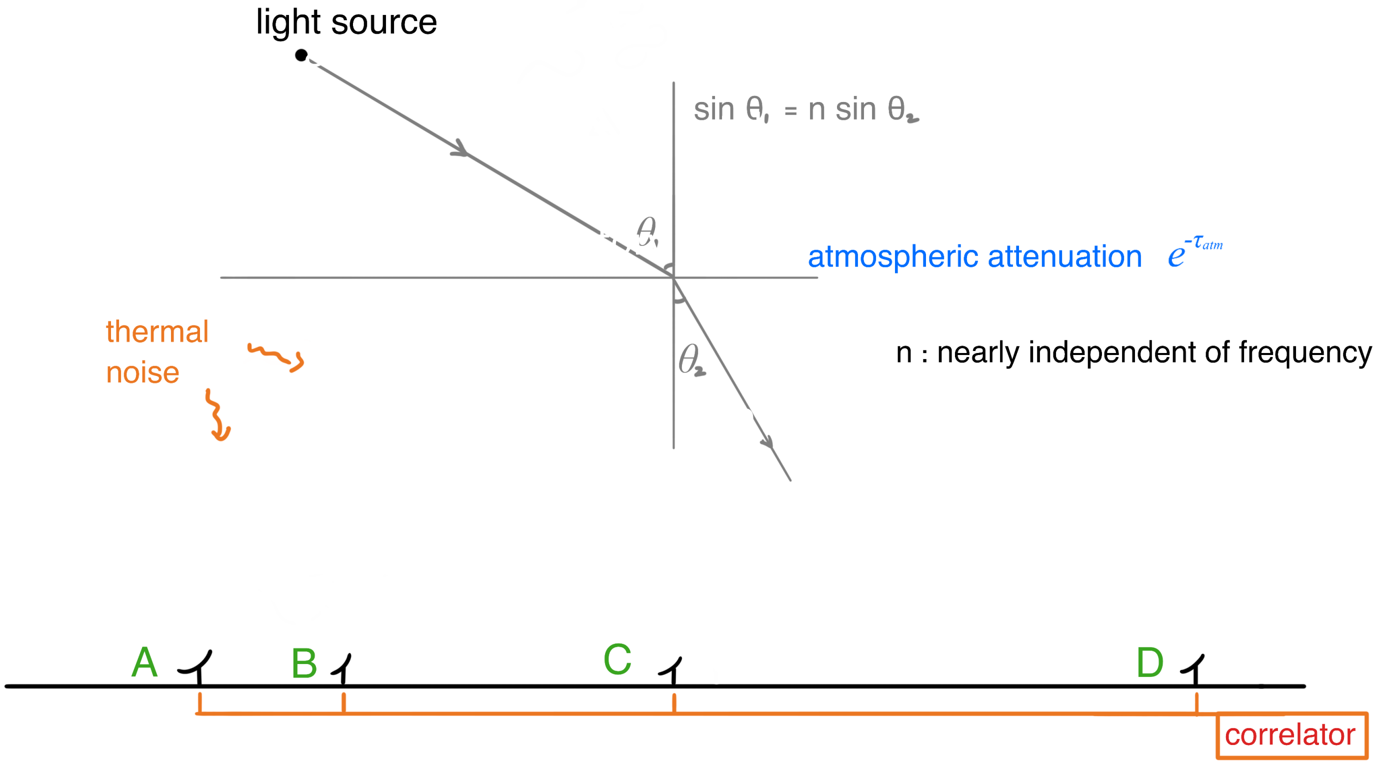
How a radio or (sub)millimeter interferometric observatory look like



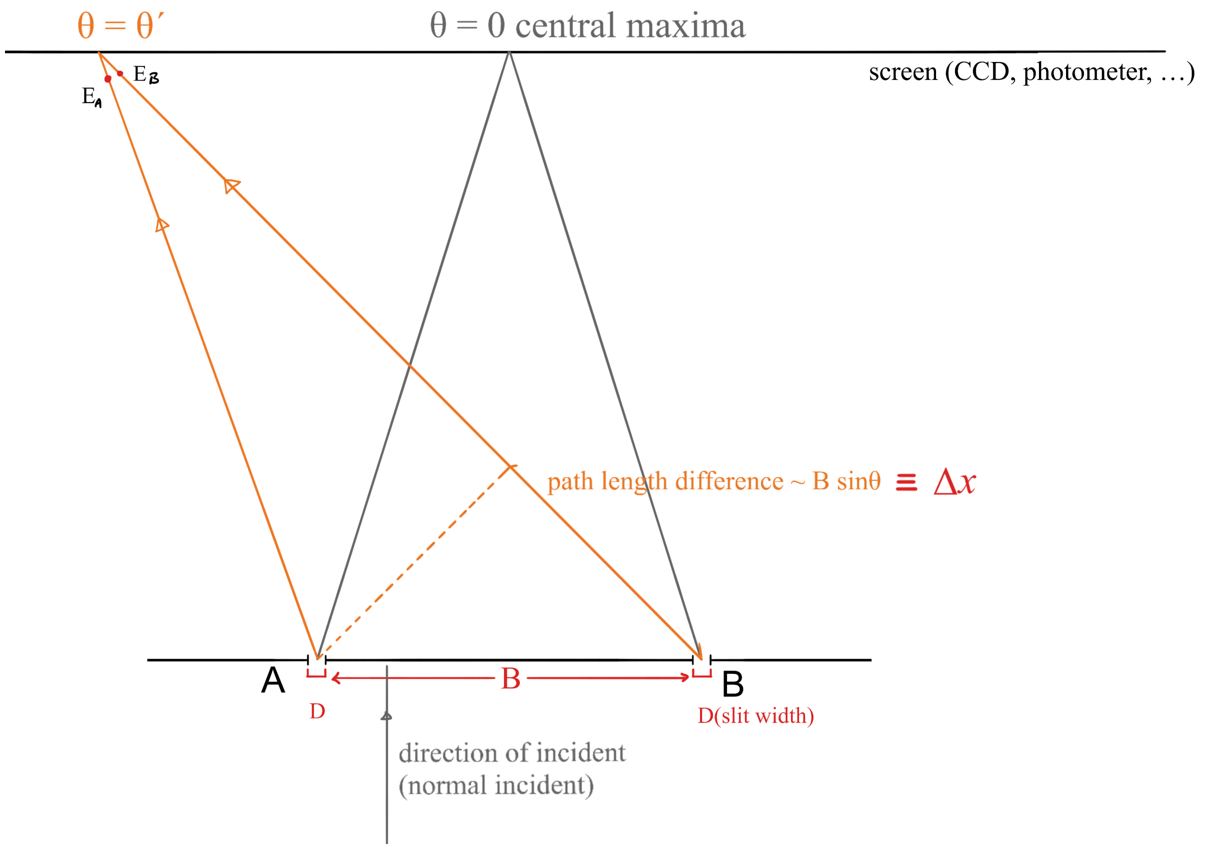
How a radio or (sub)millimeter interferometric observatory look like



干涉儀直接測量的物理量為Fourier transformed的intensity distribution



測量及成像之數學原理與光柵雷同。從最簡單情形，一維雙狹縫之成像來介紹。



1. **EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$** $\quad k = \dfrac{2\pi}{\lambda}$
2. **Energy flux density $\propto E_0^2$**

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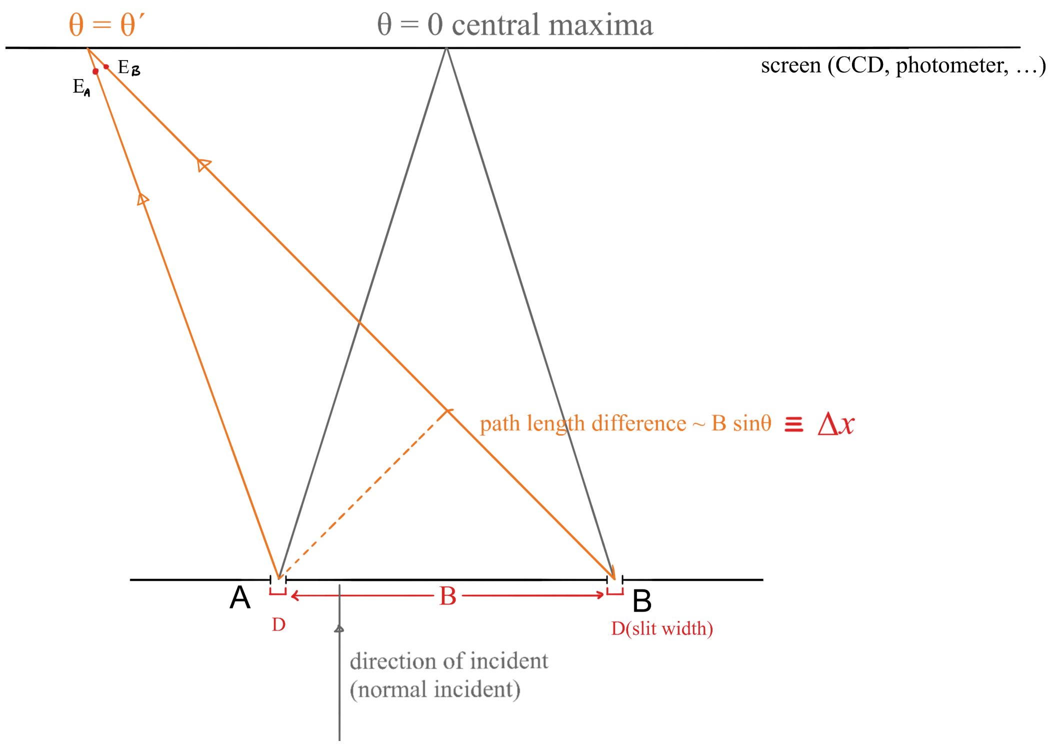

Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$\propto [E_A + E_B]^2$$
$$= \tilde{P}(\theta)[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2$$
$$= \tilde{P}(\theta)\{[\cos(kx - \omega t + \phi_0)]^2 + [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 + 2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)\}$$

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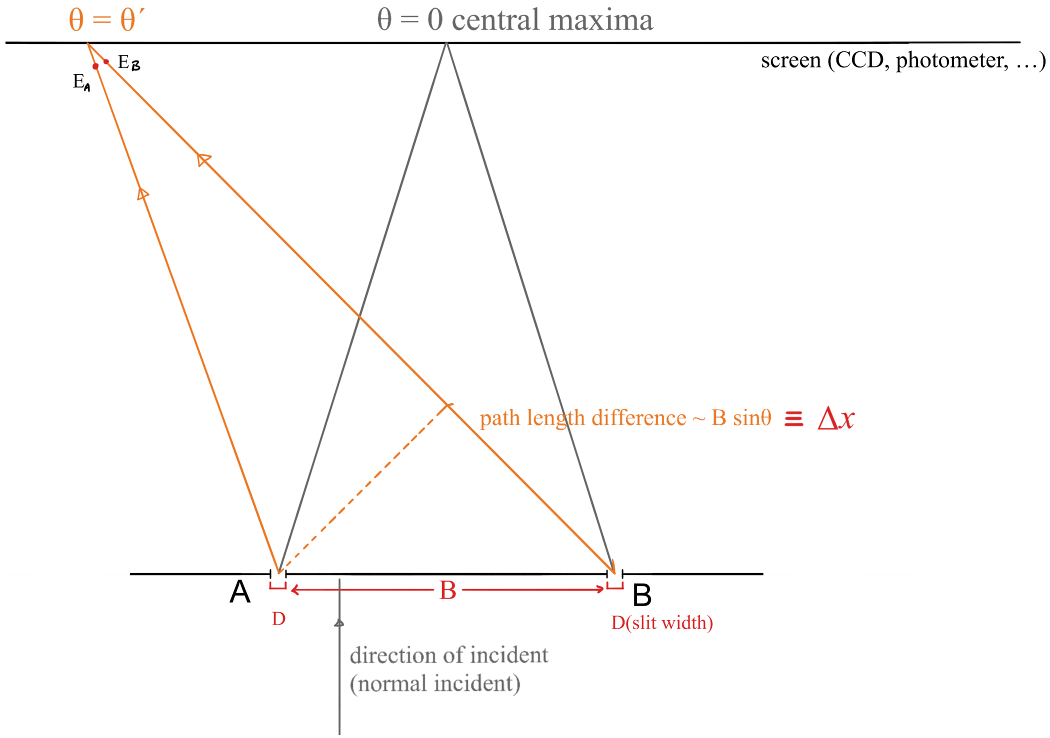

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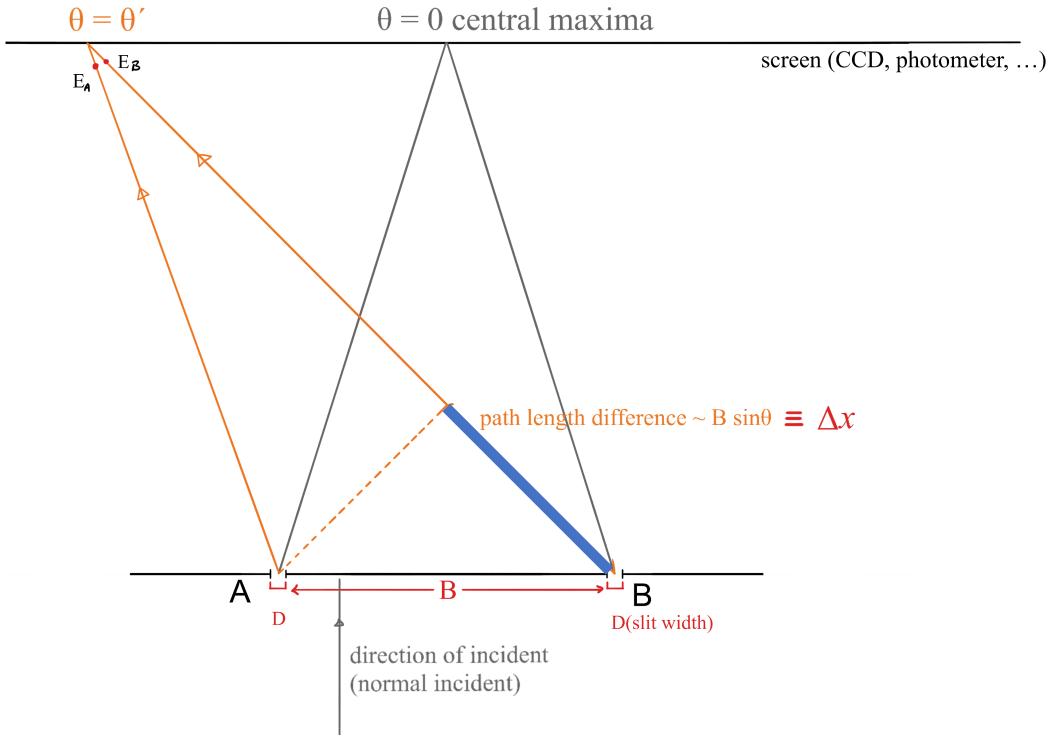

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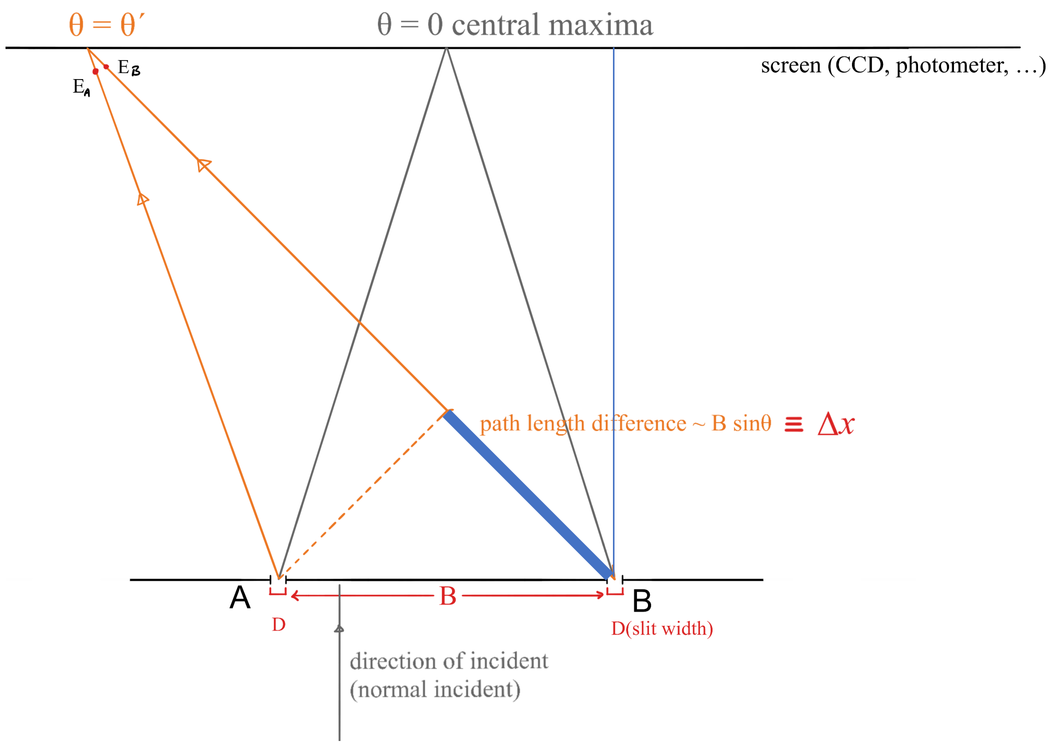

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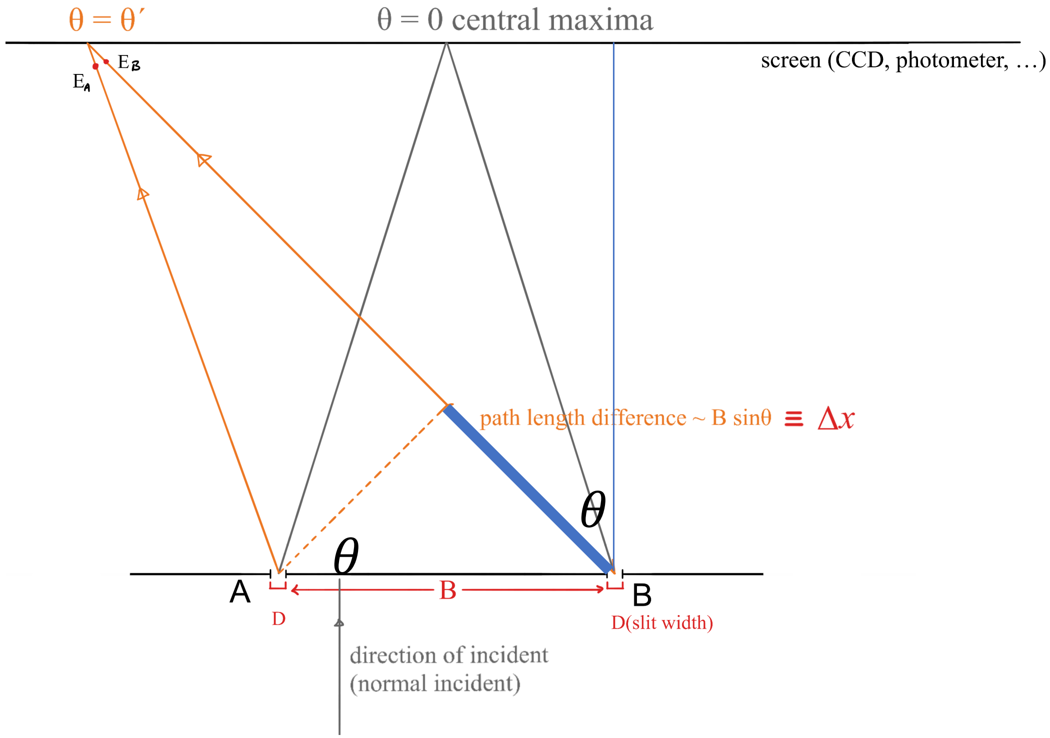

Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$
\begin{aligned}
&\propto [E_A + E_B]^2 \\
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&= \tilde{P}(\theta)\{[\cos(kx - \omega t + \phi_0)]^2 \\
&\quad + [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\
&\quad + 2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)\}
\end{aligned}
$$

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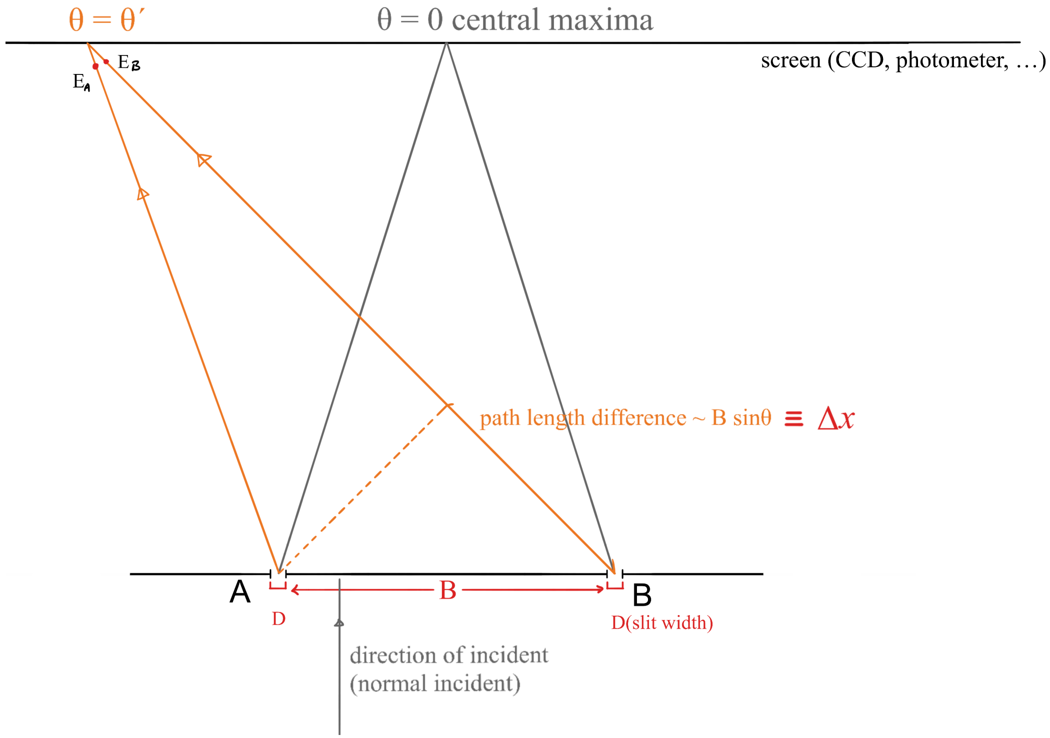

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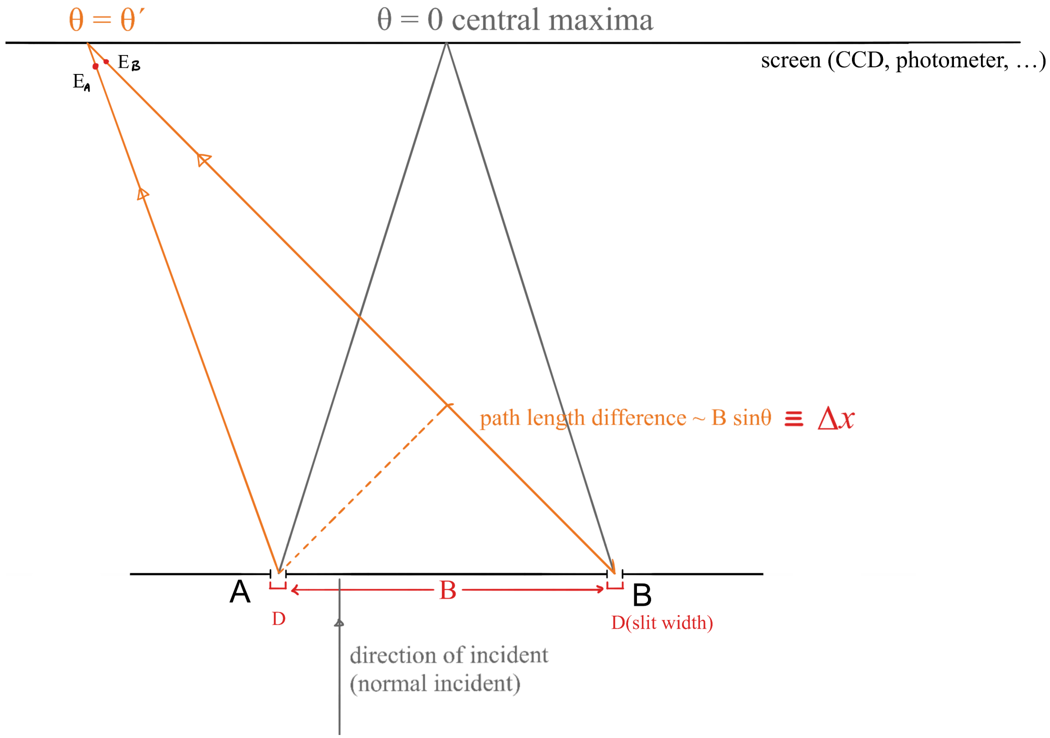

Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$
\begin{aligned}
&\propto [E_A + E_B]^2 \\
&= \cancel{\tilde{P}(\theta)}[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\
&= \cancel{\tilde{P}(\theta)}\{[\cos(kx - \omega t + \phi_0)]^2 \\
&\quad + [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \\
&\quad + 2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)\}
\end{aligned}
$$

$$
<\cdot> \equiv \lim_{\Delta t \to \infty} \frac{1}{\Delta t} \int_{-\frac{1}{2}\Delta t}^{\frac{1}{2}\Delta t} \cdot \; dt
$$

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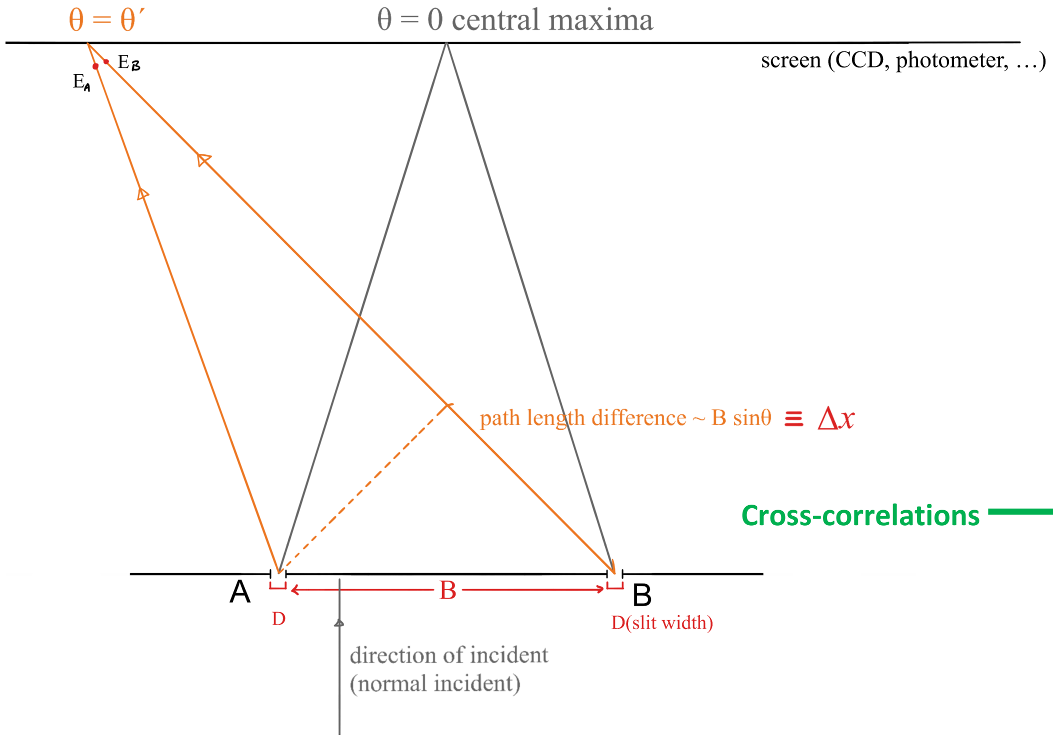

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Auto-correlations

Cross-correlations

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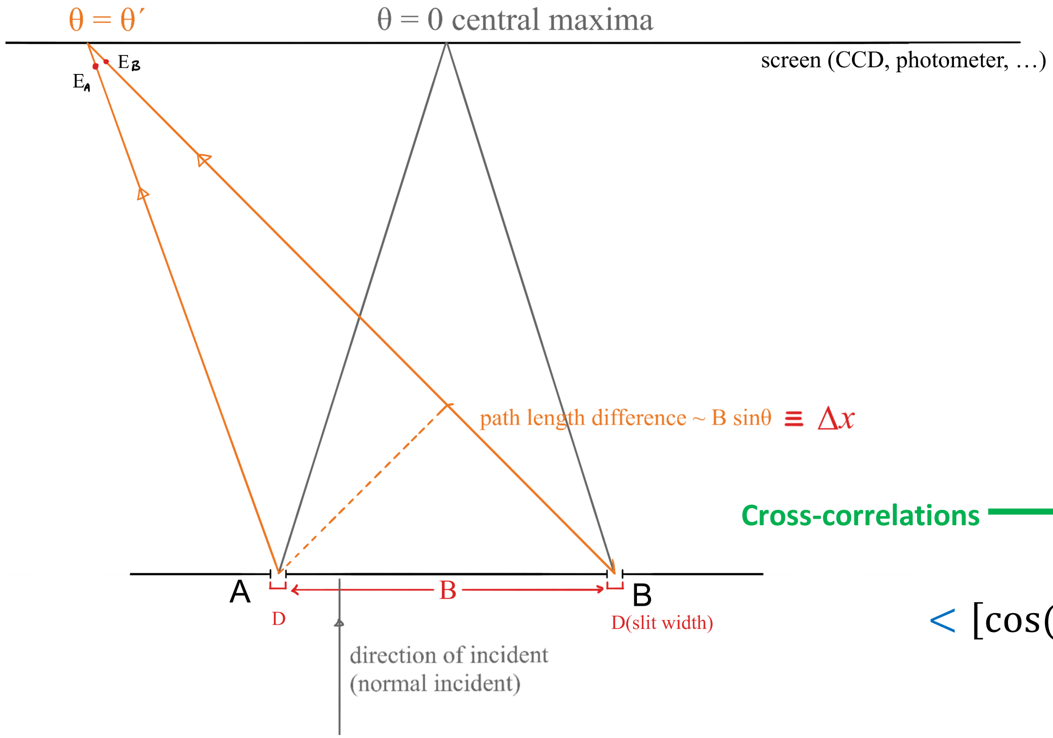

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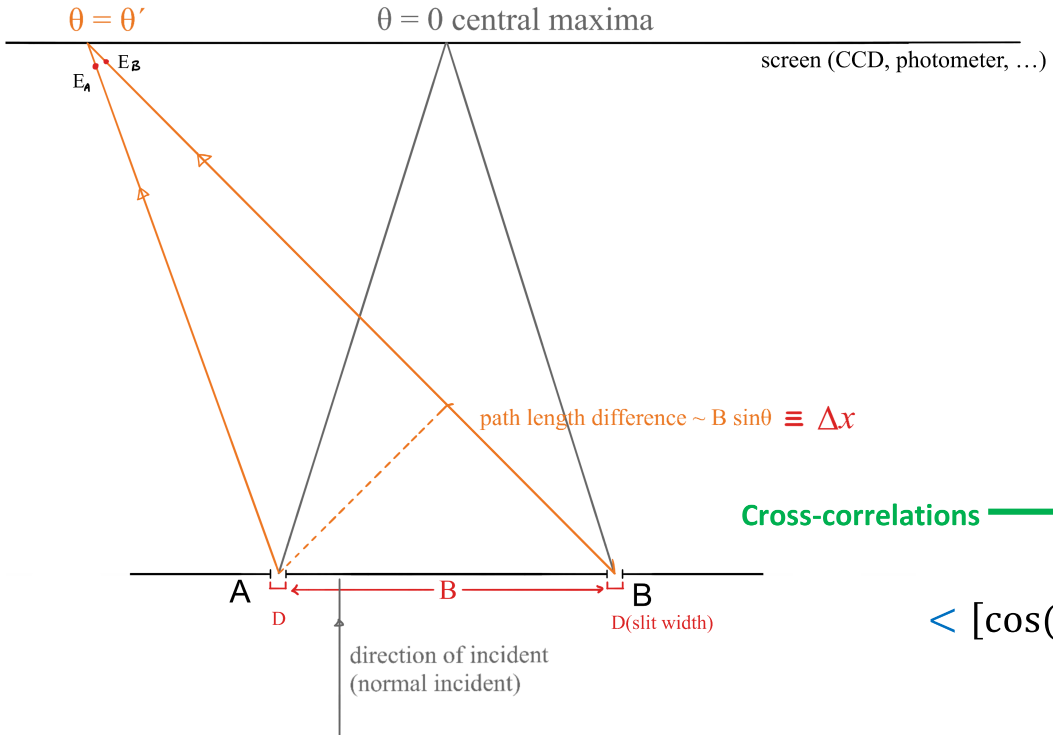

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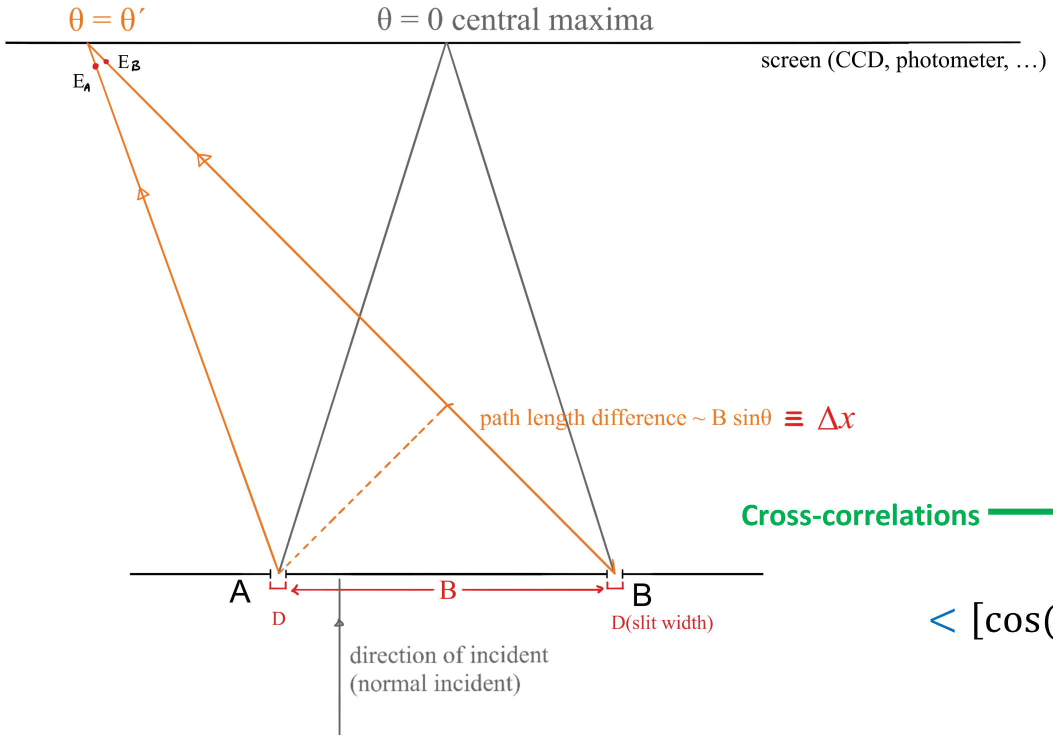

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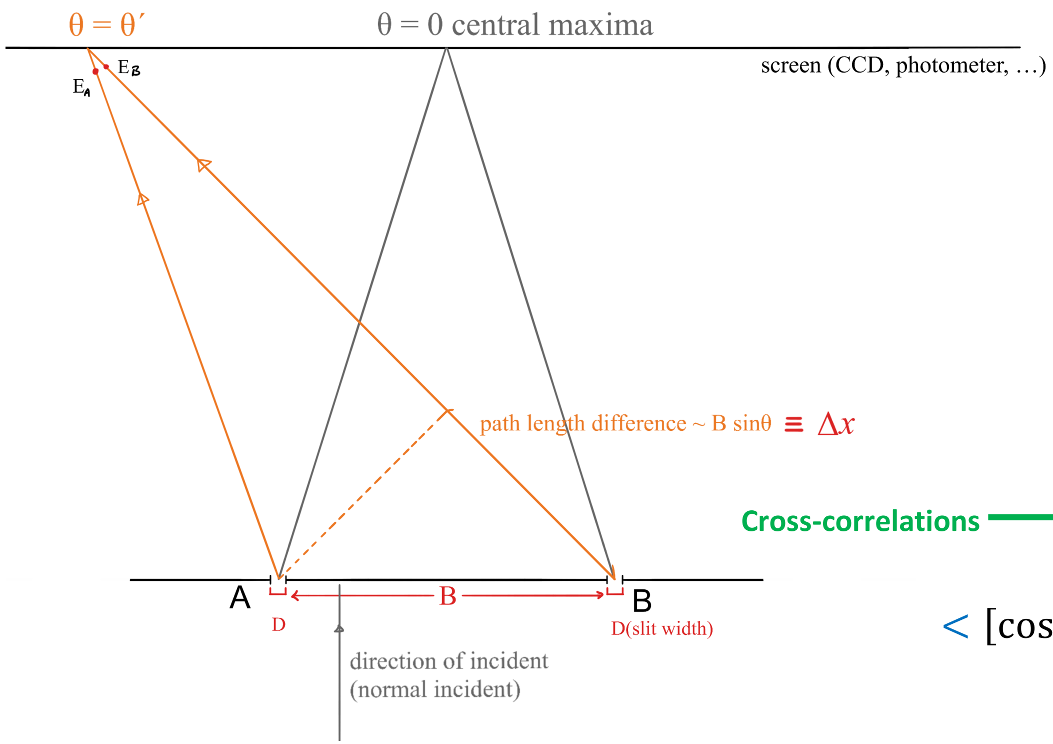

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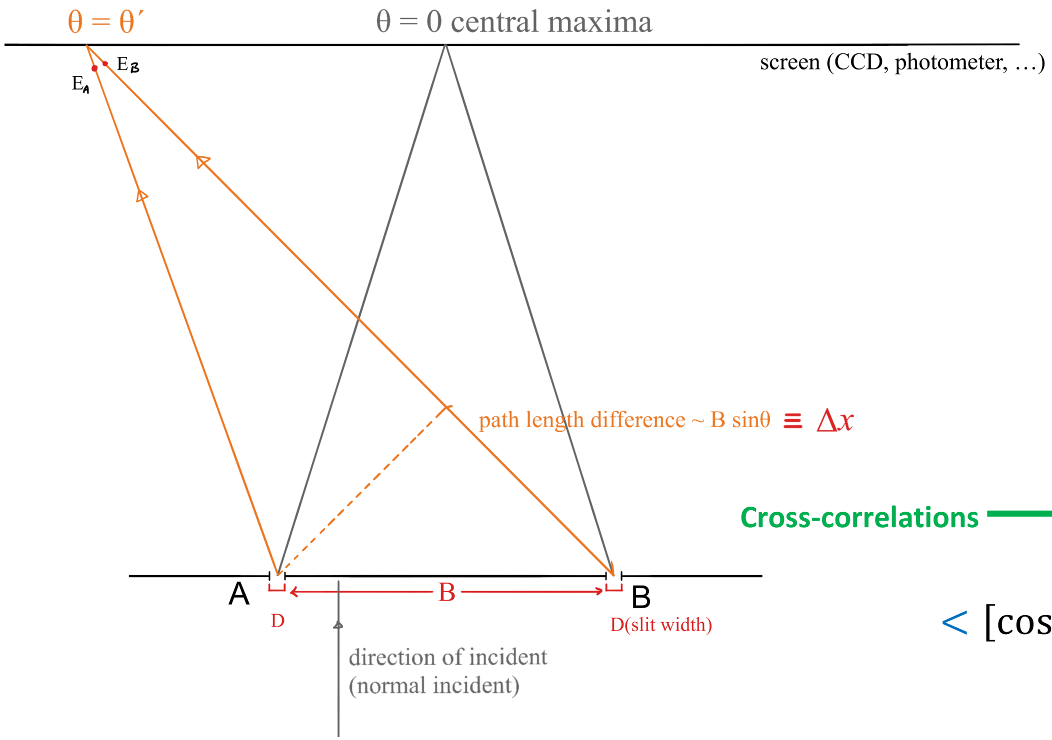

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$$\left.\begin{aligned} &= 1/2 \\ &+ 1/2 \end{aligned}\right] \text{Auto-correlations}$$

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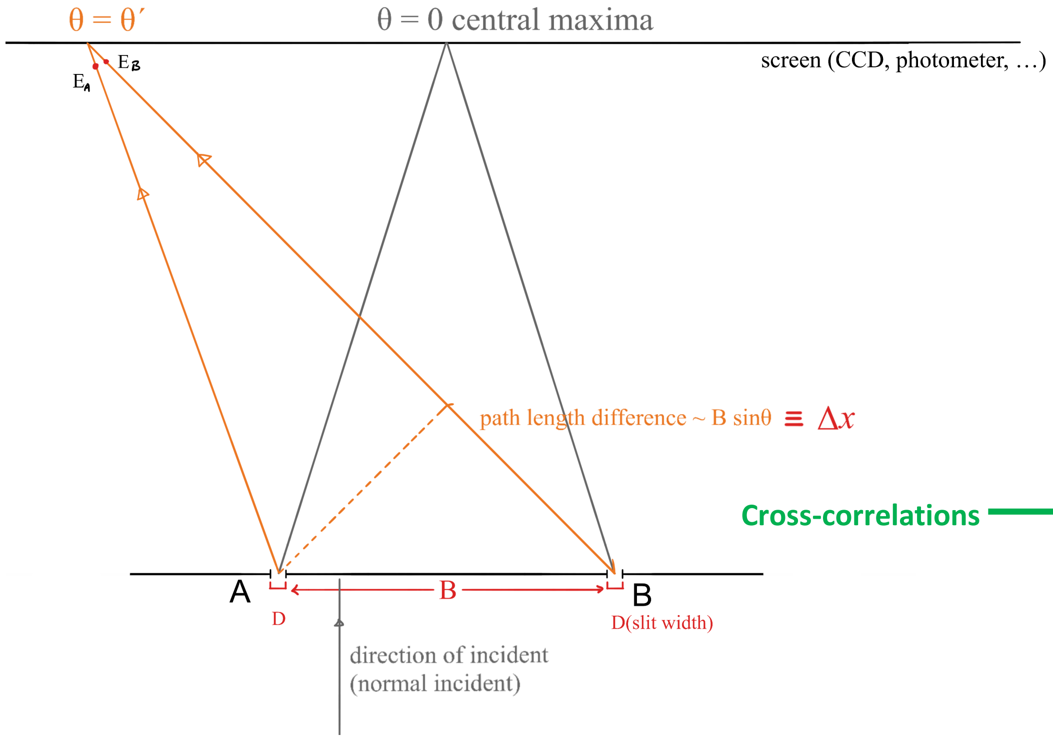

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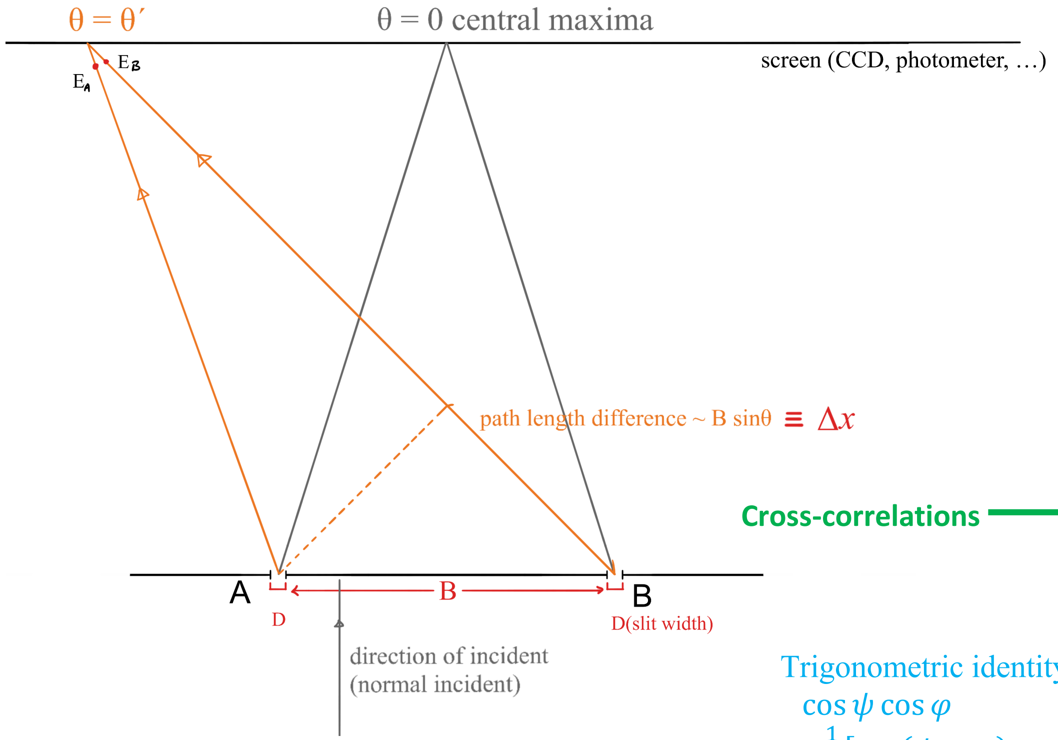

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Auto-correlations

Cross-correlations $\rightarrow + < \boxed{2\underbrace{\cos(kx - \omega t + \phi_0)}_{\psi} \underbrace{\cos(k(x + \Delta x) - \omega t + \phi_0)}_{\varphi}} >$

Trigonometric identity:

$$\cos\psi \cos\varphi = \frac{1}{2}[\cos(\psi - \varphi) + \cos(\psi + \varphi)]$$

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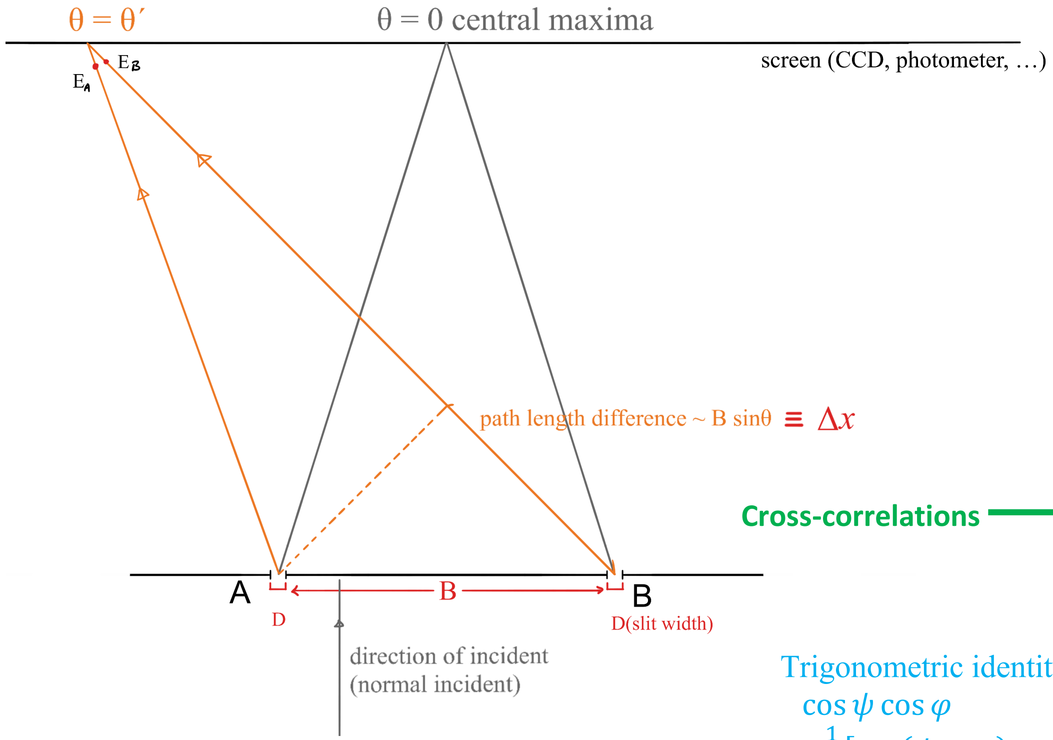

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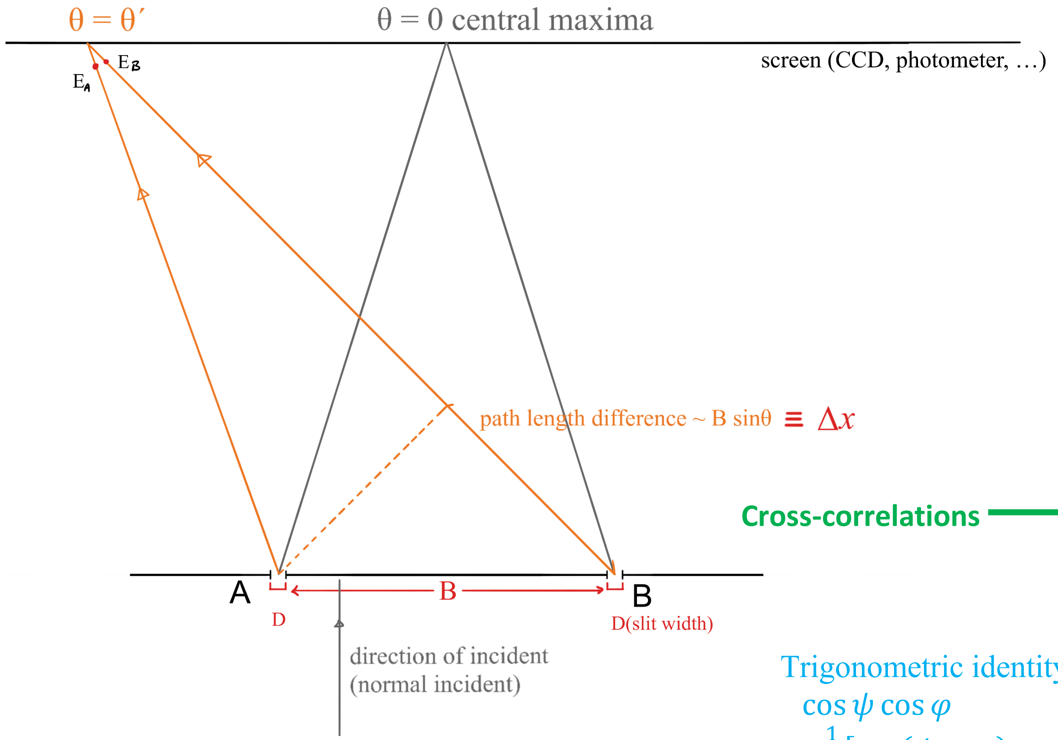

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$$\cos\psi \cos\varphi = \frac{1}{2}[\cos(\psi - \varphi) + \cos(\psi + \varphi)]$$

Minima: $\cos(k\Delta x) = -1$
$$\Longrightarrow k\Delta x = \frac{2\pi}{\lambda} B \sin\theta = m\pi,\ m = \pm 1,3,5, \cdots$$

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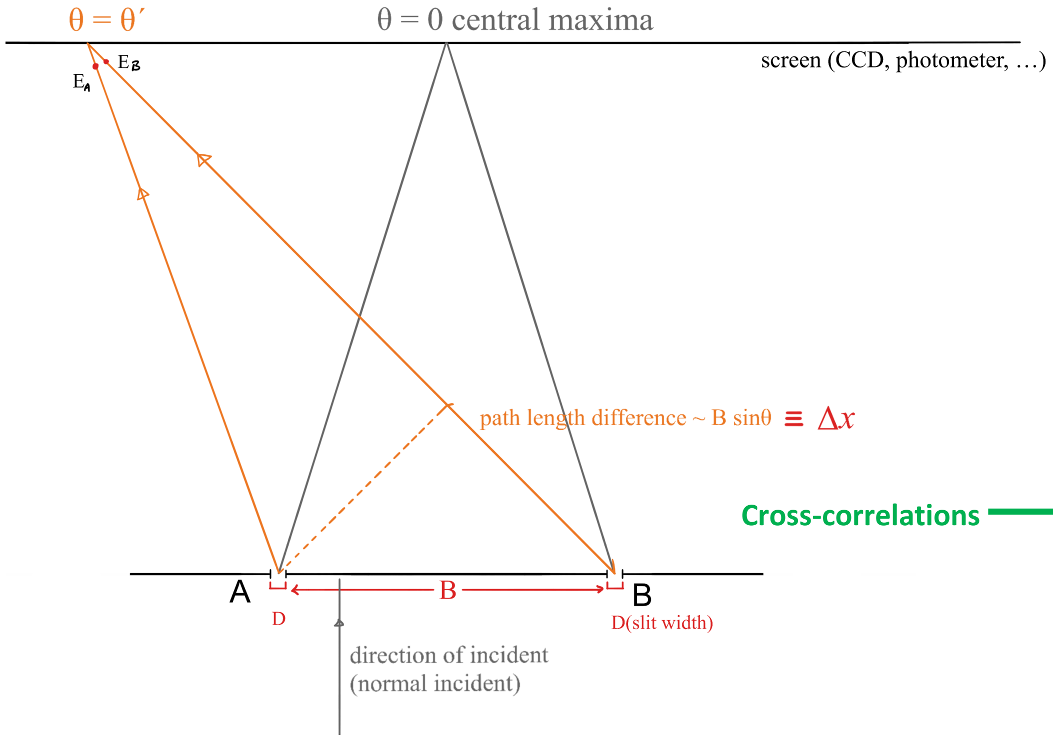

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Auto-correlations

Cross-correlations

$$+ < \boxed{2\underbrace{\cos(kx - \omega t + \phi_0)}_{\varphi} \underbrace{\cos(k(x + \Delta x) - \omega t + \phi_0)}_{\psi}} >$$

$$< [E_A + E_B]^2 > = \tilde{P}(\theta) \left[1 + \cos\left(\frac{2\pi}{\lambda} B \sin\theta\right)\right]$$
$$\sim \tilde{P}(\theta) \left[1 + \cos\left(2\pi \frac{B}{\lambda} \theta\right)\right]$$

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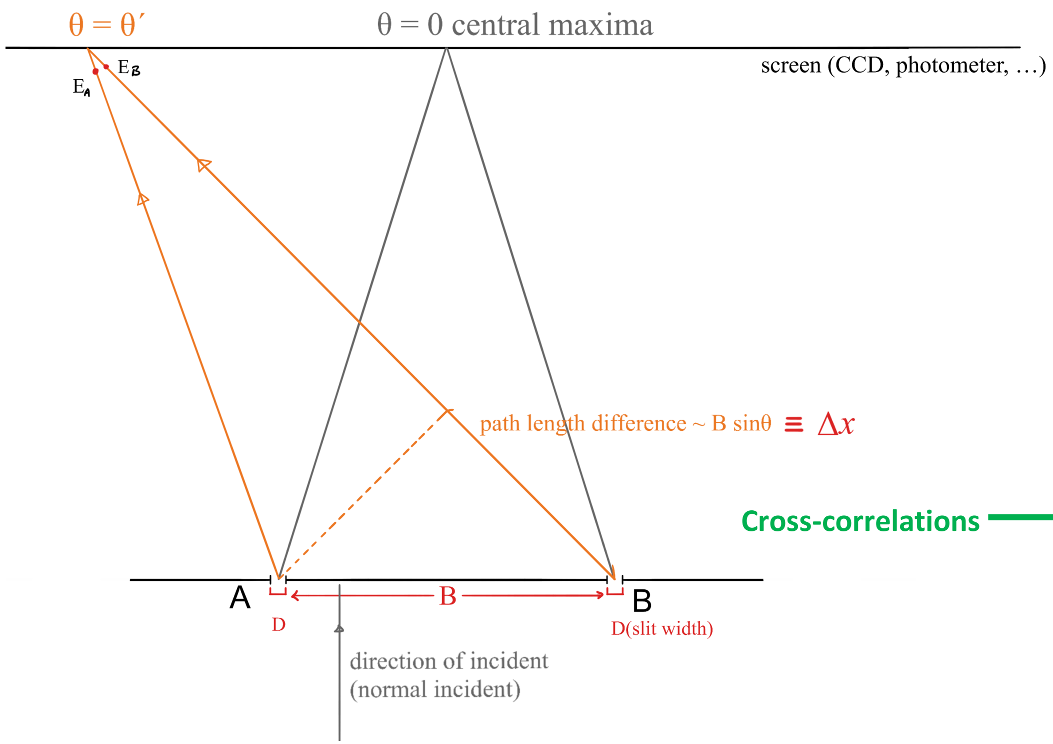

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Cross-correlations

$$+ \left\langle \boxed{2\underbrace{\cos(kx - \omega t + \phi_0)}_{\varphi}\ \underbrace{\cos(k(x + \Delta x) - \omega t + \phi_0)}_{\psi}} \right\rangle$$

$$\langle [E_A + E_B]^2 \rangle = \tilde{P}(\theta)\left[1 + \cos\left(\frac{2\pi}{\lambda} B \sin\theta\right)\right]$$

$$\sim \tilde{P}(\theta)\left[\underbrace{1}_{\text{Auto-correlations}} + \underbrace{\cos\left(2\pi \frac{B}{\lambda}\theta\right)}_{\text{Cross-correlations}}\right]$$

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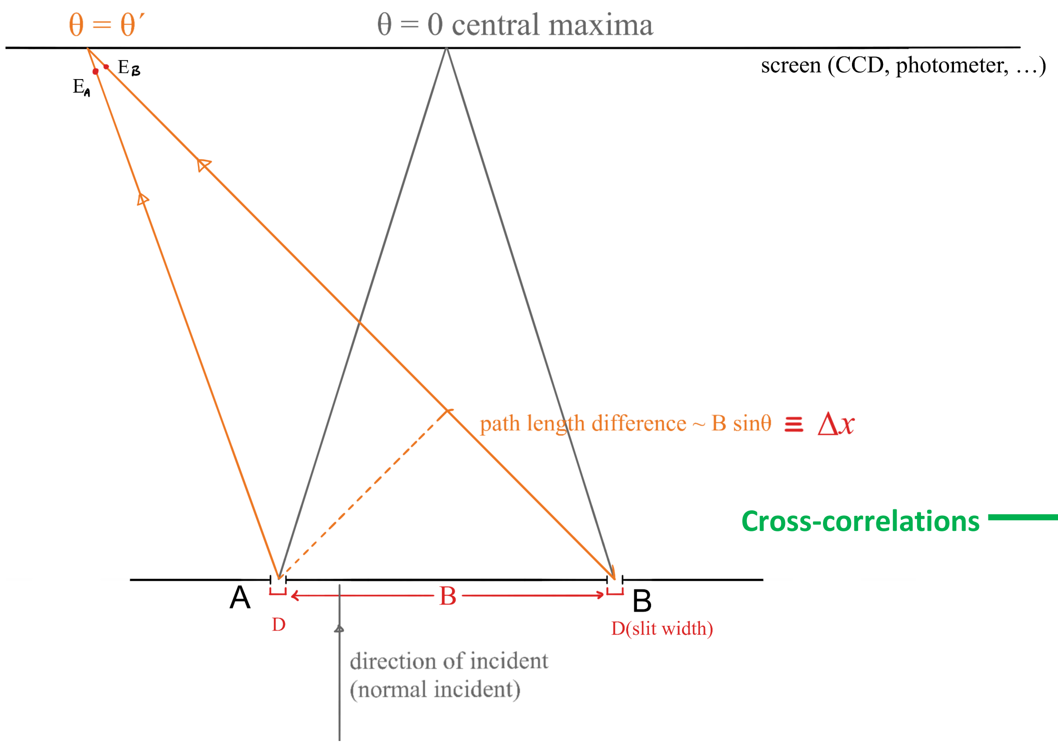

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$$\sim \tilde{P}(\theta)\left[1 + \cos\left(2\pi \frac{B}{\lambda}\theta\right)\right] \sim \tilde{P}(\theta)[1 + \cos(2\pi u \theta)]$$

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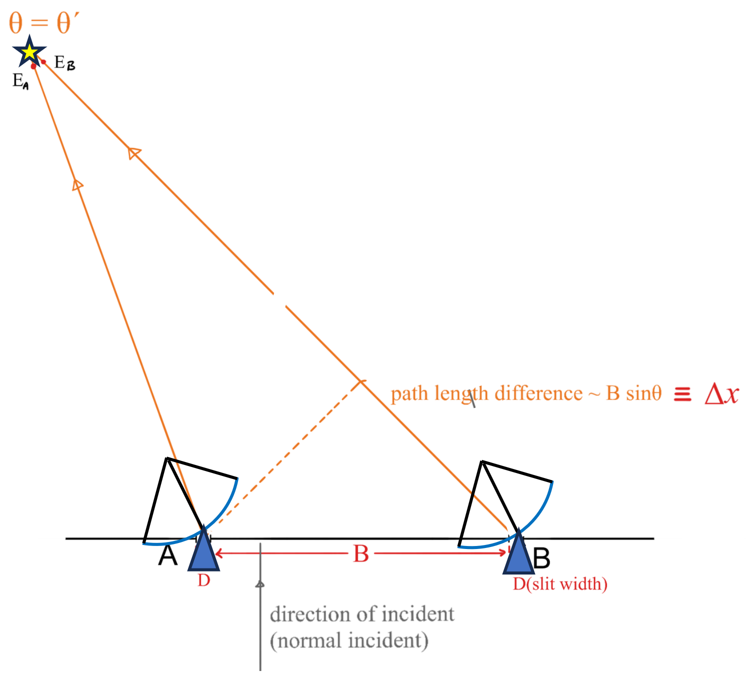

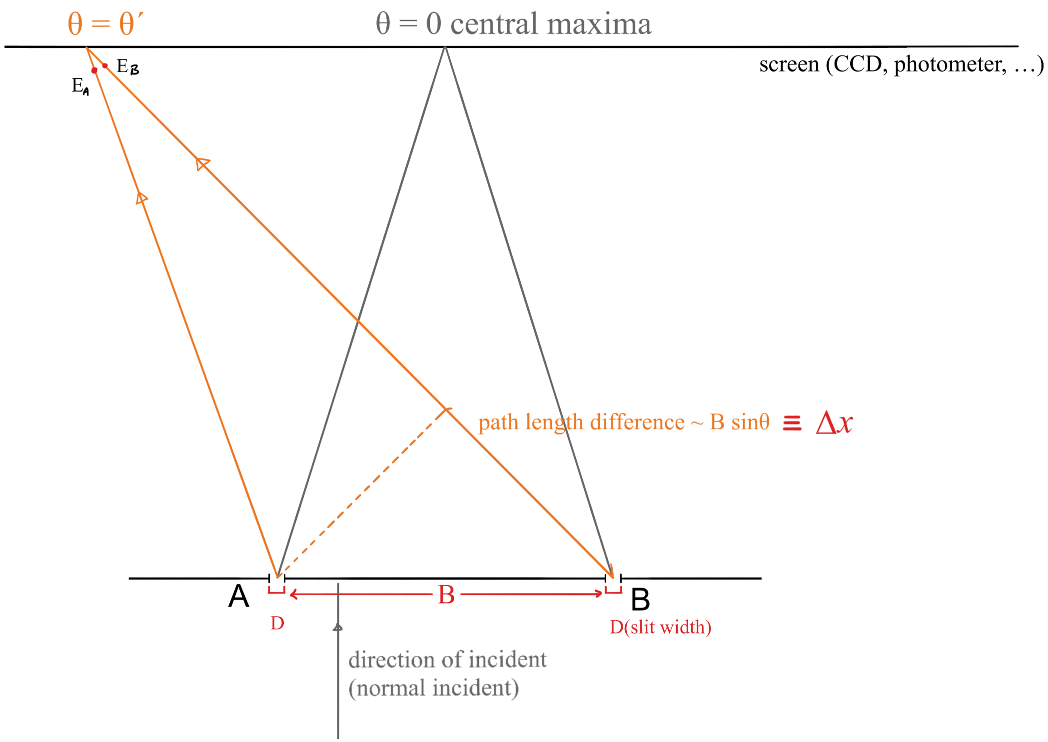

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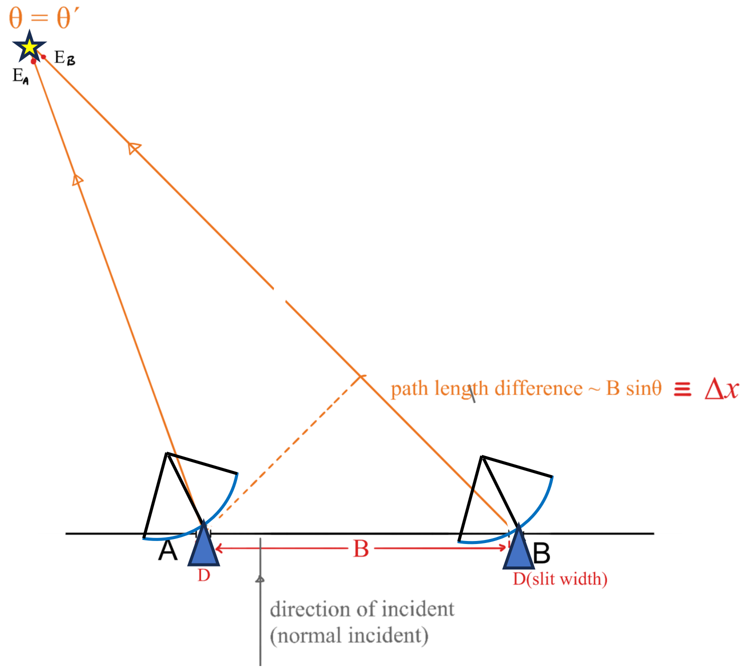

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$$+ [\cos(k(x + \Delta x) - \omega t + \phi_0)]^2$$
$$+ 2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)\}$$

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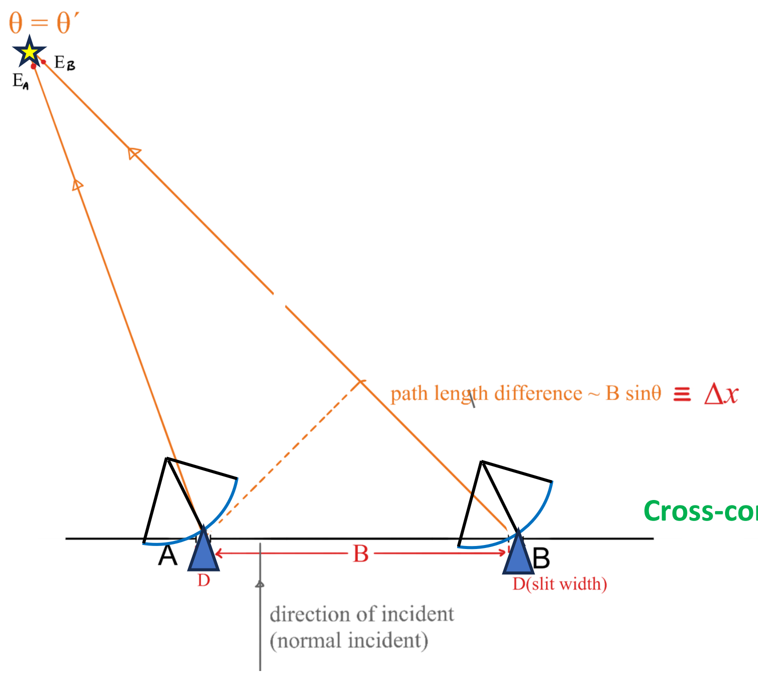

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Auto-correlations

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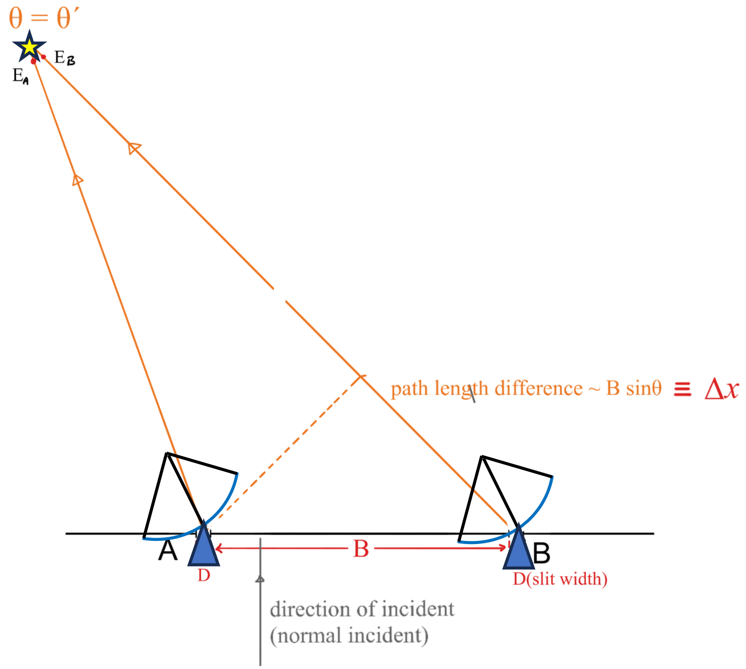

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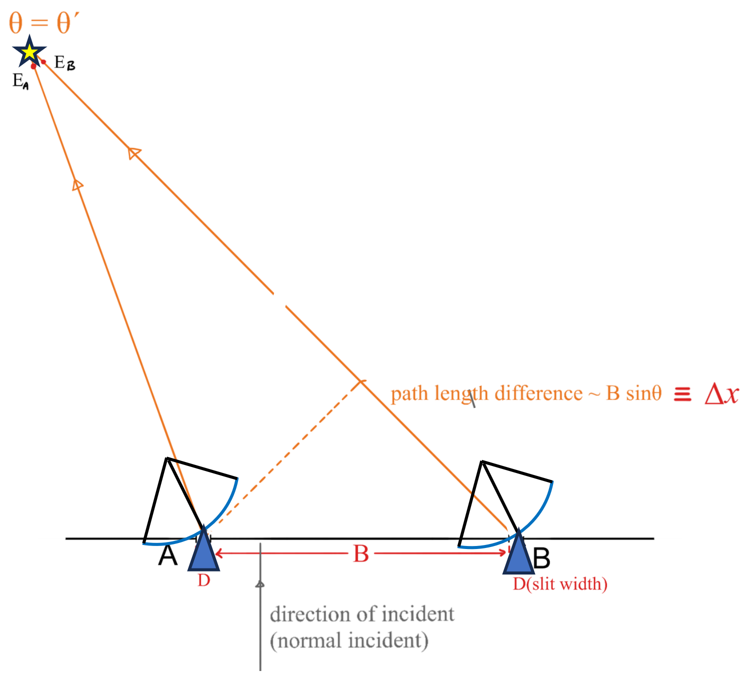
$$= \left.\begin{array}{l} 1/2 \\ + 1/2 \end{array}\right] \text{Auto-correlations}$$
$$\underbrace{+ \langle \boxed{2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)} \rangle}_{\text{Cross-correlations}}$$

$$\langle [E_A + E_B]^2 \rangle = \tilde{P}(\theta)\left[1 + \cos\left(\frac{2\pi}{\lambda} B \sin\theta\right)\right]$$
$$\sim \tilde{P}(\theta)\left[1 + \cos\left(2\pi \frac{B}{\lambda}\theta\right)\right]$$

1. <u>**EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$**</u> $\quad k = \dfrac{2\pi}{\lambda}$
2. **Energy flux density $\propto E_0^2$**

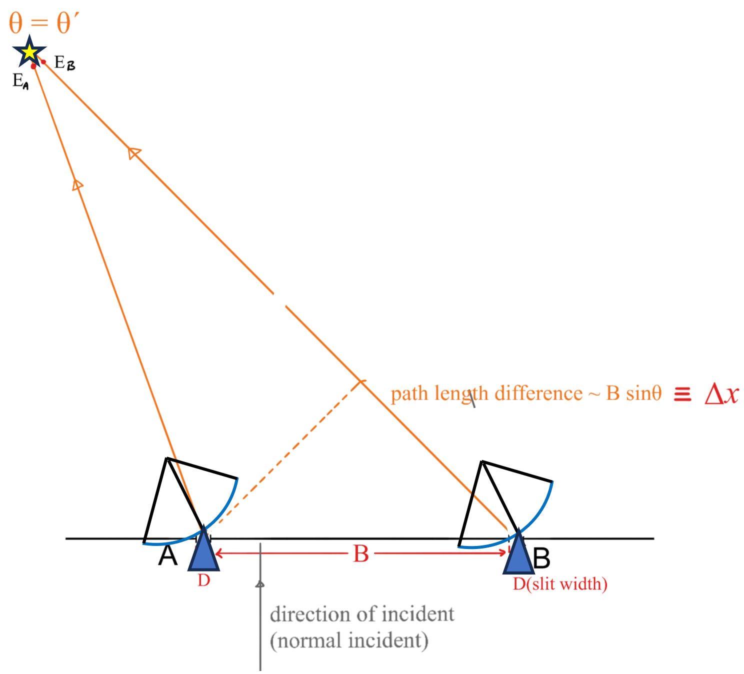


Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$
\begin{aligned}
&\propto \langle [E_A + E_B]^2 \rangle \\
&\quad = \langle [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 \rangle \\
&\quad = \left.\begin{array}{l} \ \ {}^{1}/_{2} \\ + {}^{1}/_{2} \end{array}\right\} \text{Auto-correlations} \\
&\qquad + \underbrace{\langle \boxed{2\cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0)} \rangle}_{\text{Cross-correlations}}
\end{aligned}
$$

$$
\langle [E_A + E_B]^2 \rangle \sim \tilde{P}(\theta)[\underbrace{1}_{\text{Auto-correlations}} + \cos(2\pi u \theta)]
$$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

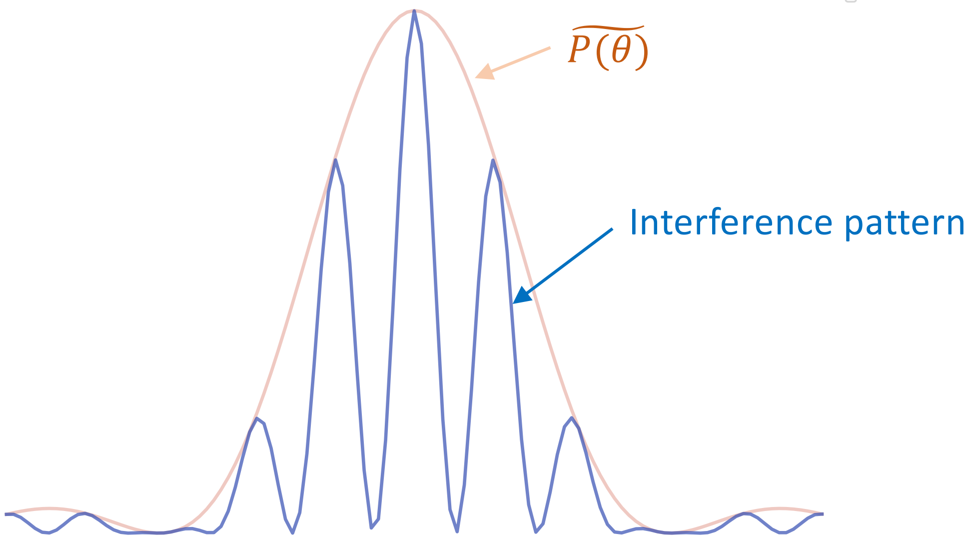


Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$\propto < [E_A + E_B]^2 >$$
$$= < [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 >$$
$$= \left.\begin{array}{l} 1/2 \\ + 1/2 \end{array}\right] \text{Auto-correlations}$$
$$\underbrace{+ < \boxed{2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)} >}_{\text{Cross-correlations}}$$

$$< [E_A + E_B]^2 > \sim \tilde{P}(\theta)[1 + \underbrace{\cos(2\pi u \theta)}_{\text{Cross-correlations}}]$$

1. <u>EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$</u> $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$



Number of incoming photons in a unit of time and a unit area (at the angle of emergence $\theta = \theta'$)

$$\propto < [E_A + E_B]^2 >$$
$$= < [\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0)]^2 >$$
$$= \left.\begin{matrix} 1/2 \\ + 1/2 \end{matrix}\right] \text{Auto-correlations}$$
$$\underbrace{+ < \boxed{2\cos(kx - \omega t + \phi_0)\cos(k(x + \Delta x) - \omega t + \phi_0)} >}_{\text{Cross-correlations}}$$

$$< [E_A + E_B]^2 > \sim \underbrace{\tilde{P}(\theta)}_{\text{Primary beam: response function of each reflector}} [1 + \cos(2\pi u \theta)]$$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $\propto < [E_A + E'_B]^2 >$

$$= < \left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2})\right]^2 >$$

$$= \left.\begin{array}{l} {}^1\!/_2 \\ + {}^1\!/_2 \end{array}\right] \text{Auto-correlations}$$

$$+ < \underbrace{2\cos(kx - \omega t + \phi_0)\cos\left(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2}\right)} >$$

$$2\cos(kx - \omega t + \phi_0)\left[\cos(k(x + \Delta x) - \omega t + \phi_0)\underbrace{\cos\frac{\pi}{2}}_{0} - \sin(k(x + \Delta x) - \omega t + \phi_0)\underbrace{\sin\frac{\pi}{2}}_{1}\right]$$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \dfrac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u \theta)]$

Sine correlation: $\propto < [E_A + E'_B]^2 >$

$$= < \left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2})\right]^2 >$$

$$= \left.\begin{matrix} {}^{1}/_{2} \\ + {}^{1}/_{2} \end{matrix}\right] \text{Auto-correlations}$$

$$+ < \underbrace{2\cos(kx - \omega t + \phi_0) \cos\left(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2}\right)}_{-2\cos(kx - \omega t + \phi_0)\sin(k(x + \Delta x) - \omega t + \phi_0)} >$$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $<[E_A + E_B]^2>_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $\propto <[E_A + E'_B]^2>$

$$= <\left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2})\right]^2>$$

$$= \left.\begin{matrix} 1/2 \\ + 1/2 \end{matrix}\right] \text{Auto-correlations}$$

$$+ < \underbrace{\underbrace{2\cos(kx - \omega t + \phi_0)\cos\left(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2}\right)}_{-2\cos(kx - \omega t + \phi_0)\sin(k(x + \Delta x) - \omega t + \phi_0)}}_{-[\sin(2kx - 2\omega t + 2\phi_0 + k\Delta x) - \sin(k\Delta x)]} >$$

Trigonometric identity:
$\cos\psi \sin\varphi$
$= \frac{1}{2}[\sin(\psi + \varphi) - \sin(\psi - \varphi)]$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \dfrac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $\propto < [E_A + E'_B]^2 >$

$$= < \left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2})\right]^2 >$$

$$\left.\begin{array}{l} = \ ^1/_2 \\ + \ ^1/_2 \end{array}\right] \text{Auto-correlations}$$

$$+ < \underbrace{\underbrace{2\cos(kx - \omega t + \phi_0)\cos\left(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2}\right)}_{-2\cos(kx - \omega t + \phi_0)\sin(k(x + \Delta x) - \omega t + \phi_0)}}_{-[\cancel{\sin(2kx - 2\omega t + 2\phi_0 + k\Delta x)} - \sin(k\Delta x)]} >$$

Zero in the <> bracket

Trigonometric identity:
$\cos\psi \sin\varphi$
$= \frac{1}{2}[\sin(\psi + \varphi) - \sin(\psi - \varphi)]$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \dfrac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $\propto < [E_A + E'_B]^2 >$

$$= < \left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2})\right]^2 >$$

$$= {}^1/_2$$
$$+ {}^1/_2$$

Auto-correlations

$$+ < \underbrace{\underbrace{2\cos(kx - \omega t + \phi_0)\cos\left(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2}\right)}_{-2\cos(kx - \omega t + \phi_0)\sin(k(x + \Delta x) - \omega t + \phi_0)}}_{[\sin(k\Delta x)]} >$$

Trigonometric identity:
$\cos\psi \sin\varphi$
$= \frac{1}{2}[\sin(\psi + \varphi) - \sin(\psi - \varphi)]$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $\propto < [E_A + E'_B]^2 >$

$$= < \left[\cos(kx - \omega t + \phi_0) + \cos(k(x + \Delta x) - \omega t + \phi_0 + \tfrac{\pi}{2})\right]^2 >$$

$$= \ ^1/_2 \\ + \ ^1/_2$$

Auto-correlations

$$+ < \underbrace{\underbrace{2\cos(kx - \omega t + \phi_0) \cos\left(k(x + \Delta x) - \omega t + \phi_0 + \frac{\pi}{2}\right)}_{-2\cos(kx - \omega t + \phi_0) \sin(k(x + \Delta x) - \omega t + \phi_0)}}_{\sin(k\Delta x) = \sin(2\pi u\theta)} >$$

Trigonometric identity:
$\cos\psi \sin\varphi$
$= \frac{1}{2}[\sin(\psi + \varphi) - \sin(\psi - \varphi)]$

1. EM-wave at long-distance limit: plane wave $E = E_0 \cos(kx - wt + \phi_0)$ $\quad k = \frac{2\pi}{\lambda}$
2. Energy flux density $\propto E_0^2$

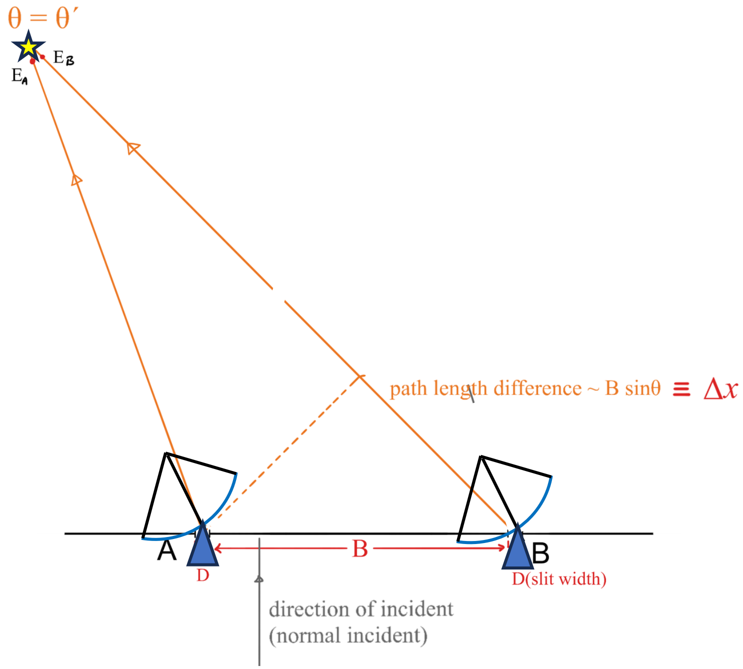
Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u \theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u \theta)]$

Cosine correlation and sine correlation are the direct measurement of a two-element radio interferometer

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

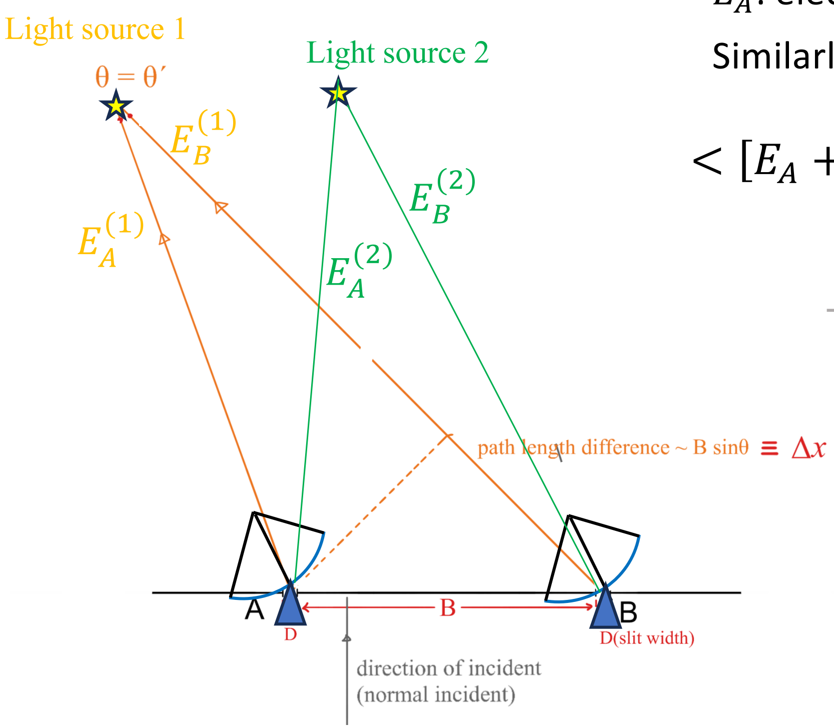


When there is one target source

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there are two incoherent sources



E_A: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

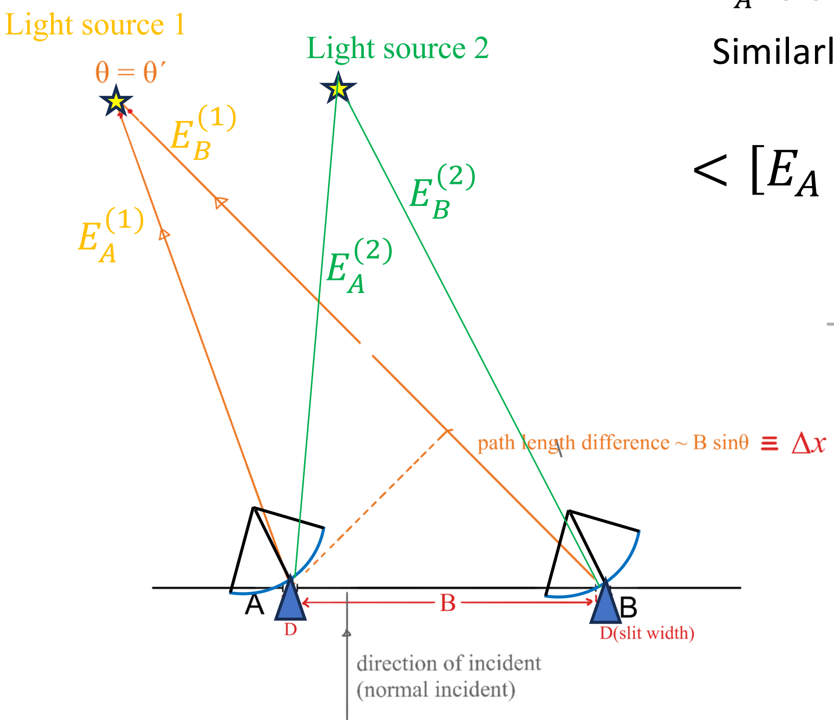
Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \; = < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)}E_B^{(1)} >$$
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)}E_B^{(2)} >$$
$$+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >$$

Cosine correlation: $<[E_A+E_B]^2>_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $<[E_A+E_B]^2>_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

When there are two incoherent sources



E_A: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

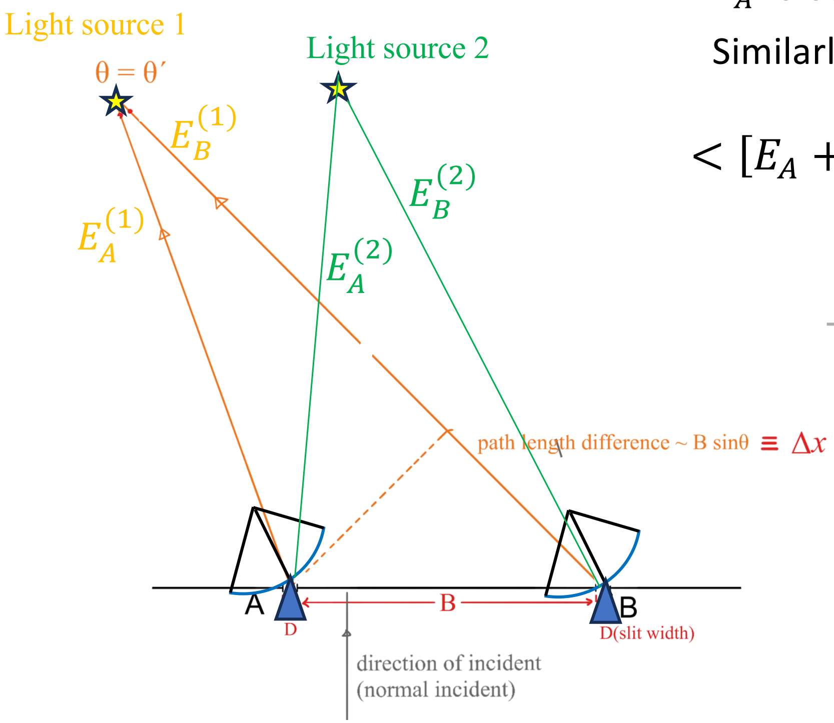
Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$<[E_A+E_B]^2> \; = \; < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)}E_B^{(1)} >$$
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)}E_B^{(2)} >$$
$$+ < 2E_A^{(1)}E_A^{(2)} > + < 2E_A^{(1)}E_B^{(2)} > + < 2E_B^{(1)}E_A^{(2)} > + < 2E_B^{(1)}E_B^{(2)} >$$

Cosine correlation: $<[E_A + E_B]^2>_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $<[E_A + E_B]^2>_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

When there are two incoherent sources



E_A: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$<[E_A + E_B]^2> \; = \; <{E_A^{(1)}}^2> + <{E_B^{(1)}}^2> + <2E_A^{(1)}E_B^{(1)}>$$

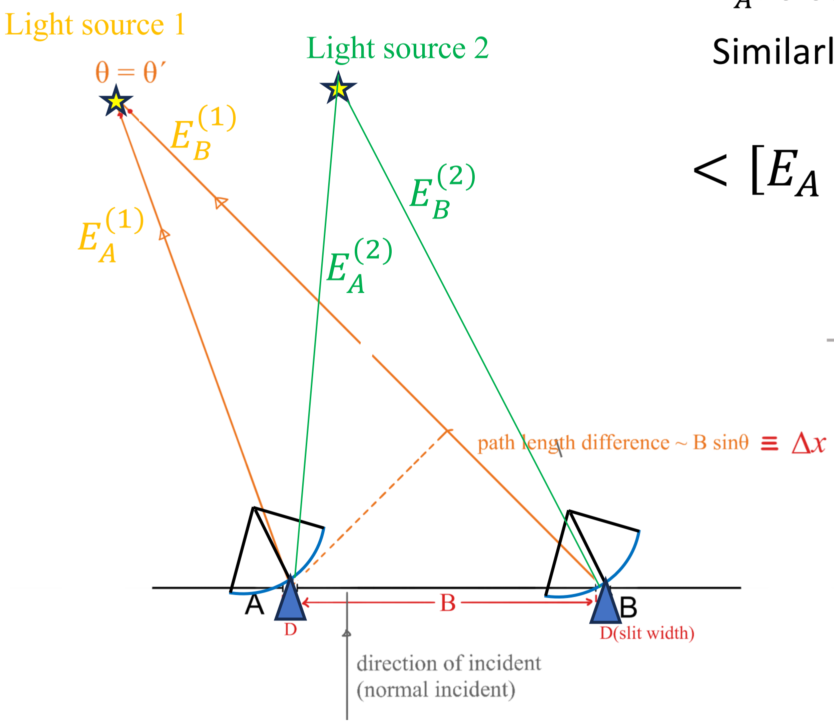
$$+ <{E_A^{(2)}}^2> + <{E_B^{(2)}}^2> + <2E_A^{(2)}E_B^{(2)}>$$

$$+ <2E_A^{(1)}E_A^{(2)}> + <2E_A^{(1)}E_B^{(2)}> + <2E_B^{(1)}E_A^{(2)}> + <2E_B^{(1)}E_B^{(2)}>$$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u \theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u \theta)]$

When there are two incoherent sources



E_A: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

$$< [E_A + E_B]^2 > \; = \; < {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)} E_B^{(1)} >$$

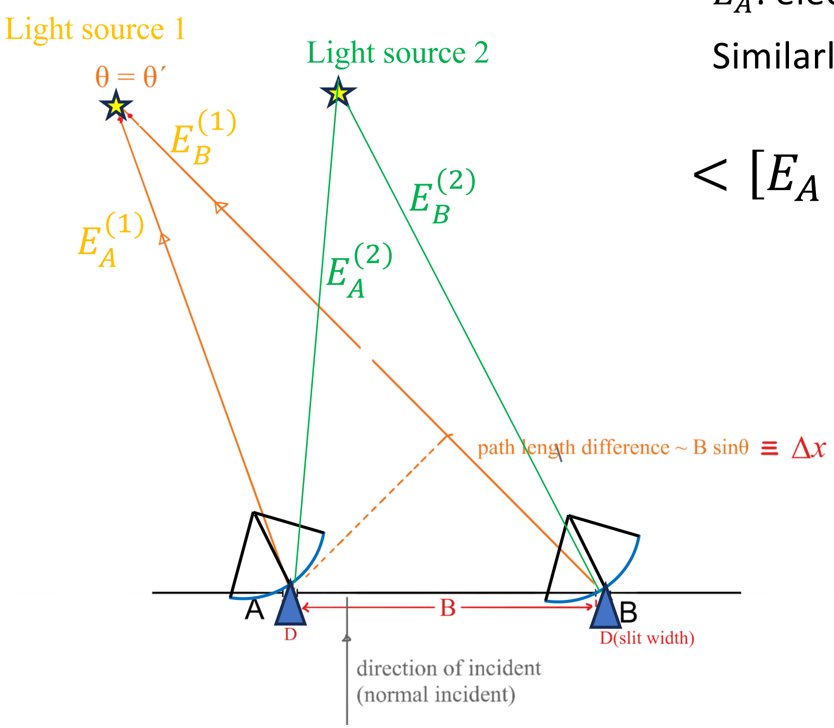
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)} E_B^{(2)} >$$

$$\underbrace{\cancel{+ < 2E_A^{(1)} E_A^{(2)} > + < 2E_A^{(1)} E_B^{(2)} > + < 2E_B^{(1)} E_A^{(2)} > + < 2E_B^{(1)} E_B^{(2)} >}}_{\text{Definition of incoherent sources}}$$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there are two incoherent sources



E_A: electric field received at antenna A $= E_A^{(1)} + E_A^{(2)}$,

Similarly, $E_B = E_B^{(1)} + E_B^{(2)}$

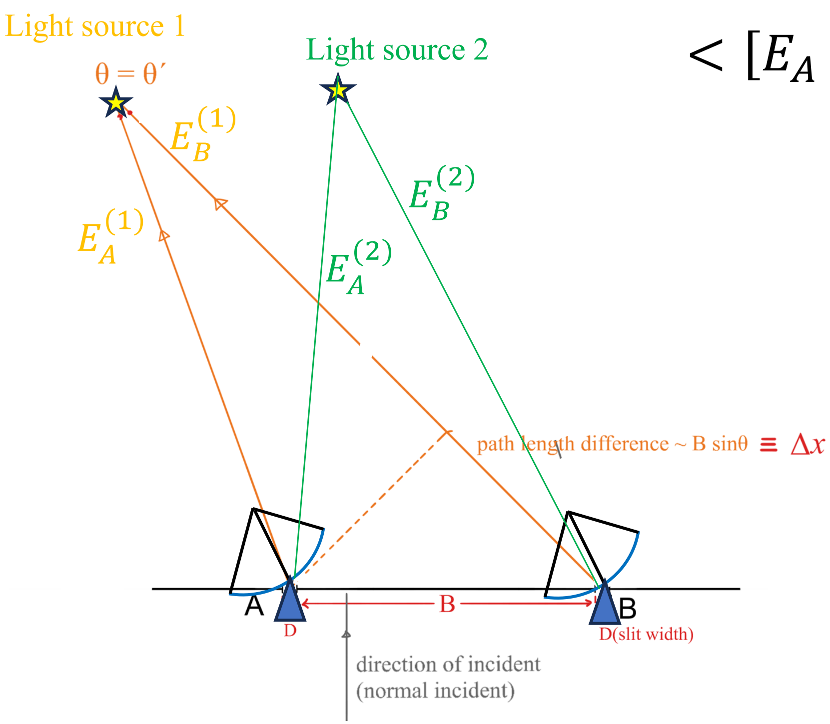
$$< [E_A + E_B]^2 > \;= \;< {E_A^{(1)}}^2 > + < {E_B^{(1)}}^2 > + < 2E_A^{(1)}E_B^{(1)} >$$
$$+ < {E_A^{(2)}}^2 > + < {E_B^{(2)}}^2 > + < 2E_A^{(2)}E_B^{(2)} >$$

Correlation of multiple incoherent sources can be evaluated by summing over the contribution of individual sources.

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there are more incoherent sources



$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \sum_{i=0}^{N-1} \widetilde{P(\theta_i{}')} \ [\underbrace{A_i}\cos(2\pi u\theta_i{}')]$$

A coefficient to describe how bright is source i

Dimension: flux density

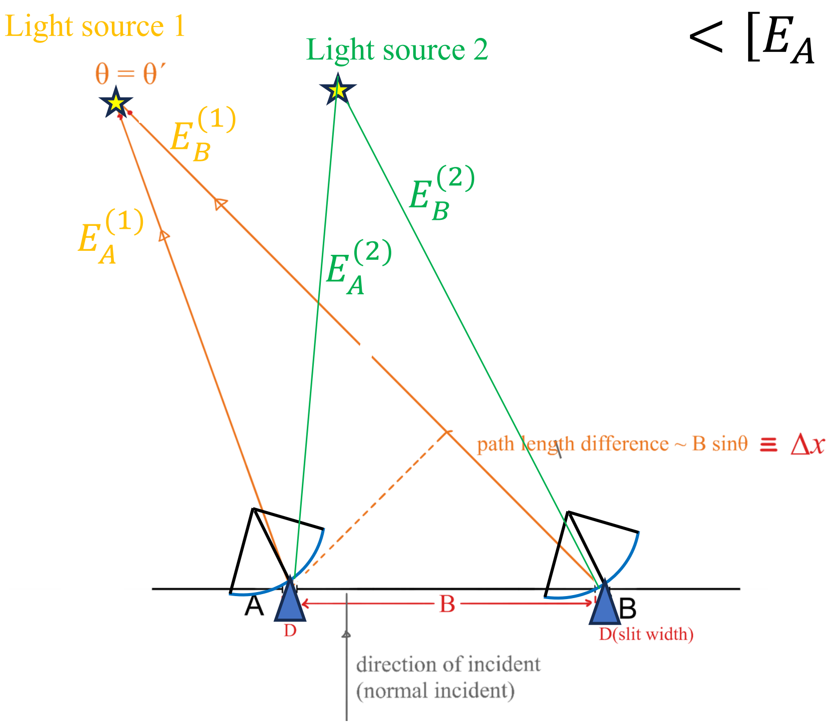
Looking at cross correlations at this moment

(i.e., neglecting auto-correlation terms)

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources



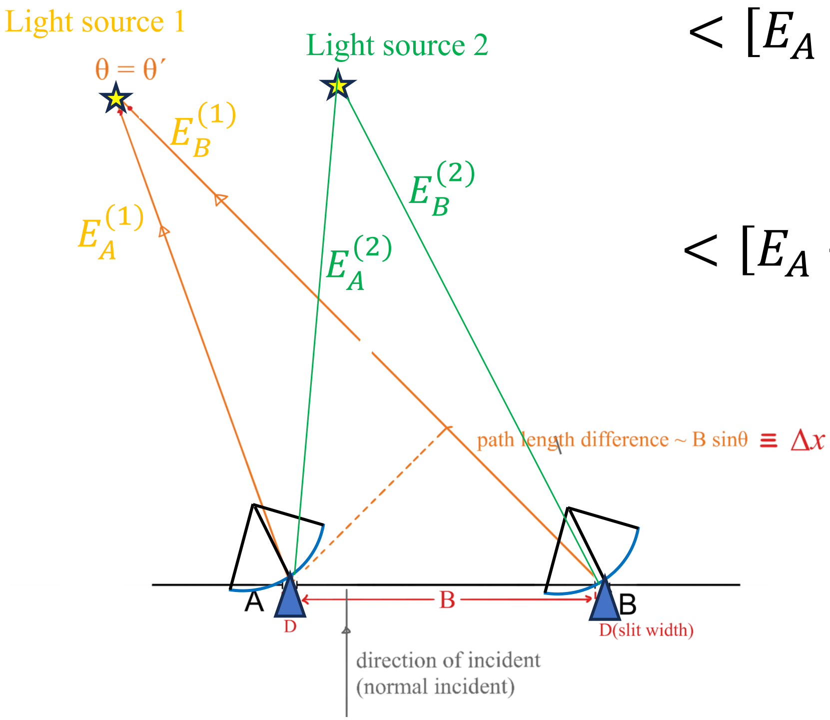
$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)} \cos(2\pi u\theta)\, d\theta$$

A coefficient to describe the intensity of the target source at incident angle θ
Dimension: intensity

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1 + \cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1 + \sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources



$$< [E_A + E_B]^2 >_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)}\cos(2\pi u\theta)\, d\theta$$

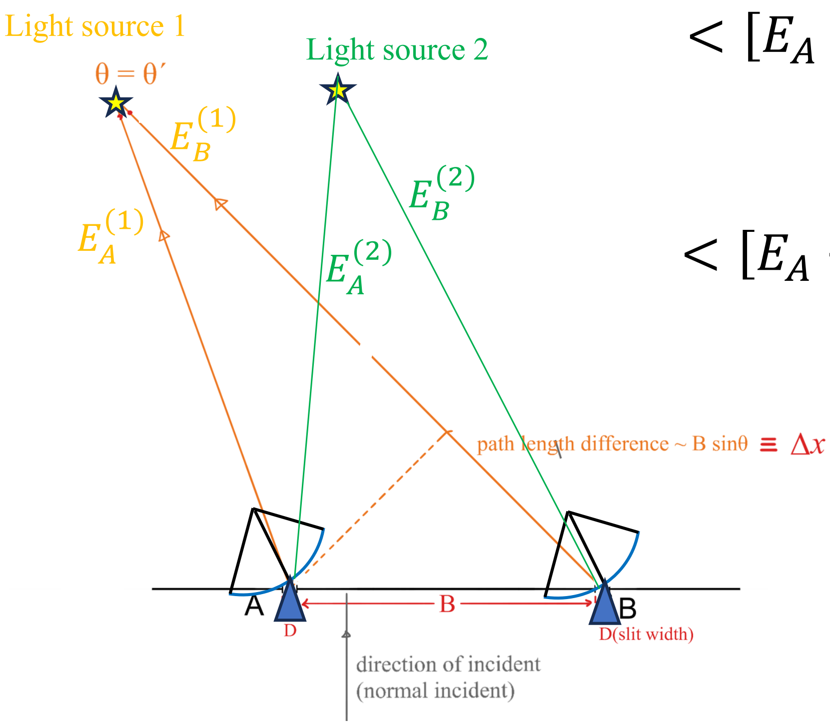
A coefficient to describe the intensity of the target source at incident angle θ

$$< [E_A + E_B]^2 >_{sin}^{cross} \propto \int \tilde{P}(\theta)A(\theta)\sin(2\pi u\theta)\, d\theta$$

Cosine correlation: $<[E_A+E_B]^2>_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $<[E_A+E_B]^2>_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

When there is a continuous distribution of incoherent sources



$$<[E_A+E_B]^2>_{cos}^{cross} \propto \int \tilde{P}(\theta)\underbrace{A(\theta)}\cos(2\pi u\theta)\,d\theta$$

A coefficient to describe the intensity of the target source at incident angle θ

$$<[E_A+E_B]^2>_{sin}^{cross} \propto \int \tilde{P}(\theta)A(\theta)\sin(2\pi u\theta)\,d\theta$$

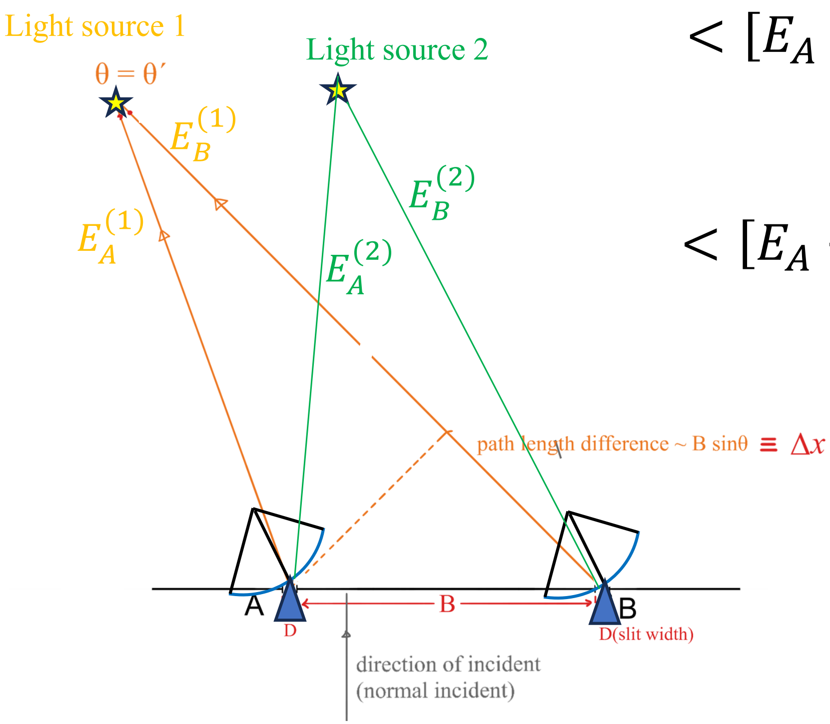
$$\text{Complex visibility } V_{AB} = <[E_A+E_B]^2>_{cos}^{cross} + i<[E_A+E_B]^2>_{sin}^{cross}$$

Cosine correlation: $<[E_A+E_B]^2>_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $<[E_A+E_B]^2>_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

Complex visibility $V_{AB} = <[E_A+E_B]^2>_{cos}{}^{cross} + i<[E_A+E_B]^2>_{sin}{}^{cross}$

Intensity distribution of a point-like source: $A(\theta) = \delta(\theta-\theta')$



$$<[E_A+E_B]^2>_{cos}{}^{cross} = \int \tilde{P}(\theta)A(\theta)\cos(2\pi u\theta)\,d\theta$$

$$= \tilde{P}(\theta')\cos(2\pi u\theta')$$

$$<[E_A+E_B]^2>_{sin}{}^{cross} = \tilde{P}(\theta')\sin(2\pi u\theta')$$

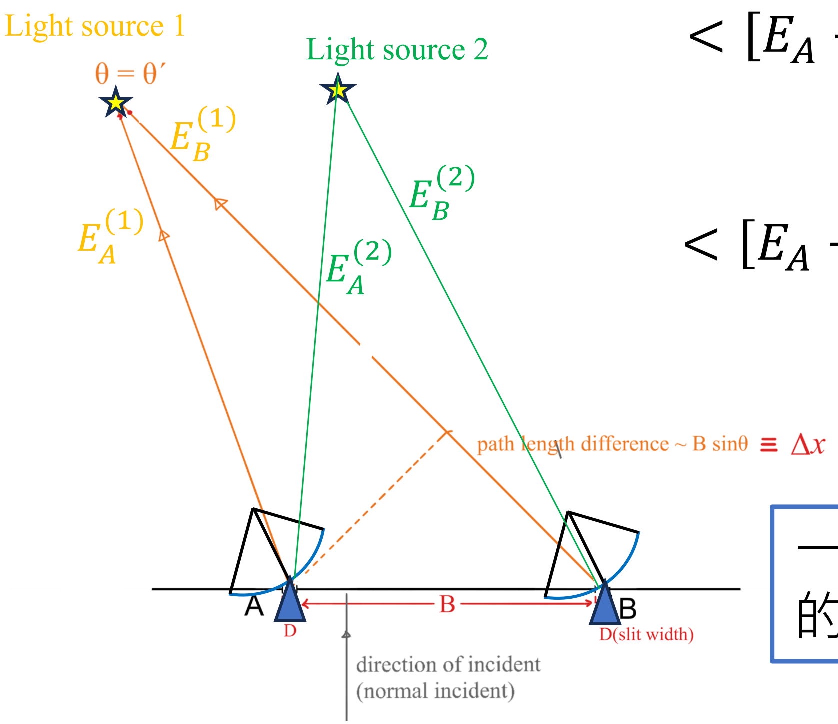
Complex visibility $V_{AB} = \tilde{P}(\theta')\cos(2\pi u\theta') + i\tilde{P}(\theta')\sin(2\pi u\theta') = \underbrace{\tilde{P}(\theta')}_{\text{Visibility amplitude}} e^{i\underbrace{2\pi u\theta'}_{\text{Visibility phase}}}$

Cosine correlation: $< [E_A + E_B]^2 >_{cos} \sim \tilde{P}(\theta)[1+\cos(2\pi u\theta)]$

Sine correlation: $< [E_A + E_B]^2 >_{sin} \sim \tilde{P}(\theta)[1+\sin(2\pi u\theta)]$

Complex visibility $V_{AB} = < [E_A + E_B]^2 >_{cos}{}^{cross} + i < [E_A + E_B]^2 >_{sin}{}^{cross}$

Intensity distribution of a point-like source: $A(\theta) = \delta(\theta - \theta')$



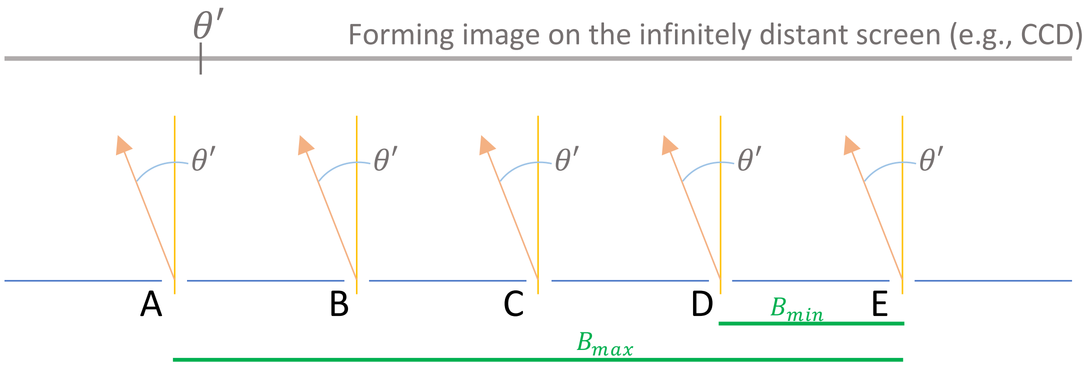
$$< [E_A + E_B]^2 >_{cos}{}^{cross} = \int \tilde{P}(\theta) A(\theta) \cos(2\pi u\theta)\, d\theta$$

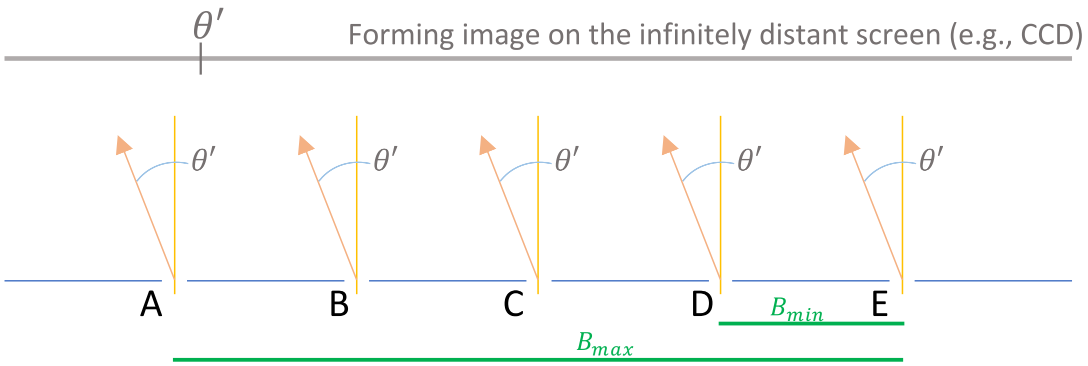
$$= \tilde{P}(\theta')\cos(2\pi u\theta')$$

$$< [E_A + E_B]^2 >_{sin}{}^{cross} = \tilde{P}(\theta')\sin(2\pi u\theta')$$

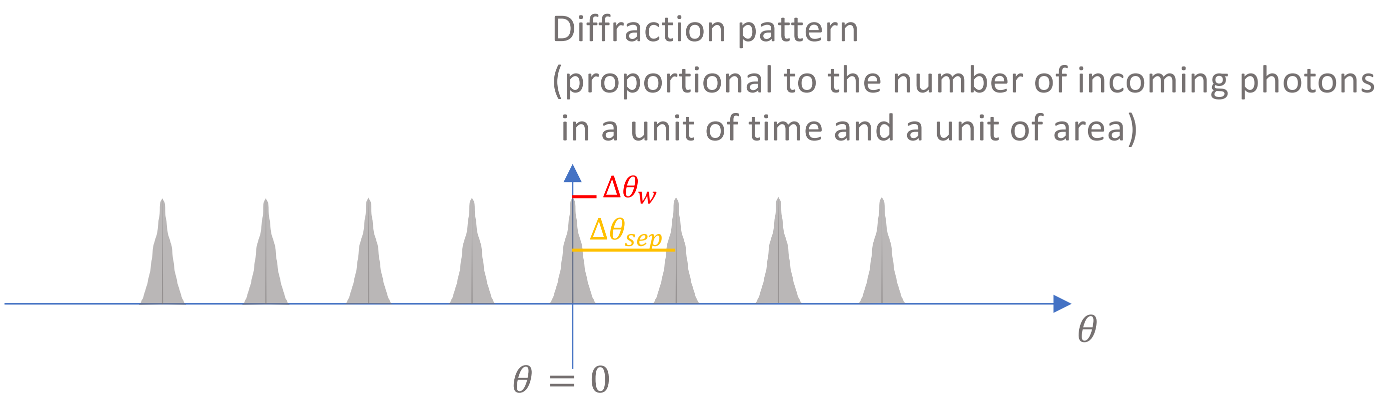
$$\underline{\text{Complex visibility } V_{AB}} = \tilde{P}(\theta')\cos(2\pi u\theta') + i\tilde{P}(\theta')\sin(2\pi u\theta') = \underbrace{\tilde{P}(\theta')}_{\text{Visibility amplitude}} e^{i\underbrace{2\pi u\theta'}_{\text{Visibility phase}}}$$

一組基線(baseline)
的測量

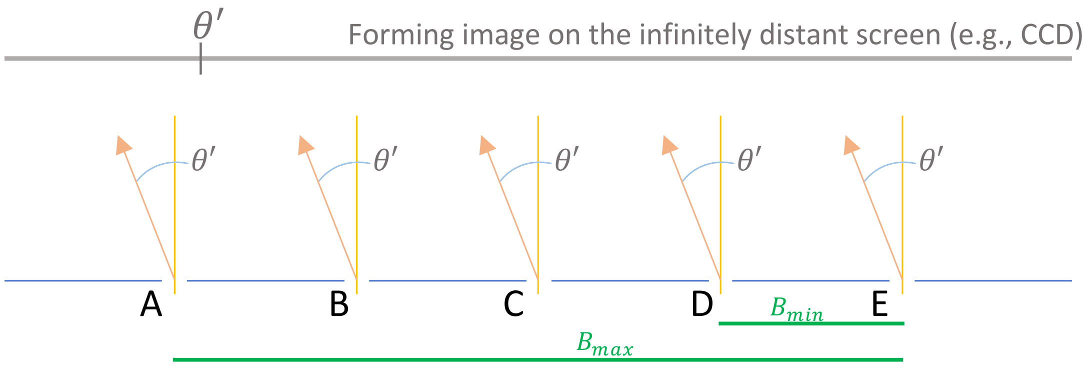

θ'
Forming image on the infinitely distant screen (e.g., CCD)
θ'
θ'
θ'
θ'
θ'
A
B
C
D
E
B_{min}
B_{max}



θ'
Forming image on the infinitely distant screen (e.g., CCD)
θ' θ' θ' θ' θ'
A B C D E
B_{min}
B_{max}



Diffraction pattern
(proportional to the number of incoming photons
in a unit of time and a unit of area)
$\Delta\theta_w$
$\Delta\theta_{sep}$
θ
$\theta = 0$

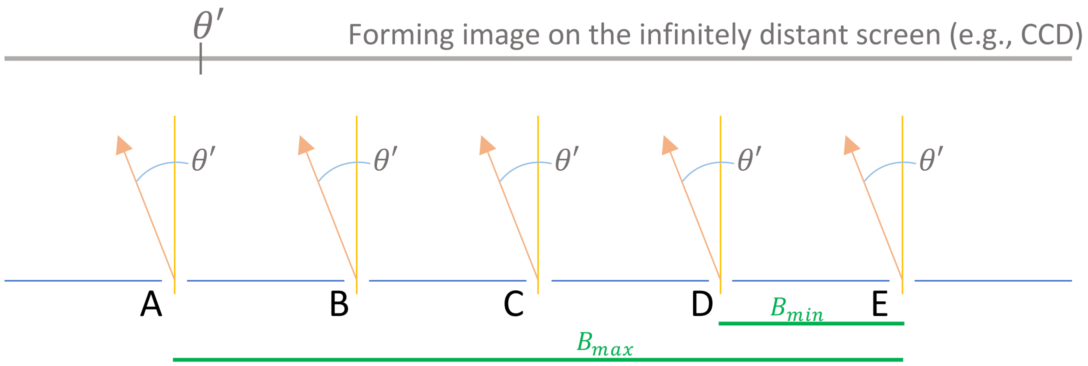


Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 >$$

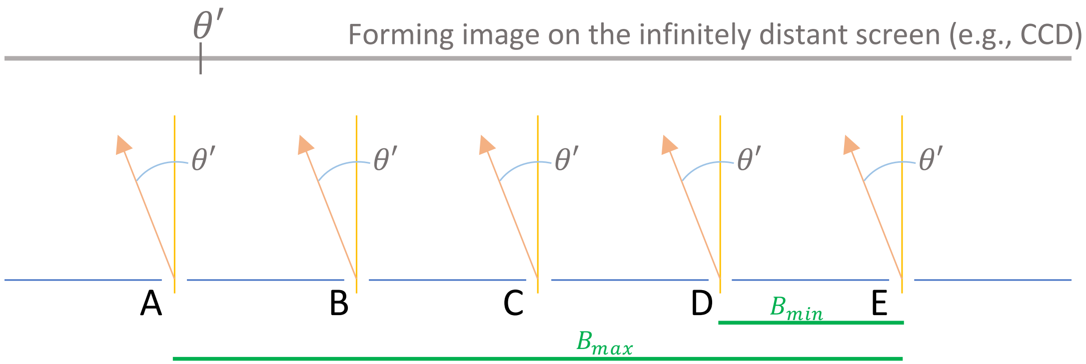


Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

$$\begin{aligned} <(E_A + E_B + E_C + E_D + E_E + \cdots)^2> = & < {E_A}^2 > + < {E_B}^2 > + < {E_C}^2 > + < {E_D}^2 > + < {E_E}^2 > \\ & + < 2E_A E_B > + < 2E_A E_C > + < 2E_A E_D > + < 2E_A E_E > \\ & + < 2E_B E_C > + < 2E_B E_D > + < 2E_B E_E > \\ & + < 2E_C E_D > + < 2E_C E_E > \\ & + < 2E_D E_E > \end{aligned}$$



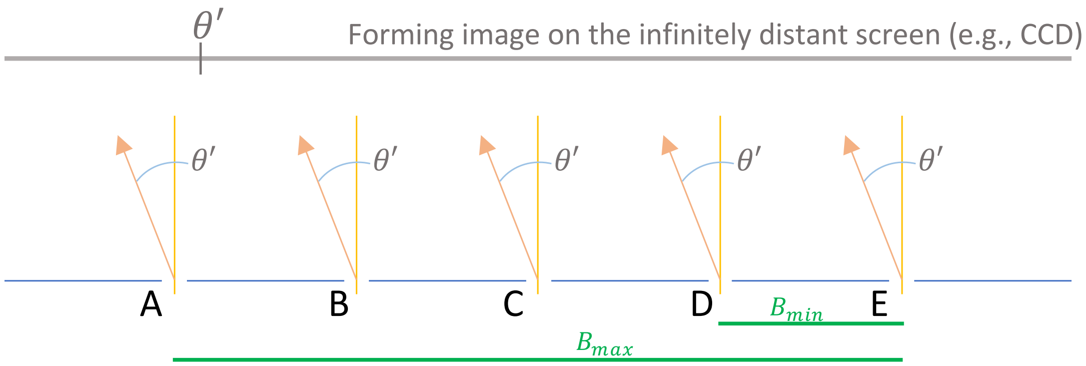
Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > = \overbrace{< E_A{}^2 > + < E_B{}^2 > + < E_C{}^2 > + < E_D{}^2 > + < E_E{}^2 >}^{\text{Auto-correlation}}$$

$$\text{Cross-correlation}\left\{\begin{aligned} &+< 2E_AE_B > + < 2E_AE_C > + < 2E_AE_D > + < 2E_AE_E > \\ &+< 2E_BE_C > + < 2E_BE_D > + < 2E_BE_E > \\ &+< 2E_CE_D > + < 2E_CE_E > \\ &+< 2E_DE_E > \end{aligned}\right.$$



Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

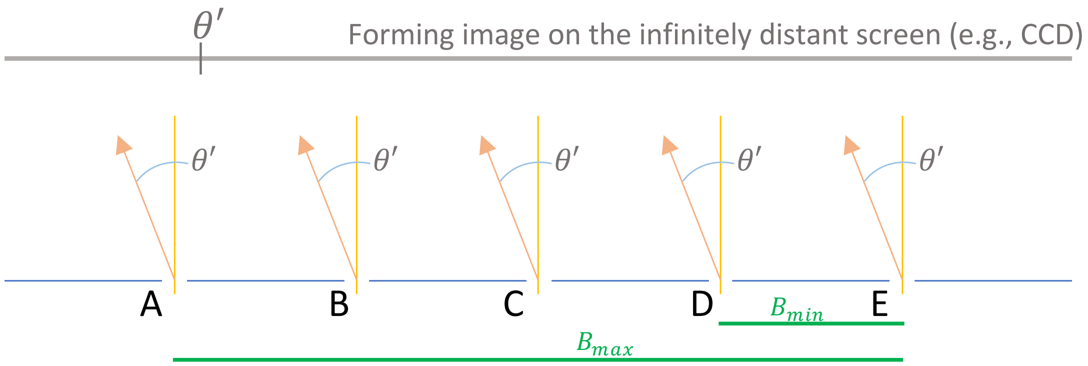
Auto-correlation

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > = \overbrace{< {E_A}^2 > + < {E_B}^2 > + < {E_C}^2 > + < {E_D}^2 > + < {E_E}^2 >}$$

Cross-correlation

$$+ < 2E_AE_B > + < 2E_AE_C > + < 2E_AE_D > + < 2E_AE_E >$$
$$+ < 2E_BE_C > + < 2E_BE_D > + < 2E_BE_E >$$
$$+ < 2E_CE_D > + < 2E_CE_E >$$
$$+ < 2E_DE_E >$$

4 slit-pairs at shortest separation



Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

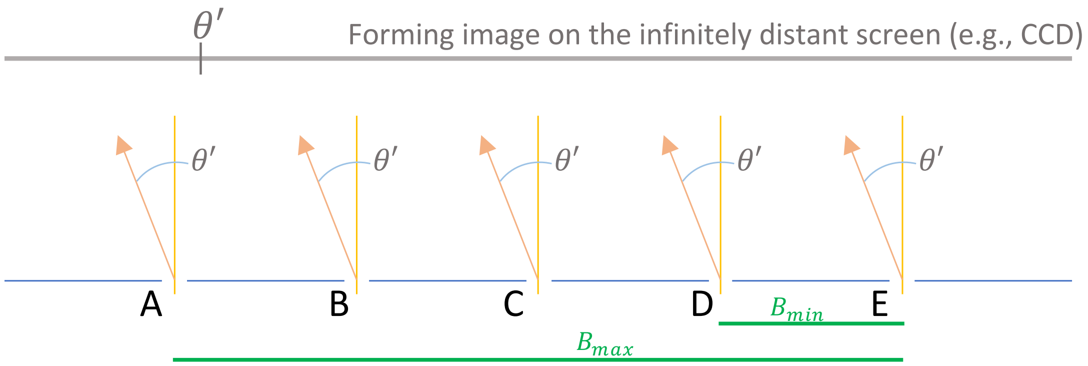
Auto-correlation

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2> =\overbrace{< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 >}$$

Cross-correlation

$$\left\{\begin{array}{l} +< 2E_AE_B >+< 2E_AE_C >+< 2E_AE_D >+< 2E_AE_E > \\ +< 2E_BE_C >+< 2E_BE_D >+< 2E_BE_E > \\ +< 2E_CE_D >+< 2E_CE_E > \\ +< 2E_DE_E > \end{array}\right.$$

3 slit-pairs at the 2nd shortest separation



Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

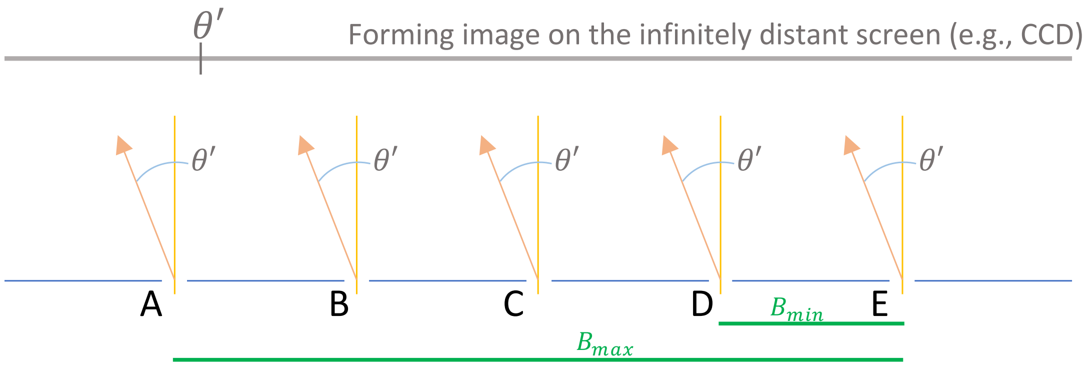
$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > = \overbrace{< {E_A}^2 > + < {E_B}^2 > + < {E_C}^2 > + < {E_D}^2 > + < {E_E}^2 >}^{\text{Auto-correlation}}$$

Cross-correlation

$$+ < 2E_AE_B > + < 2E_AE_C > + \boxed{< 2E_AE_D >} + < 2E_AE_E >$$
$$+ < 2E_BE_C > + < 2E_BE_D > + \boxed{< 2E_BE_E >}$$
$$+ < 2E_CE_D > + < 2E_CE_E >$$
$$+ < 2E_DE_E >$$

2 slit-pairs at the 3rd shortest separation



Evaluation of diffraction pattern

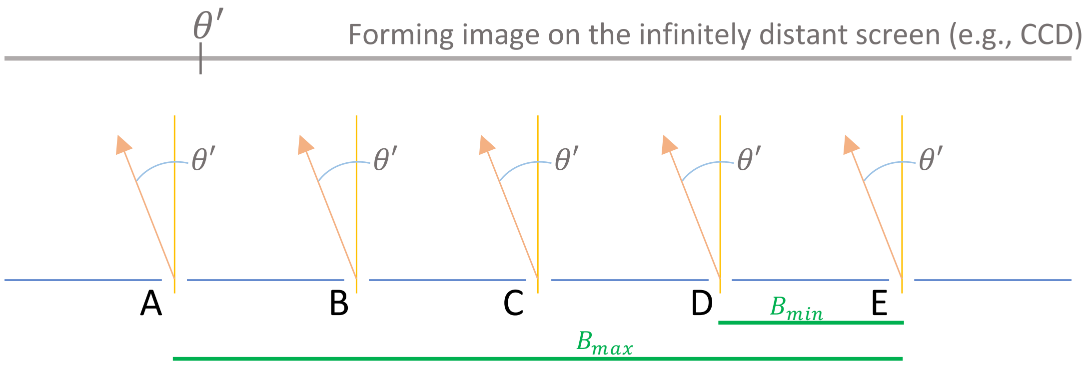
(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

$$\langle (E_A + E_B + E_C + E_D + E_E + \cdots)^2 \rangle = \overbrace{\langle {E_A}^2 \rangle + \langle {E_B}^2 \rangle + \langle {E_C}^2 \rangle + \langle {E_D}^2 \rangle + \langle {E_E}^2 \rangle}^{\text{Auto-correlation}}$$

$$\text{Cross-correlation}\left\{\begin{aligned} &+\langle 2E_AE_B \rangle + \langle 2E_AE_C \rangle + \langle 2E_AE_D \rangle + \boxed{\langle 2E_AE_E \rangle} \\ &+\langle 2E_BE_C \rangle + \langle 2E_BE_D \rangle + \langle 2E_BE_E \rangle \\ &+\langle 2E_CE_D \rangle + \langle 2E_CE_E \rangle \\ &+\langle 2E_DE_E \rangle \end{aligned}\right.$$

1 slit-pair at longest separation



Evaluation of diffraction pattern

(proportional to the number of incoming photons in a unit of time and a unit of area)

$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > = \overbrace{< {E_A}^2 > + < {E_B}^2 > + < {E_C}^2 > + < {E_D}^2 > + < {E_E}^2 >}^{\text{Auto-correlation}}$$

Cross-correlation

$$\left\{\begin{array}{l} + < 2E_A E_B > + < 2E_A E_C > + < 2E_A E_D > + \boxed{< 2E_A E_E >} \\ + < 2E_B E_C > + < 2E_B E_D > + < 2E_B E_E > \\ + < 2E_C E_D > + < 2E_C E_E > \\ + < 2E_D E_E > \end{array}\right.$$

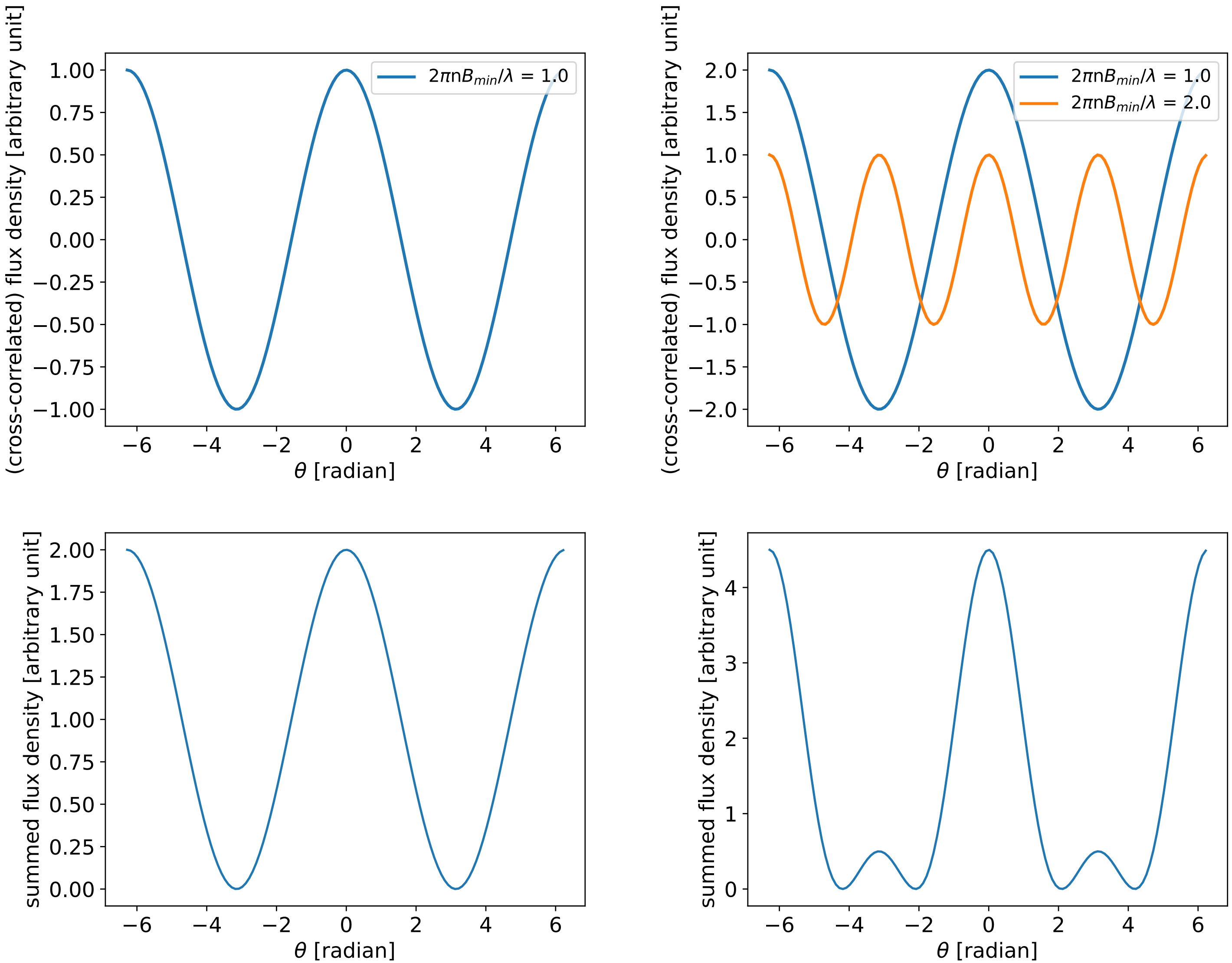
1 slit-pair at longest separation

Form of each cross-term

$$< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) >$$
$$= < \cos(k\Delta x) > + < \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \qquad \Delta x = nB_{min} \sin\theta'$$
$$= \cos\left(\frac{2\pi}{\lambda} nB_{min} \sin\theta'\right) \sim \cos\left(\frac{2\pi}{\lambda} nB_{min}\theta'\right)$$

Trigonometric identity:
$\cos\psi \cos\varphi$
$= \frac{1}{2}[\cos(\psi - \varphi) + \cos(\psi + \varphi)]$

Assuming a diffraction grating with 2 and 3 slits



Flux density:

$\cos(knB_{min}\theta) = \cos\left(2\pi \frac{nB_{min}}{\lambda}\theta\right)$,
$n = 1, 2, 3, 4, 5, \ldots$, (N-1)

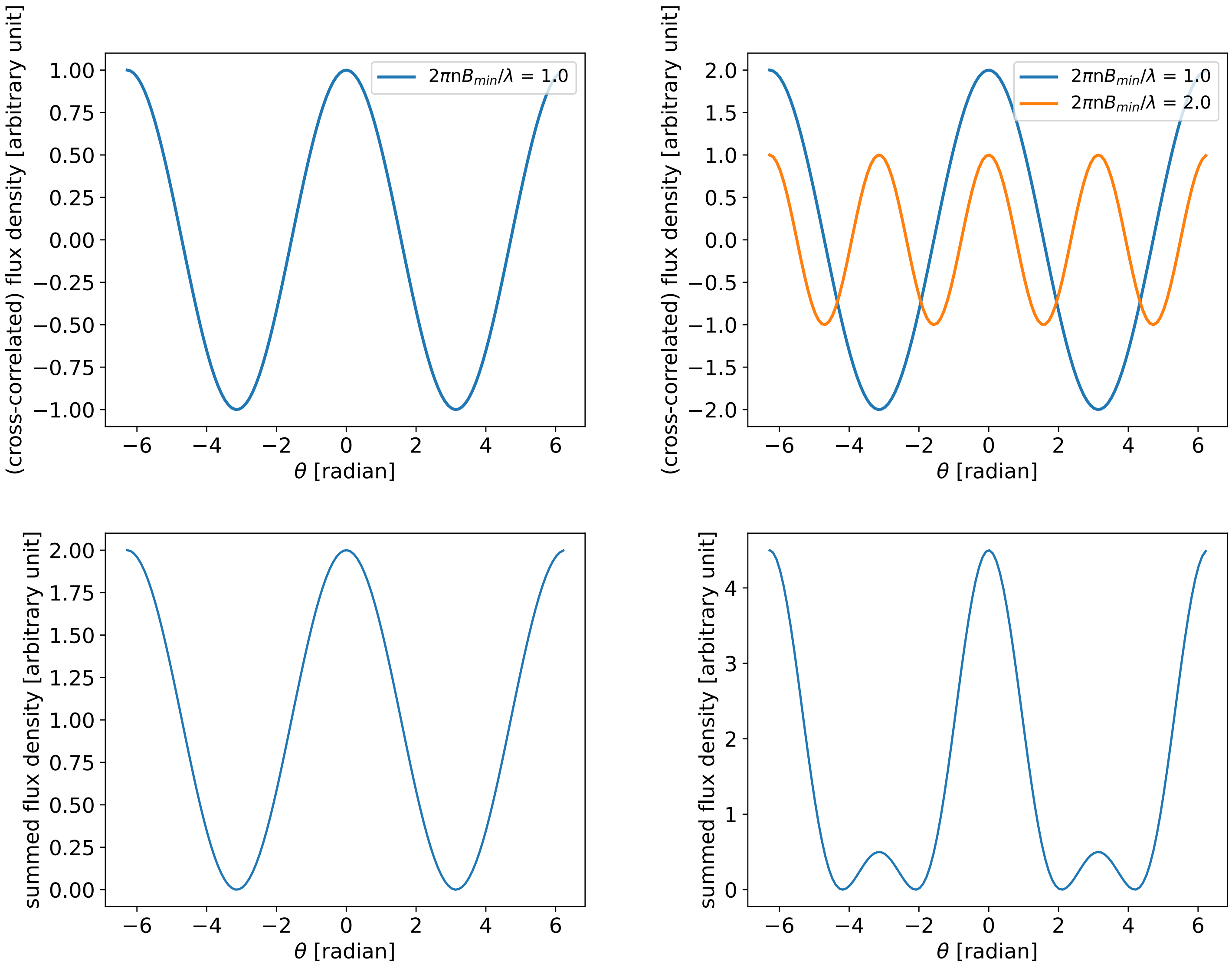
Summed flux density:

$\sum_n (N-n)\cos(knB_{min}\theta)$ + auto-corr.

Properties:

1. For any n, $\theta = 0$ is always the location of the central maximum (i.e., constructive interference)

Assuming a diffraction grating with 2 and 3 slits



Flux density:

$\cos(knB_{min}\theta) = \cos\left(2\pi \frac{nB_{min}}{\lambda}\theta\right)$,
$n = 1, 2, 3, 4, 5, \ldots$, (N-1)

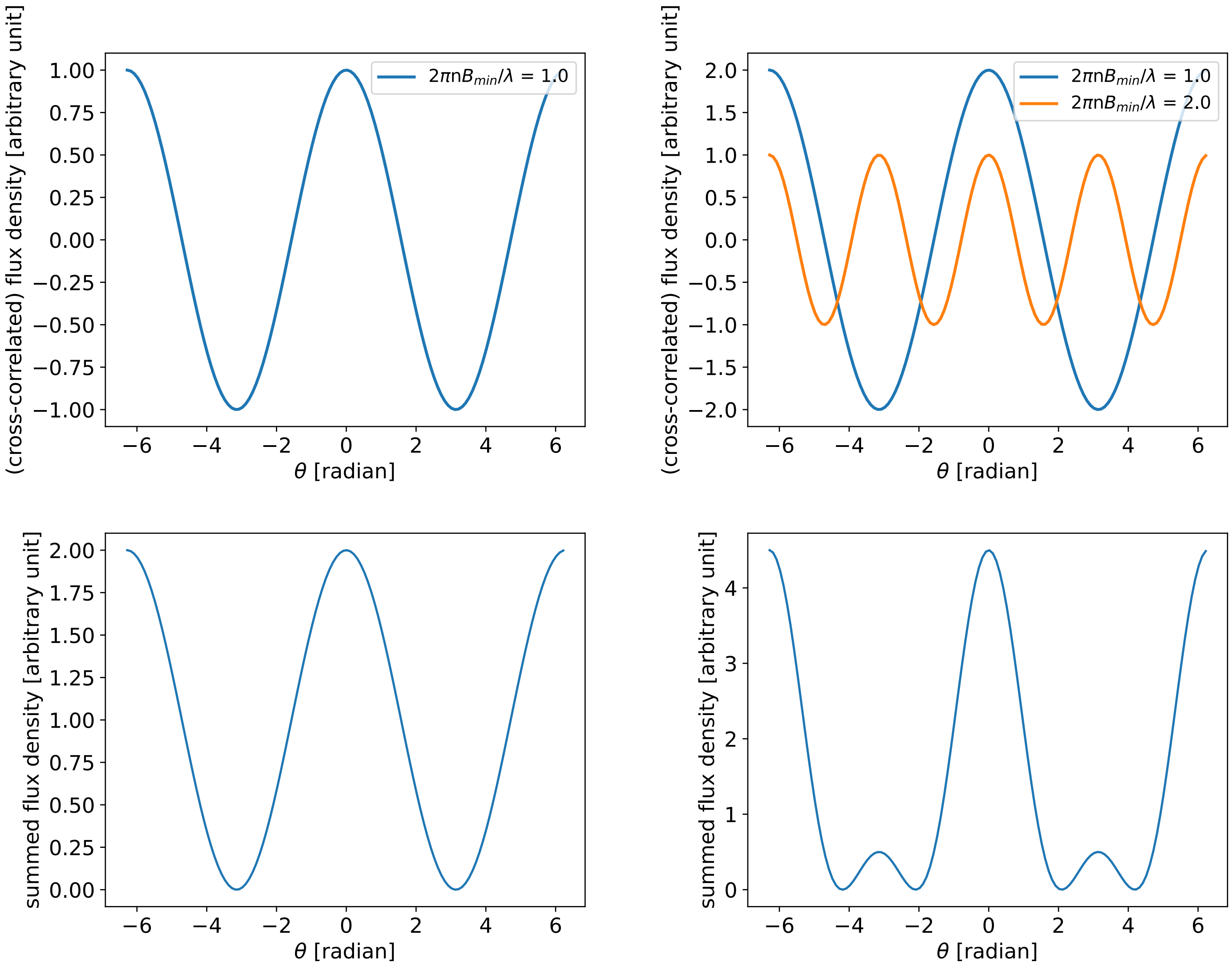
Summed flux density:

$\sum_n (N-n)\cos(knB_{min}\theta)$ + auto-corr.

Properties:

1. For any n, $\theta = 0$ is always the location of the central maximum (i.e., constructive interference)
2. The locations of the maxima for the $n = 1$ slit-pairs are always the locations of the maxima for any other pairs of slits.

Assuming a diffraction grating with 2 and 3 slits



Flux density:

$\cos(knB_{min}\theta) = \cos\left(2\pi \frac{nB_{min}}{\lambda}\theta\right)$,
$n = 1, 2, 3, 4, 5, \ldots$, (N-1)

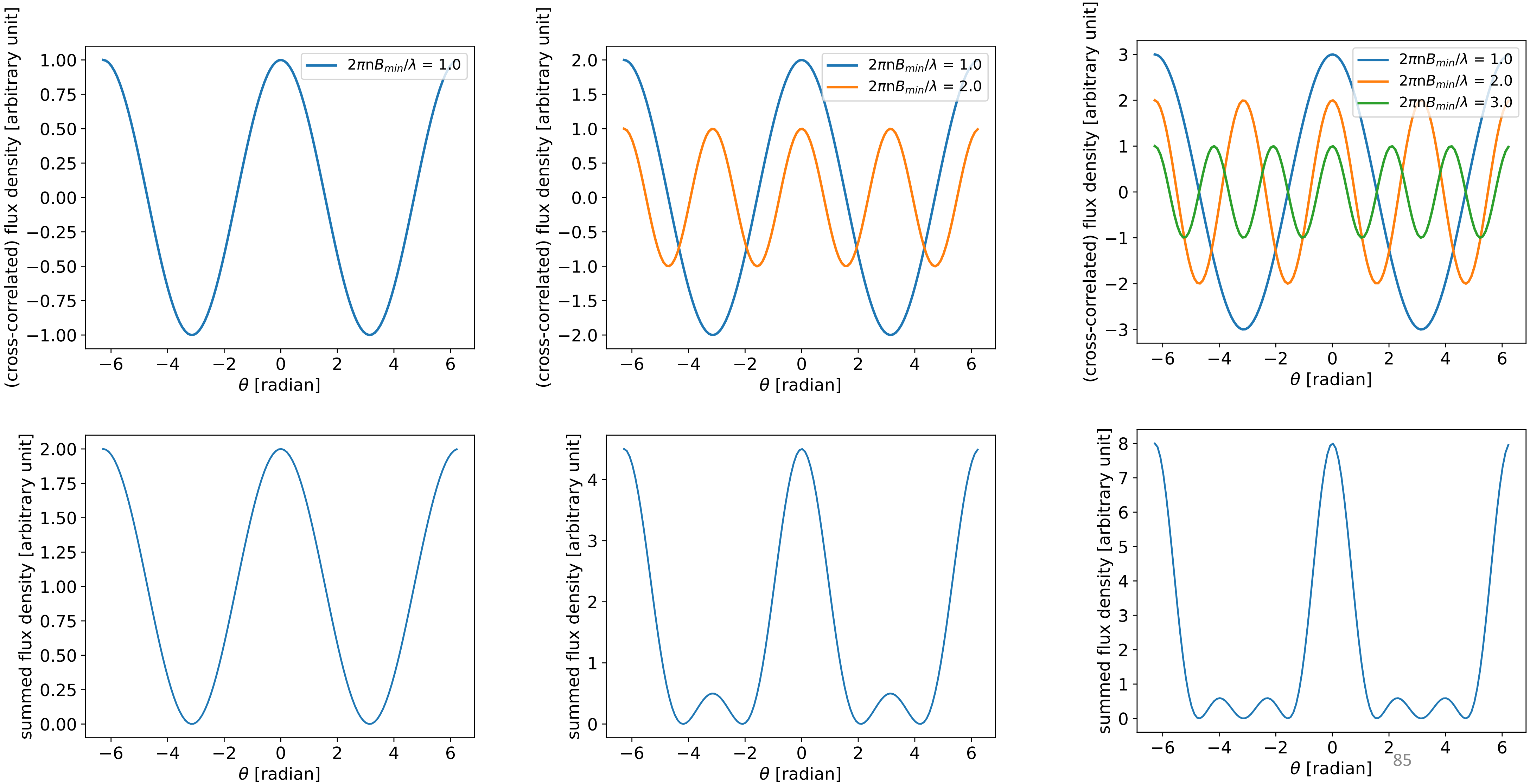
Summed flux density:

$\sum_n (N-n)\cos(knB_{min}\theta)$ + auto-corr.

Properties:

1. For any n, $\theta = 0$ is always the location of the central maximum (i.e., constructive interference)
2. The locations of the maxima for the $n = 1$ slit-pairs are always the locations of the maxima for any other pairs of slits.
3. Between the maxima of the n=1 pairs of slits, the destructive interference makes the photon flux density approach 0. (i.e., the cross-correlation terms cancel each other).

Assuming a diffraction grating with 2, 3, and 4 slits



Assuming a diffraction grating with 2, 3, and 10 slits

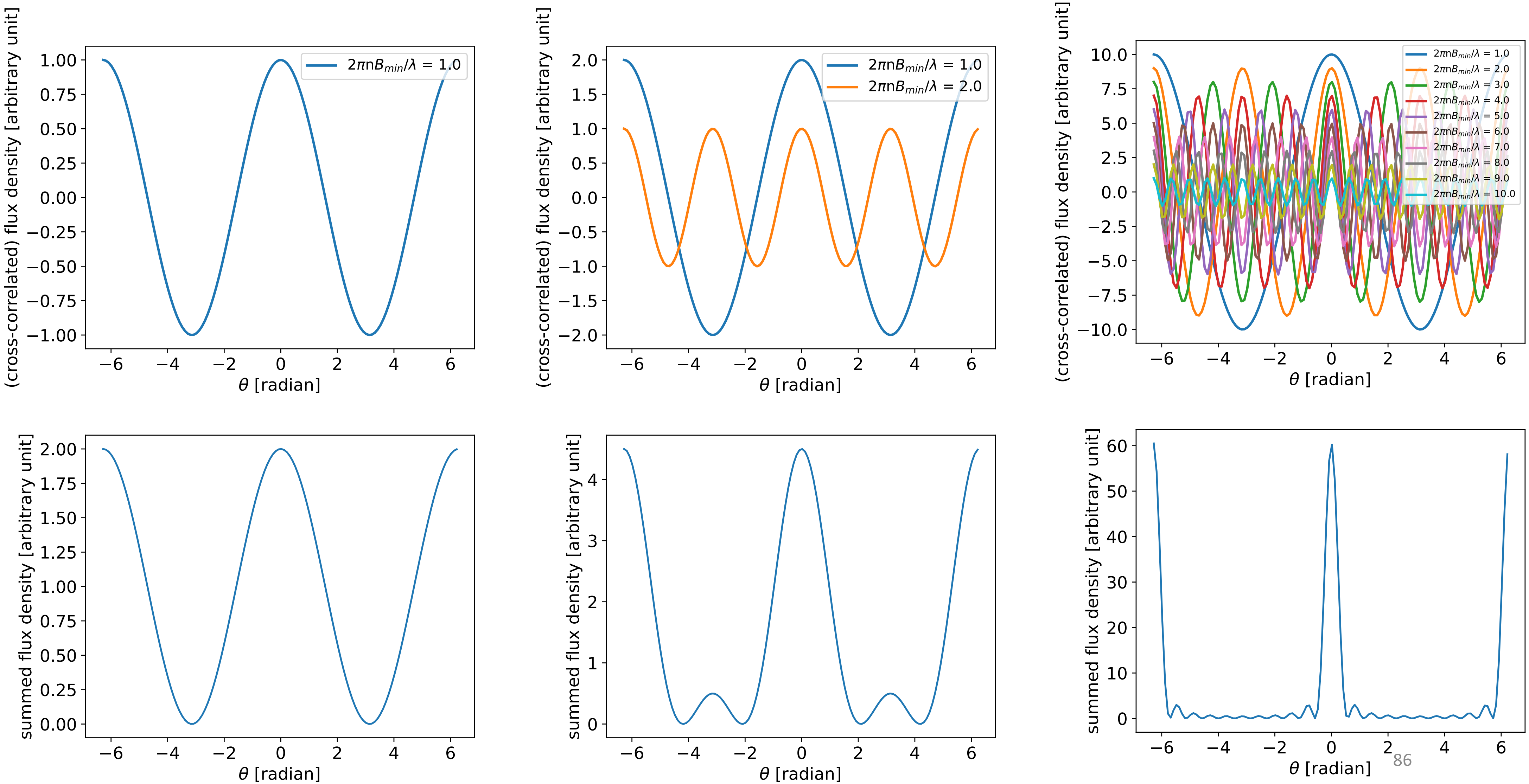
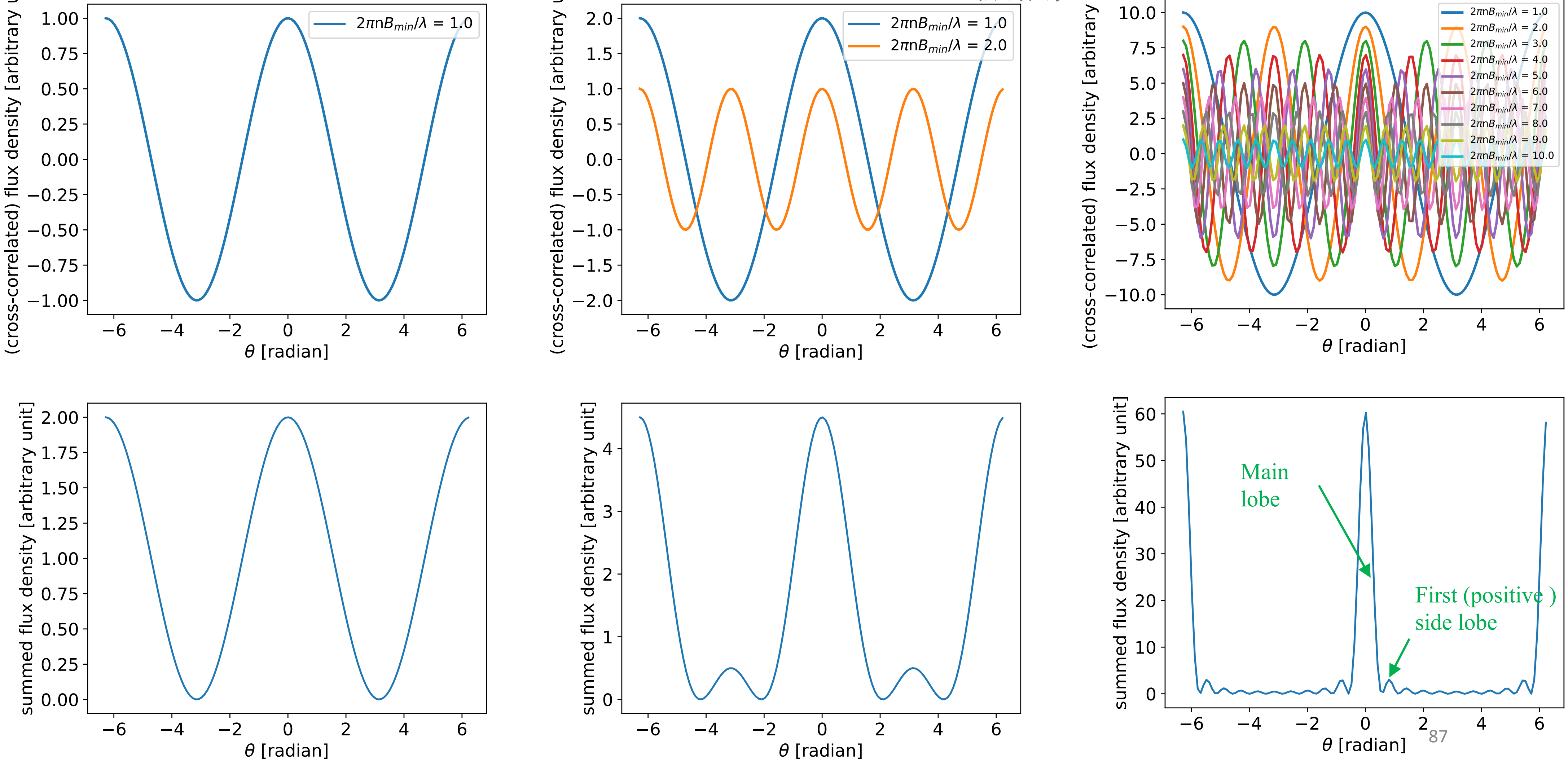
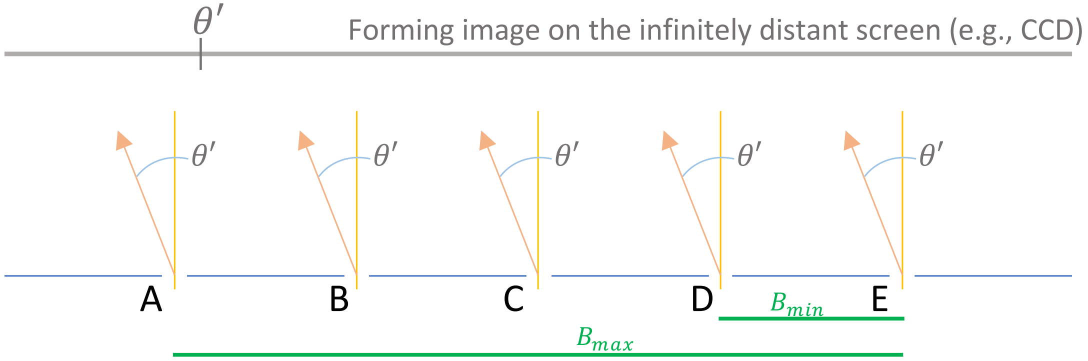
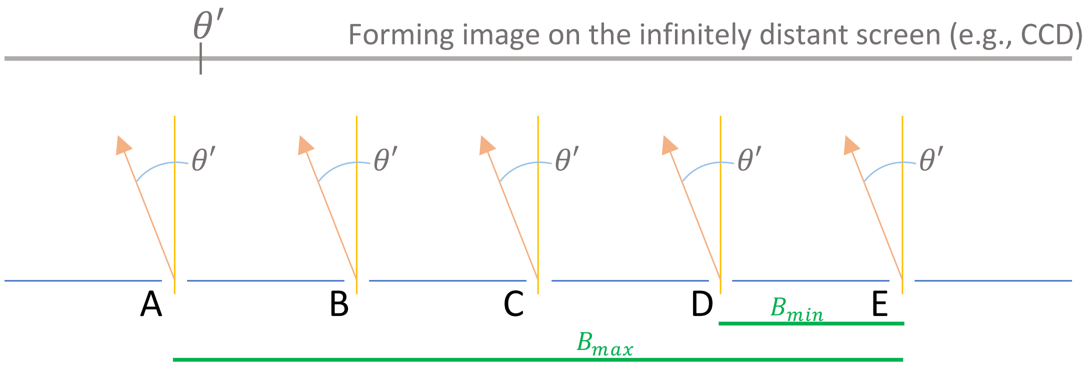


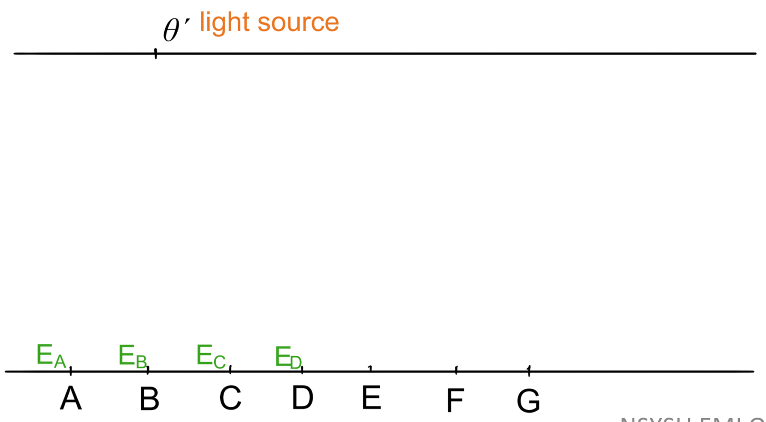
Image is formed by co-adding $< 2E_iE_j >$ (什麼是 $< 2E_iE_j >$? 空間方向cosine函式具特定波長及振幅)

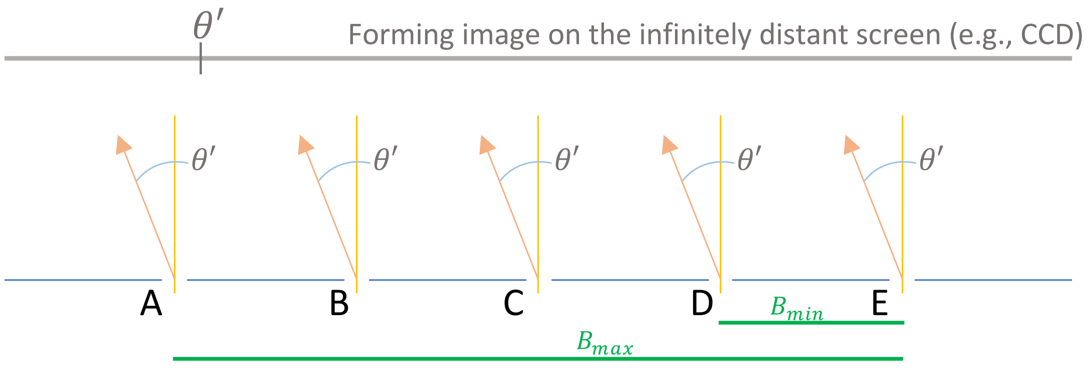



θ'
Forming image on the infinitely distant screen (e.g., CCD)
θ'
θ'
θ'
θ'
θ'
A
B
C
D
B_{min}
E
B_{max}

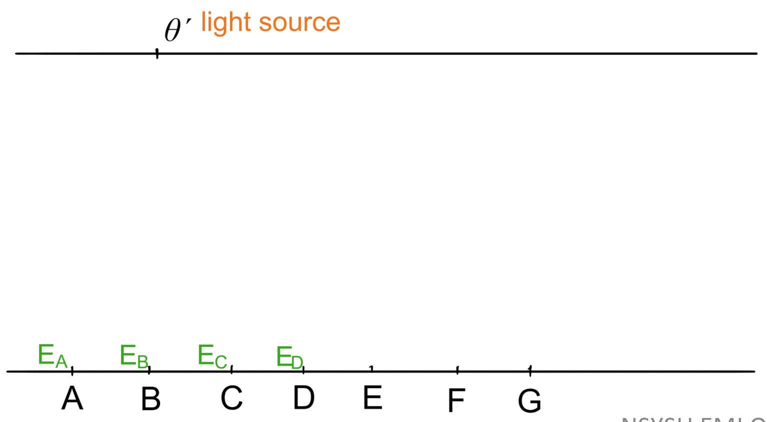


Interferometric array observation towards a infinitely distant, monochromatic source





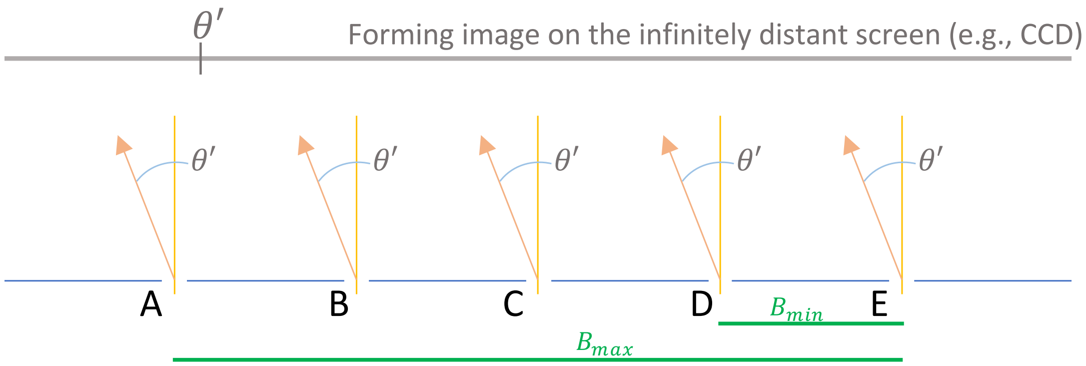
Interferometric array observation towards a infinitely distant, monochromatic source



$$E_A = E_0\cos(kx - \omega t + \phi_0)$$

With correlators, we can evaluate each of the following terms:

$$
\begin{aligned}
&< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > \\
&=< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 > \\
&\quad +< 2E_AE_B >+< 2E_AE_C >+< 2E_AE_D >+< 2E_AE_E > \\
&\quad +< 2E_BE_C >+< 2E_BE_D >+< 2E_BE_E > \\
&\quad +< 2E_CE_D >+< 2E_CE_E > \\
&\quad +< 2E_DE_E >
\end{aligned}
$$

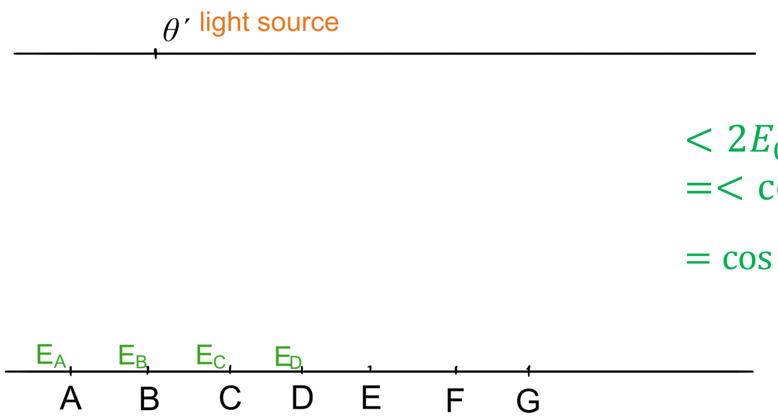


Interferometric array observation towards a infinitely distant, monochromatic source

$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

With correlators, we can evaluate each of the following terms:

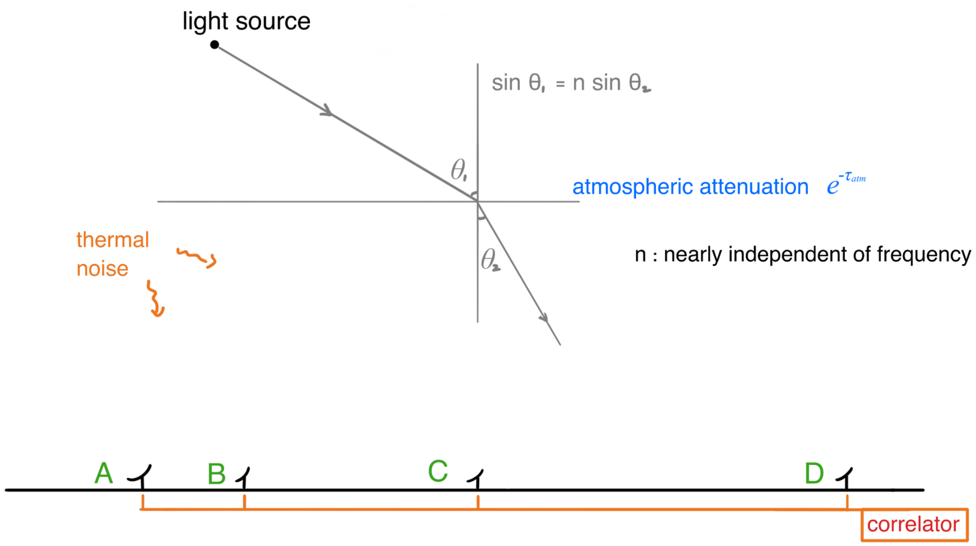
$$< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 >$$
$$=< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 >$$
$$+< 2E_A E_B >+< 2E_A E_C >+< 2E_A E_D >+< 2E_A E_E >$$
$$+< 2E_B E_C >+< 2E_B E_D >+< 2E_B E_E >$$
$$+< 2E_C E_D >+< 2E_C E_E >$$
$$+< 2E_D E_E >$$



Form of each cross-term

$$< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) >$$
$$=< \cos(k\Delta x) >+< \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \qquad \Delta x = nB_{min} \sin\theta'$$
$$= \cos\left(\frac{2\pi}{\lambda} nB_{min} \sin\theta'\right) \sim \cos\left(\frac{2\pi}{\lambda} nB_{min}\theta'\right)$$

1D linear radio interferometer



$E_A = E_0 \cos(kx - \omega t + \phi_0)$

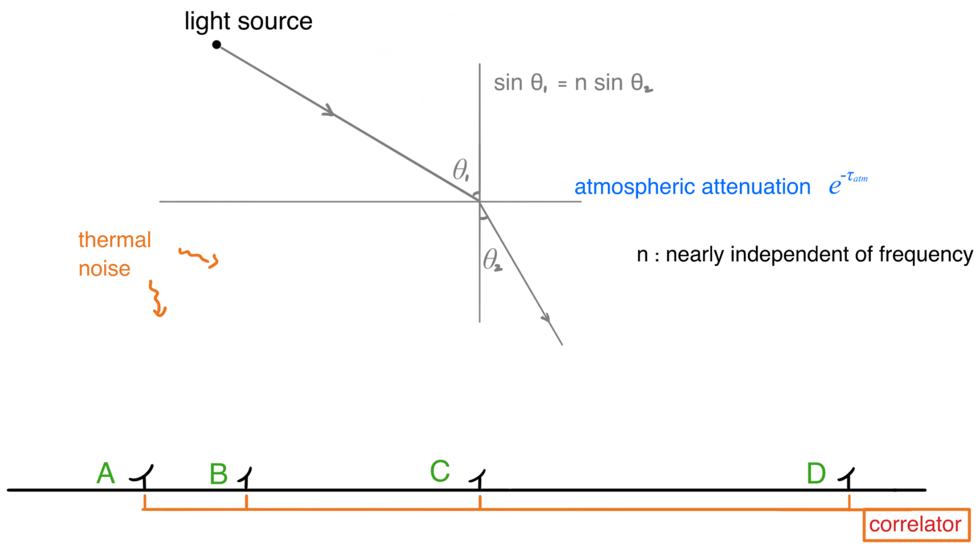
With correlators, we can evaluate each of the following terms:

$$
\begin{aligned}
&< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > \\
&=< {E_A}^2 >+< {E_B}^2 >+< {E_C}^2 >+< {E_D}^2 >+< {E_E}^2 > \\
&\quad +< 2E_A E_B >+< 2E_A E_C >+< 2E_A E_D >+< 2E_A E_E > \\
&\quad +< 2E_B E_C >+< 2E_B E_D >+< 2E_B E_E > \\
&\quad +< 2E_C E_D >+< 2E_C E_E > \\
&\quad +< 2E_D E_E >
\end{aligned}
$$

General expression for the cross (cosine) correlation of antennae i and j:

$$
\begin{aligned}
&< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) > \\
&=< \cos(k\Delta x) >+< \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \qquad \Delta x = B_{ij} \sin\theta' \\
&= \cos\left(\frac{2\pi}{\lambda} B_{ij} \sin\theta'\right) \sim \cos\left(\frac{2\pi}{\lambda} B_{ij} \theta'\right)
\end{aligned}
$$

1D linear radio interferometer



$$E_A = E_0 \cos(kx - \omega t + \phi_0)$$

With correlators, we can evaluate each of the following terms:

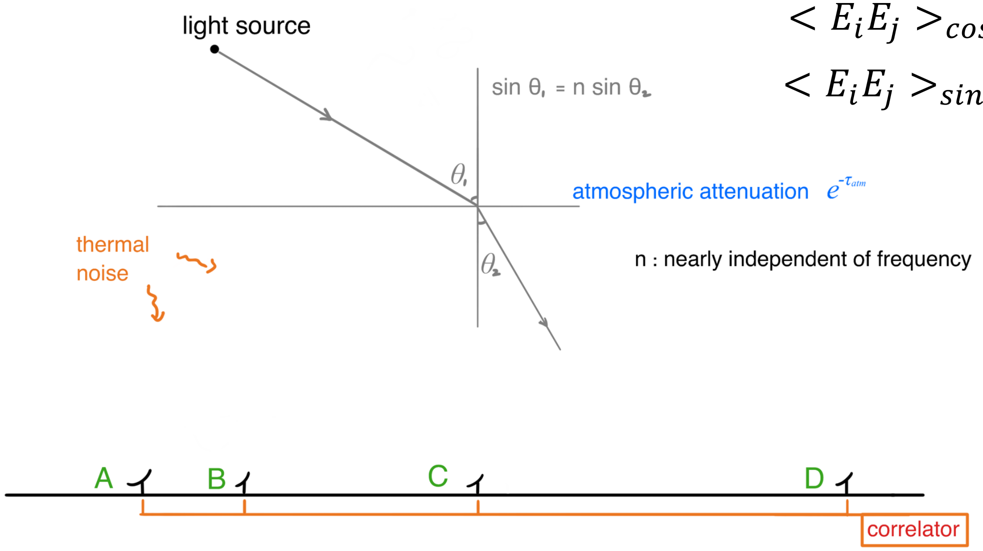
$$\begin{aligned}
&< (E_A + E_B + E_C + E_D + E_E + \cdots)^2 > \\
&= < {E_A}^2 > + < {E_B}^2 > + < {E_C}^2 > + < {E_D}^2 > + < {E_E}^2 > \\
&\quad + < 2E_AE_B > + < 2E_AE_C > + < 2E_AE_D > + < 2E_AE_E > \\
&\quad + < 2E_BE_C > + < 2E_BE_D > + < 2E_BE_E > \\
&\quad + < 2E_CE_D > + < 2E_CE_E > \\
&\quad + < 2E_DE_E >
\end{aligned}$$

General expression for the cross (cosine) correlation of antennae i and j:

$$\begin{aligned}
&< 2{E_0}^2 \cos(kx - \omega t + \phi_0) \cos(k(x + \Delta x) - \omega t + \phi_0) > \\
&= < \cos(k\Delta x) > + < \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \\
&= \cos\left(\frac{2\pi}{\lambda} B_{ij} \sin\theta'\right) \sim \cos\left(2\pi \frac{B_{ij}}{\lambda} \theta'\right) \equiv \cos(2\pi u_{ij} \theta')
\end{aligned}$$

$$\Delta x = \underbrace{B_{ij}}_{\text{Separation of antennae i and j}} \sin\theta'$$

1D linear radio interferometer



Evaluating Complex visibility of a point source at θ' (i.e., Dirac delta function) taken from antennae i and j

$$< E_i E_j >_{cos} \cong \tilde{P}(\theta') \cos\left(\frac{2\pi}{\lambda} B_{ij}\theta'\right) = \int \delta(\theta-\theta')\, \tilde{P}(\theta)\cos\left(2\pi u_{ij}\theta\right) d\theta$$

$$< E_i E_j >_{sin} \cong \tilde{P}(\theta') \sin\left(\frac{2\pi}{\lambda} B_{ij}\theta'\right) = \int \delta(\theta-\theta')\, \underbrace{\tilde{P}(\theta)}_{\text{Response function of each single-dish reflector}}\sin\left(2\pi u_{ij}\theta\right) d\theta$$

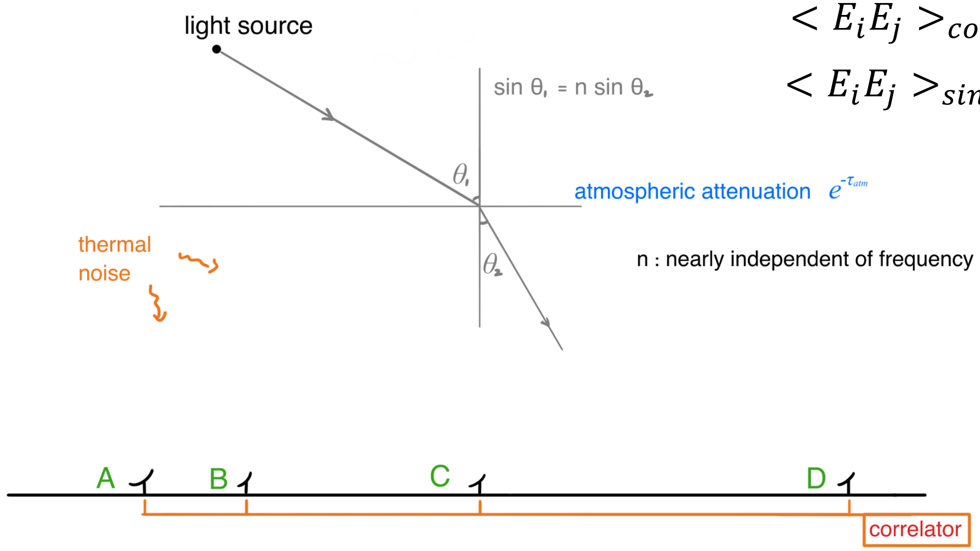
General expression for the cross (cosine) correlation of antennae i and j:

$$< 2{E_0}^2 \cos(kx - \omega t + \phi_0)\cos(k(x+\Delta x) - \omega t + \phi_0) >$$

$$=< \cos(k\Delta x) >+< \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >, \qquad \Delta x = \underbrace{B_{ij}}_{\text{Separation of antennae i and j}} \sin\theta'$$

$$= \cos\left(\frac{2\pi}{\lambda} B_{ij} \sin\theta'\right) \sim \cos\left(2\pi \frac{B_{ij}}{\lambda}\theta'\right) \equiv \cos(2\pi u_{ij}\theta')$$

1D linear radio interferometer



Evaluating Complex visibility of a point source at θ' (i.e., Dirac delta function) taken from antennae i and j

$$< E_i E_j >_{cos} \cong \tilde{P}(\theta') \cos\left(\frac{2\pi}{\lambda} B_{ij}\theta'\right) = \int \delta(\theta-\theta')\,\tilde{P}(\theta)\cos\left(2\pi u_{ij}\theta\right) d\theta$$

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General expression for the cross (cosine) correlation of antennae i and j:

$$< 2{E_0}^2 \cos(kx - \omega t + \phi_0)\cos(k(x+\Delta x) - \omega t + \phi_0) >$$
$$=< \cos(k\Delta x) > + < \cos(2kx - 2\omega t + 2\phi_0 + k\Delta x) >,$$
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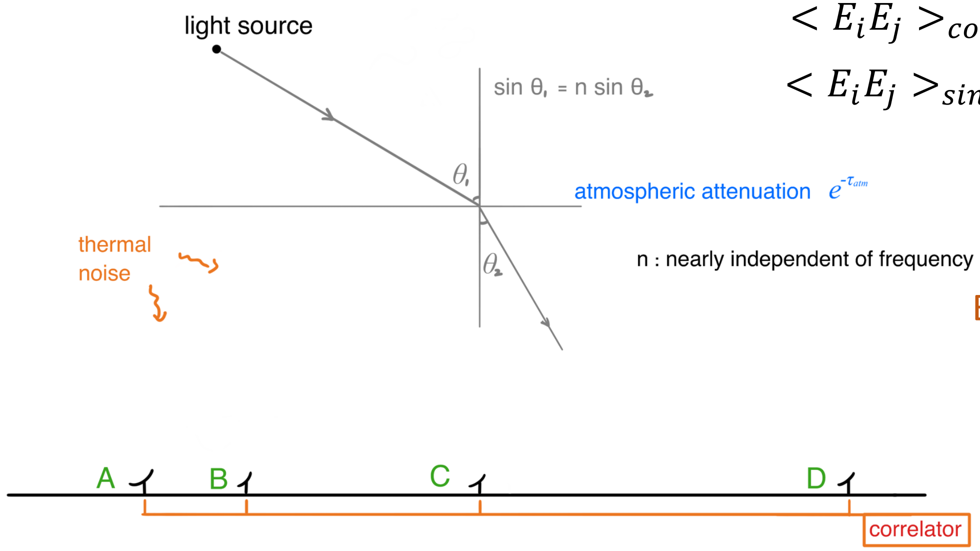
$$\Delta x = \underbrace{B_{ij}}_{\text{Separation of antennae i and j}} \sin\theta'$$

1D linear radio interferometer



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$$V_{ij} = \int \delta(\theta - \theta') \widetilde{P(\theta)} e^{i2\pi u_{ij}\theta} d\theta$$

Evaluating Complex visibility for continuous intensity distribution

$$V_{ij} = \int A(\theta) \widetilde{P(\theta)} e^{i2\pi u_{ij}\theta} d\theta$$

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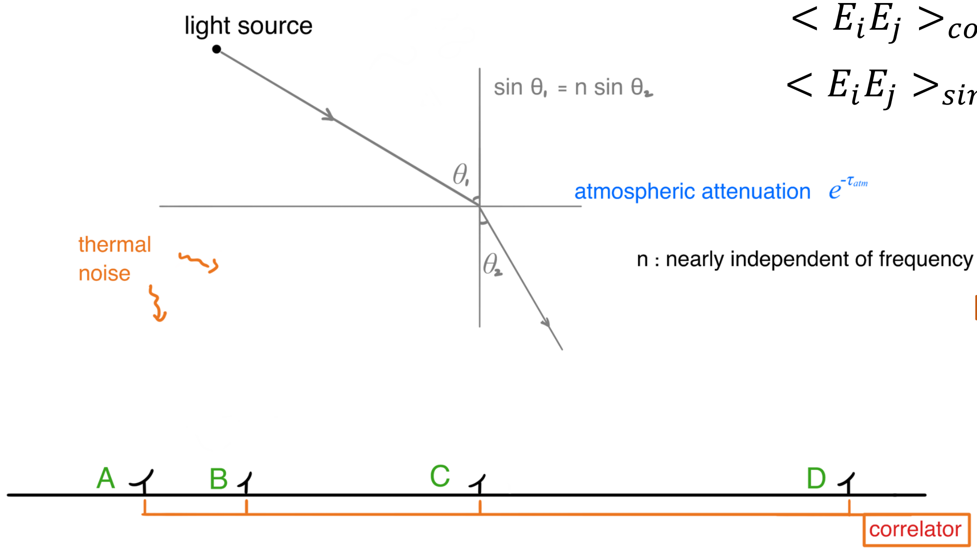
1D linear radio interferometer



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Evaluating Complex visibility for continuous intensity distribution

$$V_{ij} = \underbrace{\int A(\theta)\widetilde{P(\theta)} e^{i2\pi u_{ij}\theta} d\theta}_{\text{Mathematical form of fourier transform}}$$

Mathematical form of fourier transform

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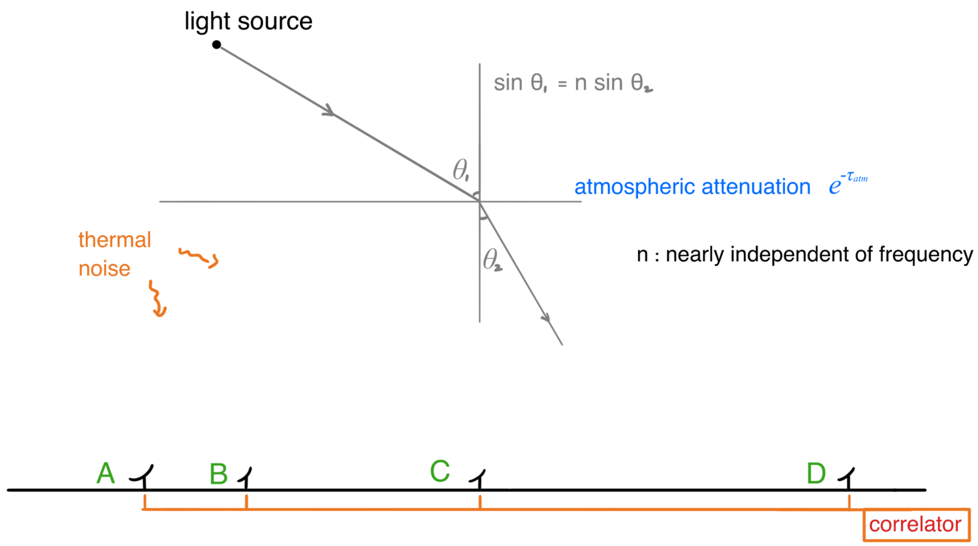
Separation of antennae i and j

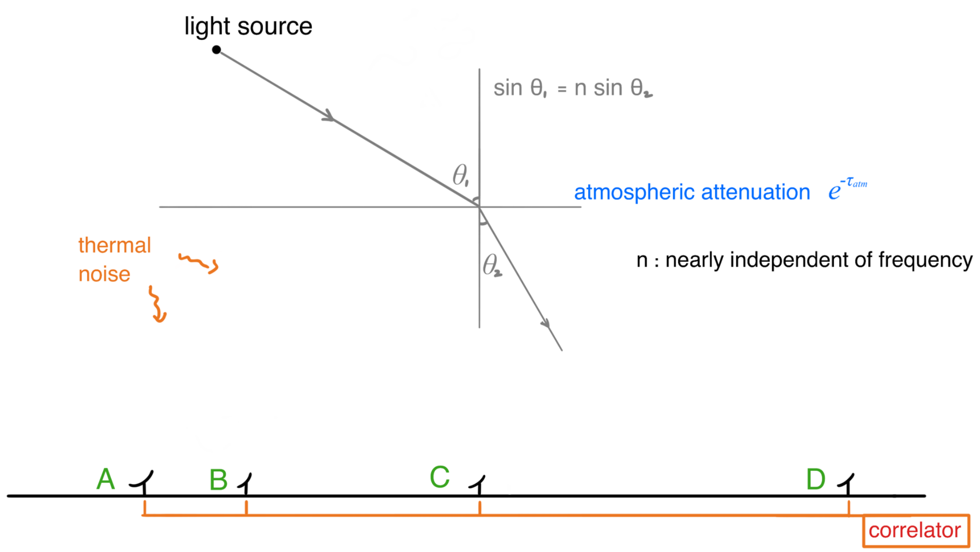
1D linear radio interferometer



Evaluating Complex visibility for continuous intensity distribution

$$V_{ij} = \int A(\theta)\widetilde{P(\theta)}e^{i2\pi u_{ij}\theta}d\theta$$

1D linear radio interferometer

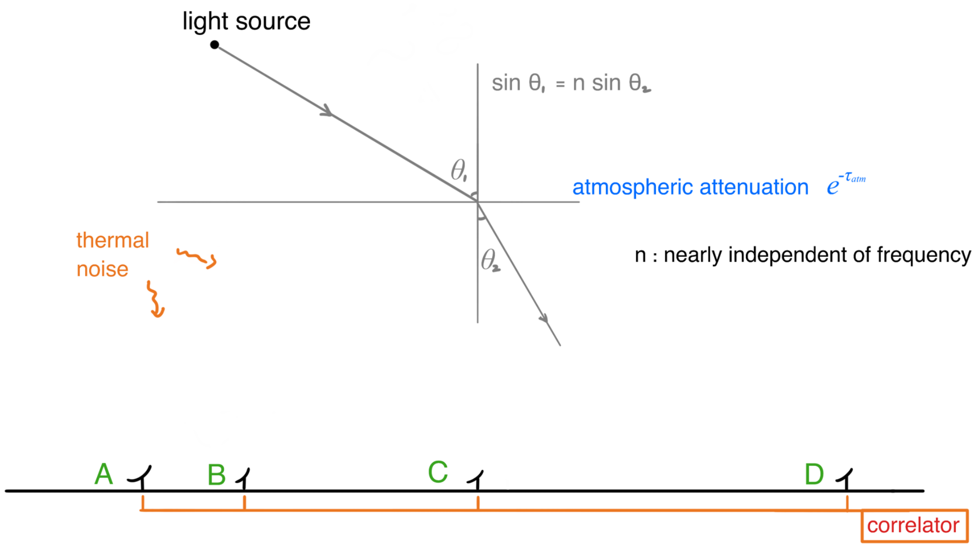


Evaluating Complex visibility for continuous intensity distribution

$$\boxed{V(u)} = \underbrace{\int A(\theta)\widetilde{P(\theta)}e^{i2\pi u\theta}d\theta}_{\text{Mathematical form of fourier transform}}$$

What we directly measure

1D linear radio interferometer



Evaluating Complex visibility for continuous intensity distribution

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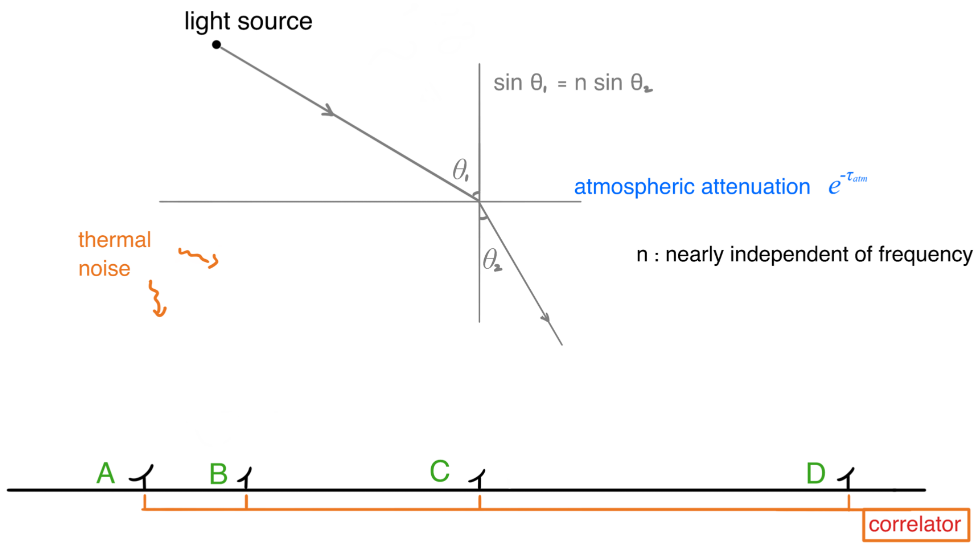
Mathematical form of fourier transform

What we are interested in

1D linear radio interferometer



Evaluating Complex visibility for continuous intensity distribution

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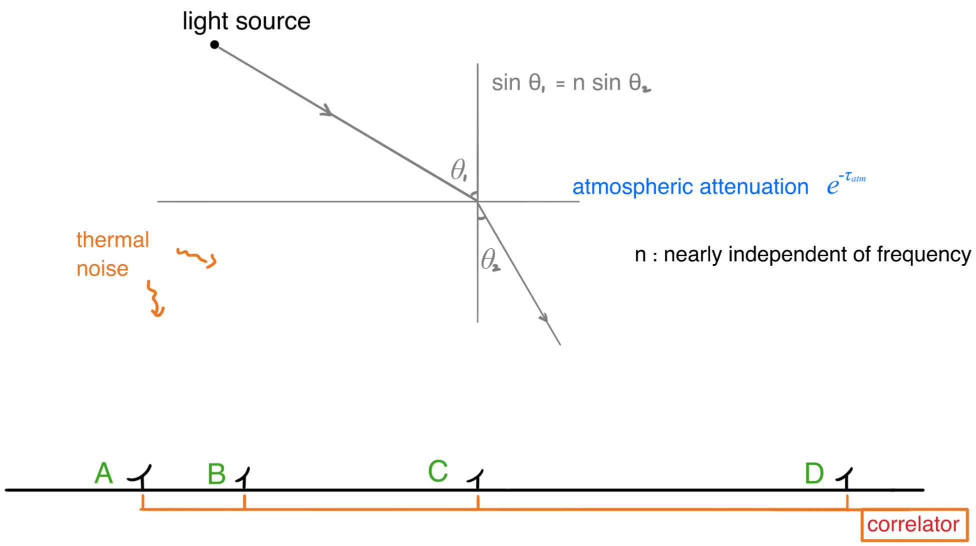
Mathematical form of fourier transform

Inverse fourier transform

$$A(\theta')\widetilde{P(\theta')} = \int V(u) e^{-i2\pi u\theta'} du$$

If $V(u)$ can be completely sampled
(i.e., measured at any u)

1D linear radio interferometer



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Evaluating Complex visibility for continuous intensity distribution

$$V(u) = \underbrace{\int \boxed{A(\theta)\widetilde{P(\theta)}}\, e^{i2\pi u\theta}\, d\theta}_{\textbf{Mathematical form of fourier transform}}$$

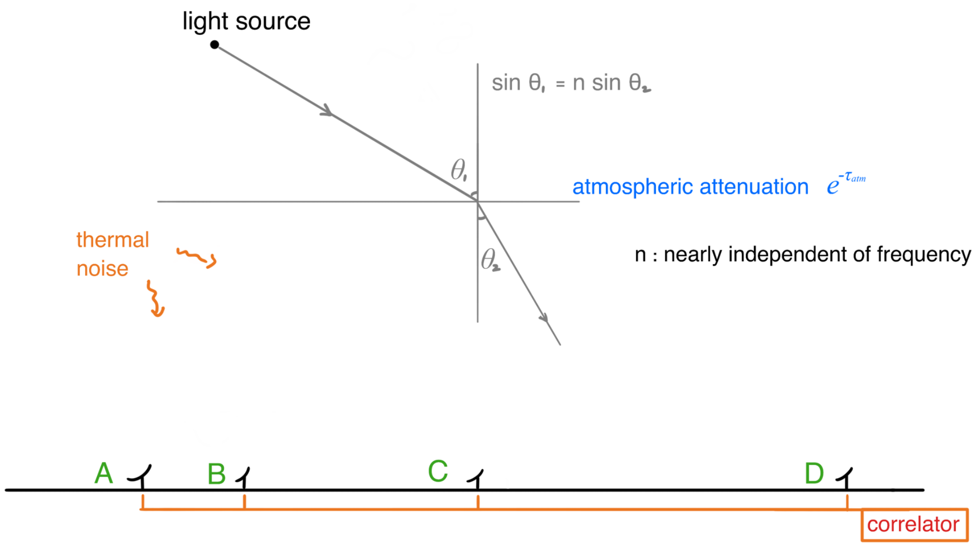
Inverse fourier transform

$$A(\theta')\widetilde{P(\theta')} = \int V(u) e^{-i2\pi u\theta'} du$$

Requiring measurements at arbitrary antenna-separations, even at infinitely small antenna-separations.

In reality, an interferometric array can only take discretized samples at certain antenna-separations. In addition, the shortest possible (projected) antenna-separation is limited by the sizes of the reflector.

1D linear radio interferometer



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Evaluating Complex visibility for continuous intensity distribution

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Mathematical form of fourier transform

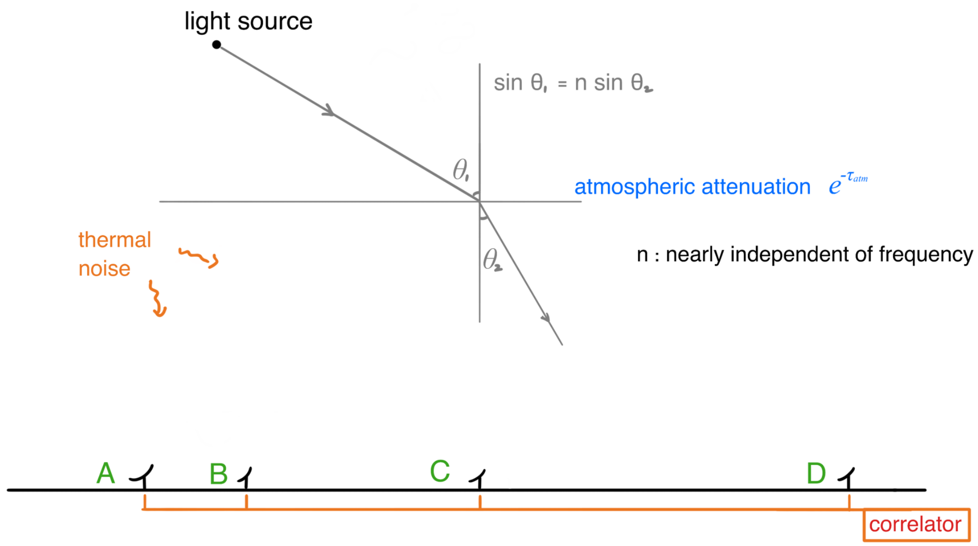
Inverse fourier transform

$$A(\theta')\widetilde{P(\theta')} = \int V(u) e^{-i2\pi u\theta'} du$$

Sampling function (at $u_k, k = 1,2,3,\ldots, M$)

$$S(u) \equiv \sum_{k=1}^{M} \delta(u - u_k)$$

1D linear radio interferometer



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Evaluating Complex visibility for continuous intensity distribution

$$V(u) = \underbrace{\int \boxed{A(\theta)\widetilde{P(\theta)}} e^{i2\pi u\theta} d\theta}_{}$$

Mathematical form of fourier transform

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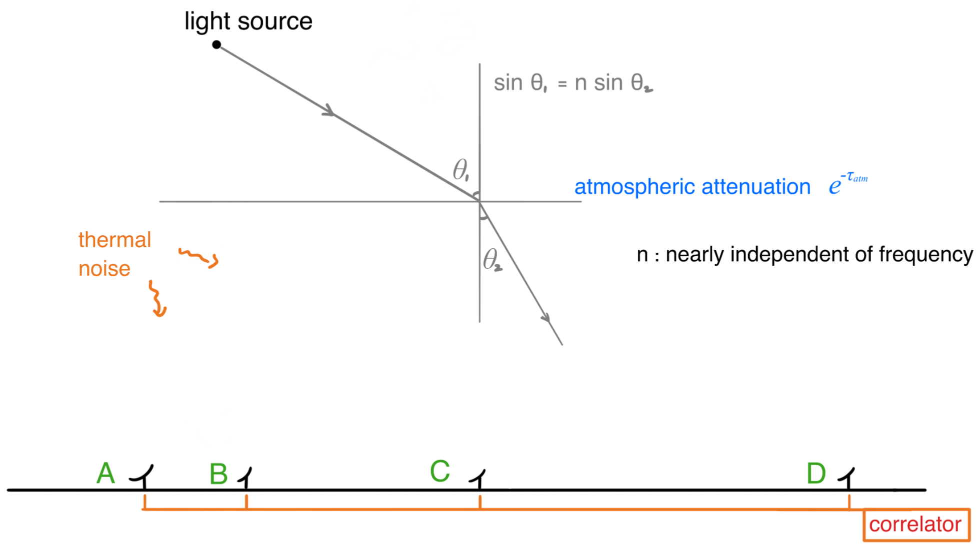
Naiive inverse fourier transform

$$\left[A(\theta')\widetilde{P(\theta')}\right]^D \equiv \int S(u)V(u) e^{-i2\pi u\theta'} du$$

1D linear radio interferometer



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Naiive inverse fourier transform

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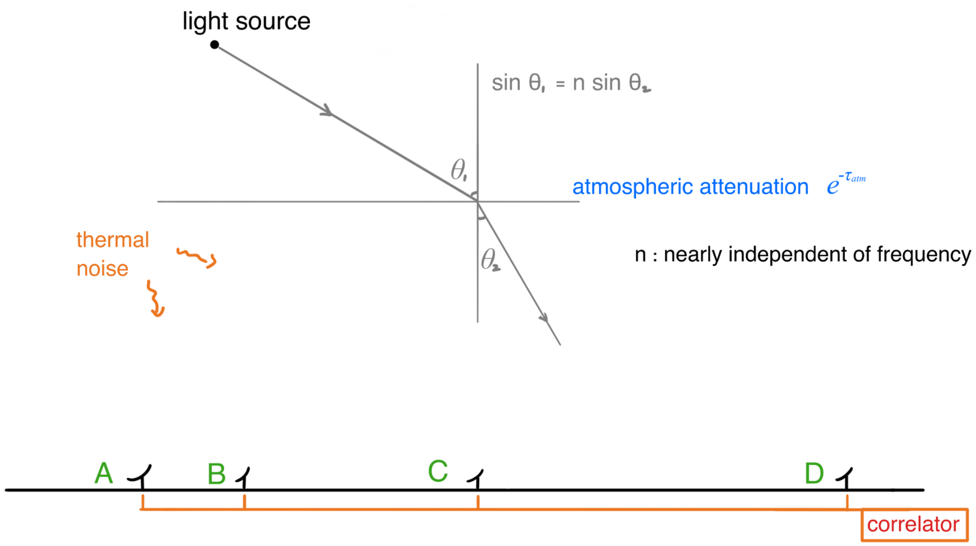
Convolution theorem

$$F.T.(f * g) = F.T.(f) \cdot F.T.(g)$$

$$F.T.(f \cdot g) = F.T.(f) * F.T.(g)$$

1D linear radio interferometer



Evaluating Complex visibility for continuous intensity distribution

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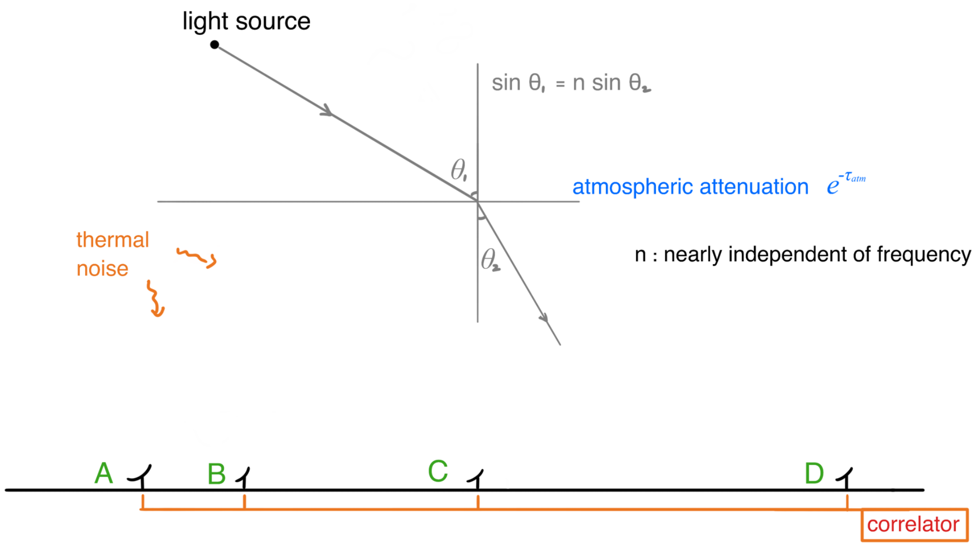

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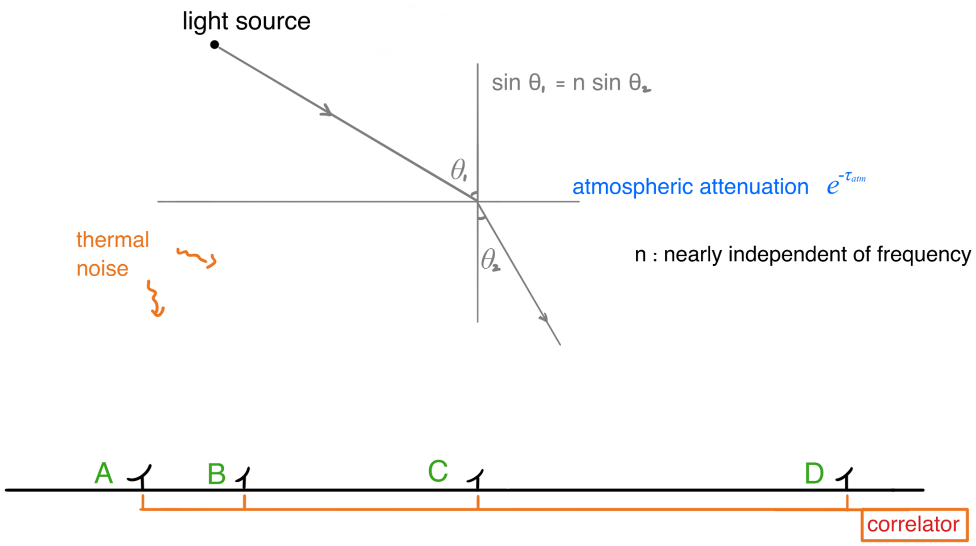

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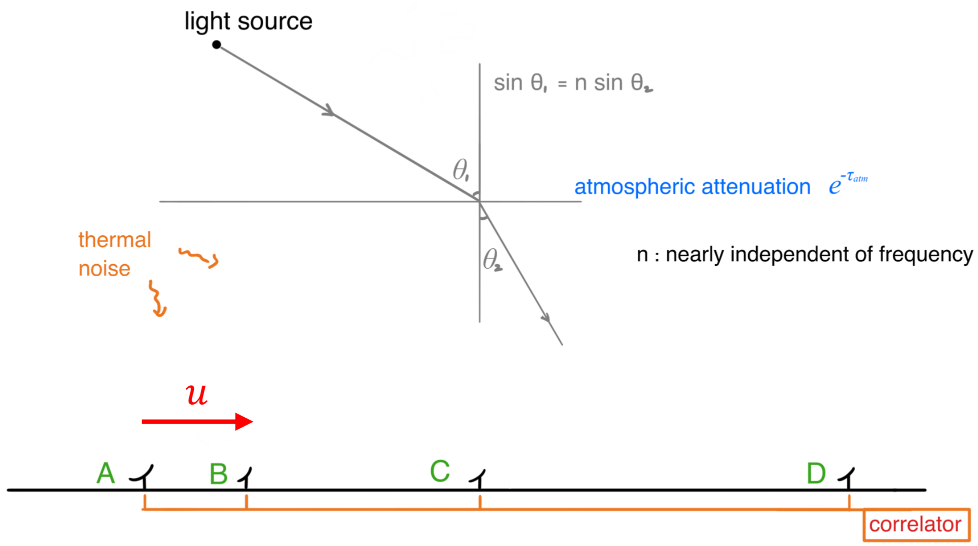

Evaluating Complex visibility for continuous intensity distribution

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Dirty beam $\quad A(\theta')\widetilde{P(\theta')}$

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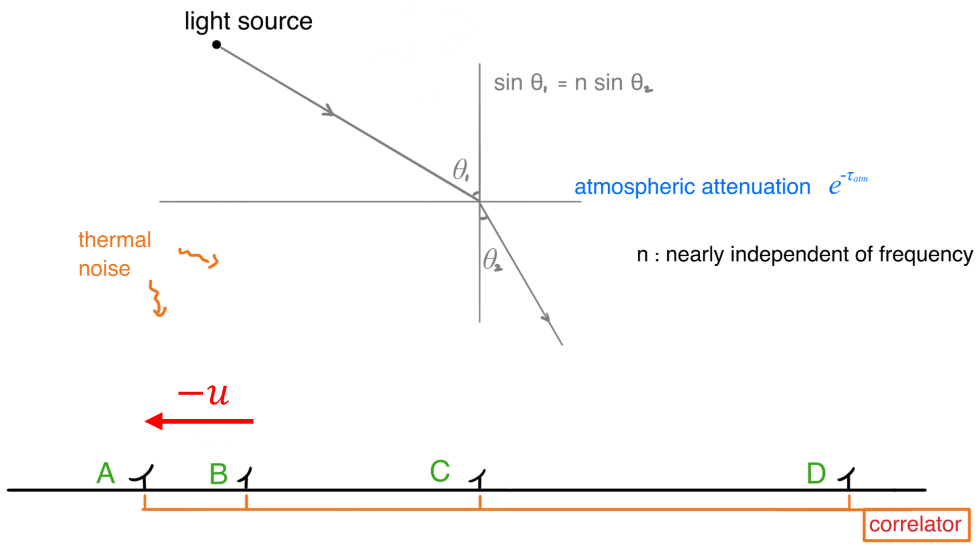
1D linear radio interferometer



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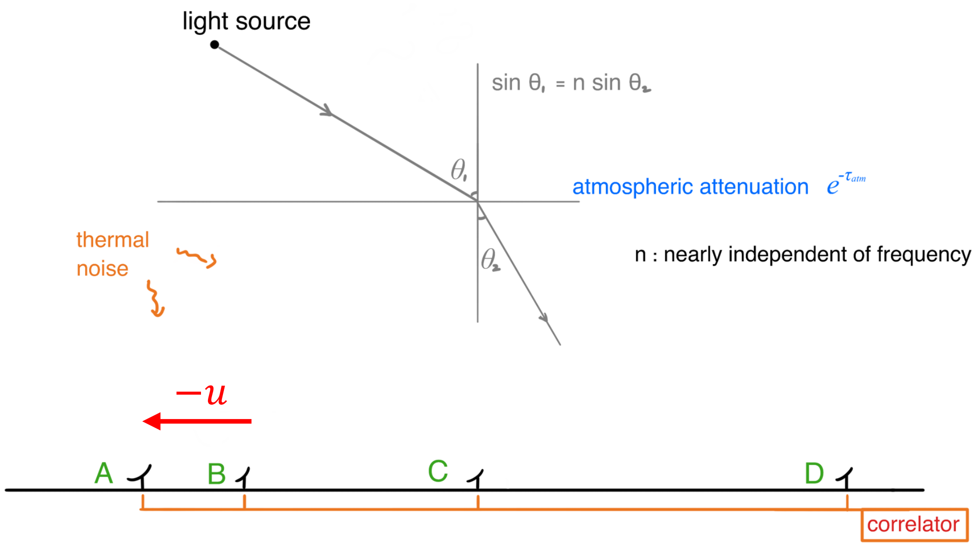

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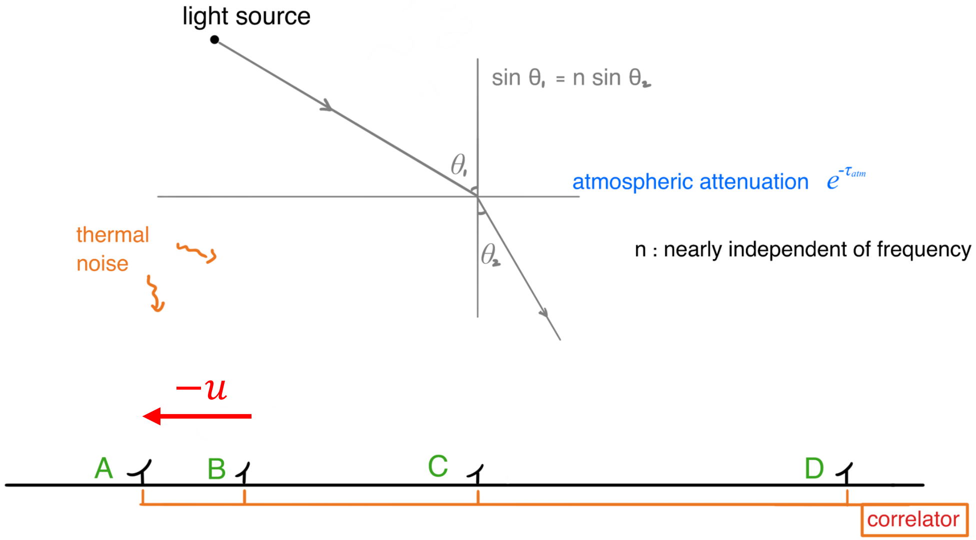

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$$\sum_{k=1}^{M} \int \delta(u-u_k)\cos(2\pi u\theta')\,du = \sum_{k=1}^{M} \cos(2\pi u_k\theta')$$

Resemblance to the diffraction pattern of the diffraction grating

Convolution theorem

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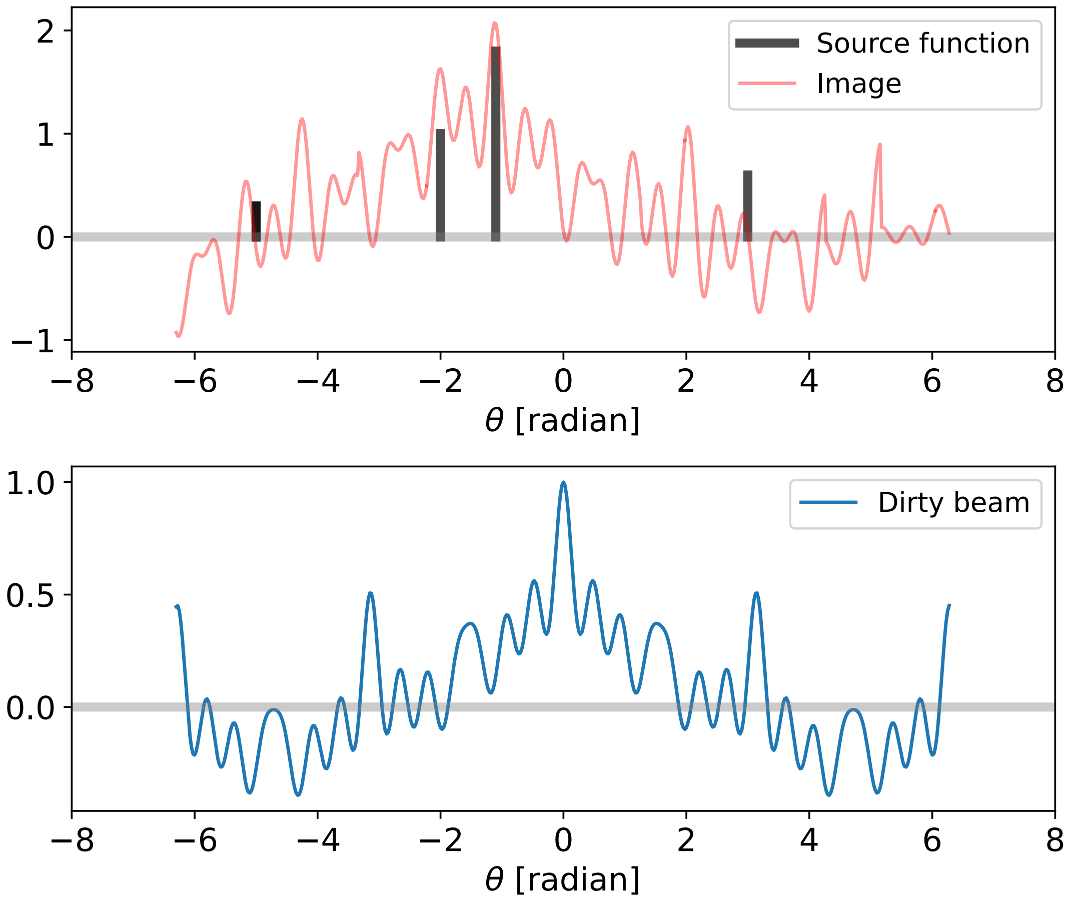

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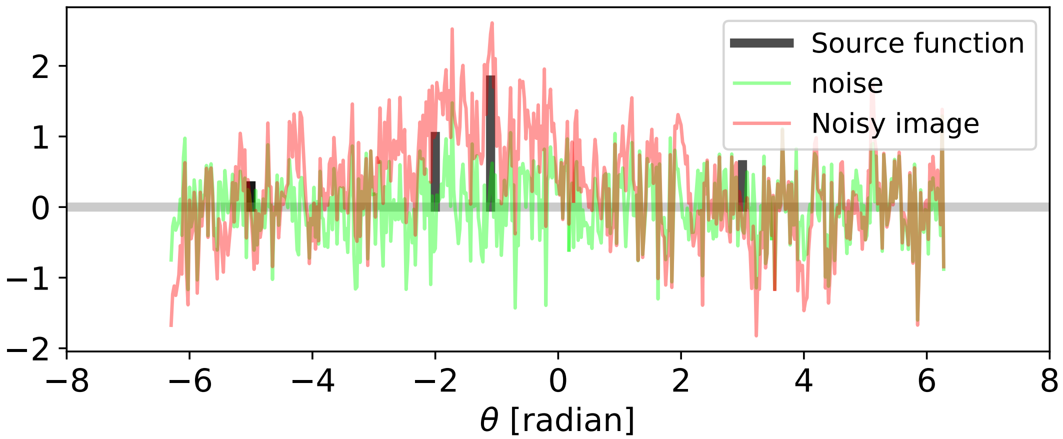

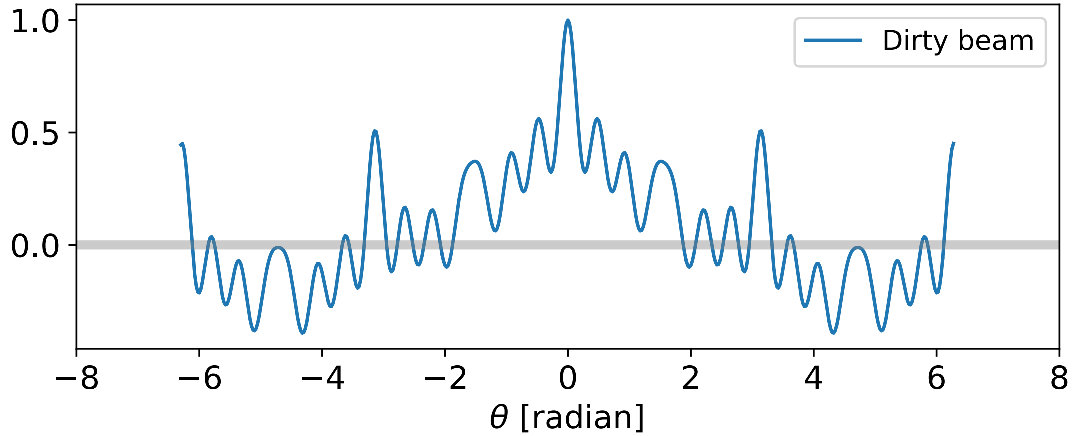

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Need to deconvolve $\left[A(\theta')\widetilde{P(\theta')}\right]^D$

The CLEAN algorithm

Ansatz: intensity distribution of target sources can be approximated by point sources and 2D Gaussians with various pre-defined sizes

Need to deconvolve $\left[A(\theta')\widetilde{P(\theta')}\right]^D$

1. Find the intensity peak in (residual) dirty image

To save time, sometimes specify to search over certain restricted areas, call **clean boxes**.

2. *Minor cycle*
Use certain conditions to find multiple clean components at a time

3. (i) Subtract $(I_m \times g) * B$ from the ungrided visibility.
(ii) Append the subtracted $(I_m \times g)$ to so called clean model

4. [Is the intensity peak in the residual image larger than a specified **cutoff**? 1~2 times the rms noise, normally.]
&&
[Not yet run more iteration than a specified limit **niter**?]

True

False

3. (i) Convolve the clean imodel *with the* clean beam, which is the a Gaussian profile obtained from fitting the central lobe of the dirty beam.
(ii) Add the residual image *to* the convolved image. This is the final product, the clean image.